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Shear strength of steel fiber-reinforced concrete beams

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ABSTRACT. This study analyzed the mechanical behavior of shear strength of steel fiber-reinforced concrete beams. Six beams subjected to shear loading were tested until failure. Additionally, prisms were tested to evaluate fiber contribution to the concrete shear strength. Steel fibers were straight, hook-ended, 35 mm long and aspect ratio equal to 65. Volumetric fractions used were 1.0 and 2.0%. The results demonstrated a great contribution from steel fibers to shear strength of reinforced concrete beams and to reduce crack width, which can reduce the amount of stirrups in reinforced concrete structures. Beam capacity was also evaluated by empirical equations, and it was found that these equations provided a high variability, while some of them have not properly predicted the ultimate shear strength of the steel fiber-reinforced concrete beams.

Keywords: empirical models, cracking, transverse reinforcement reduction.

Resistência ao cisalhamento de vigas de concreto reforçado com fibras de aço

RESUMO. Este trabalho analisa o comportamento mecânico de vigas de concreto armado reforçado com fibras de aço. Seis vigas de concreto armado submetidas à força cortante foram ensaiadas até a ruptura. Adicionalmente, foram ensaiados prismas com o objetivo de caracterizar a contribuição das fibras na resistência ao cisalhamento do concreto. As fibras de aço utilizadas eram retilíneas, com gancho nas extremidades, de 35 mm de comprimento e relação de aspecto igual a 65, e foram adicionadas nas frações volumétricas de 1.0 e 2.0%. Os resultados mostram que as fibras contribuem com o aumento da resistência ao cisalhamento das vigas de concreto armado, limitam a abertura das fissuras e podem substituir parcialmente os estribos em vigas de concreto armado Os resultados são comparados com valores obtidos de modelos empíricos, tendo sido observado grande variabilidade. Além disso, alguns desses modelos não preveem de forma adequada a resistência ao cisalhamento de vigas com concreto reforçado com fibras de aço.

Palavras-chave: modelos empíricos, fissuração, redução de armadura transversal.

Introduction

The inclusion of steel fibers in concrete, at an appropriate volume, can significantly increase its tensile strength and ductility. A random distribution and orientation of these fibers provides reinforcement of the small attenuating spacing in three dimensions, which is impossible to achieve by means of any conventional reinforcement, therefore resulting in a more uniform tensile capacity. Hence, the fibers intercept and transfer stress from the cracks formed in every direction, increasing tensile strength of a cracked matrix and shear strength of cracking surfaces. For these reasons, fibers can significantly contribute to concrete strength when subjected to shear loads.

There are several experimental studies that show positive contributions from fibers when beams are subjected to shear forces. Table 1 shows the main empirical equations used to evaluate shear strength of reinforced concrete beams with fibers without shear reinforcement.

Several experimental studies have been conducted in recent years demonstrating the efficiency of steel fibers as additional shear reinforcement in reinforced concrete beams.

The use of these fibers as shear reinforcement can decrease crack width, contributing to the control of concrete cracking. However, the results obtained are highly variable. This variability is due to the great diversity of mechanical properties of Steel Fiber-Reinforced Concrete - SFRC, specifically its tensile strength. Therefore, previous empirical models provide different results. Furthermore, most were developed for SFRC with up to 1% in fiber content (PARRA-MONTESINOS, 2006; CHOI et al., 2007; DINH et al., 2010; YAKOUB, 2011; SUSETYO et al., 2011; SLATER et al., 2012). Few tests were developed for reinforced concrete beams with 2% in steel fiber content (OH et al., 1999; CUCCHIARA et al., 2004).

Table 1. Shear strength models for FRC beams without shear reinforcement.

Investigator	Shear strength of empirical models, MPa
Narayanan; Darwish (1987)	$\tau_{cf}=e\left[0.24f_{ct,sp}+80\rho_{sl}\frac{d}{a}\right]+0.41\tau_{bf}F$ e = 1 if a/d > 2.8 or e = 2.8 d/a if a/d ≤ 2.8
Sharma (1986)	$\tau_{cf}=kf_{ct}\left(\frac{d}{a}\right)^{0.25}$ k = 1 if f_{ct} is obtained by direct tension test; k = 2/3 if f_{ct} is obtained by indirect tension test; k = 4/9 if f_{ct} is obtained using modulus of rupture.
Ashour et al. (1992)	$\tau_{cf}=\left(0.7\sqrt{f_c}+7F\right)\frac{d}{a}+17.2\rho_{sl}\frac{d}{a}$
Li et al. (1992)	$\tau_{cf}=1.25+4.68\left[\left(f_{ct,f}f_{ct,sp}\right)^{3/4}\left(\rho_{sl}\frac{d}{a}\right)^{1/3}d^{-1/3}\right]$ if *a/d* ≥ 2.5
Khuntia et al. (1999)	$\tau_{cf}=\left(0.167e+0.25F\right)\sqrt{f_c}$ e = 1 for *a/d* ≥ 2.5 or e = 2.5*d/a* ≤ 3 for *a/d* <2.5
Iman et al. (1995)	$\tau_{cf}=0.6\psi\sqrt[3]{\omega}\left[f_c^{\,0.44}+275\sqrt{\frac{\omega}{(a/d)^5}}\right]$ with, $\psi=\frac{\left[1+\sqrt{\frac{5.08}{d_a}}\right]}{\sqrt{1+\frac{d}{25d_a}}}$ and $\omega=\rho(1+4F)$
Swamy et al. (1993)	$\tau_{cf}=0.41\tau_{bf}\frac{L_f}{D_f}V_f$, with τ_{bf} = 4.04 MPa
Shin et al. (1994)	$\tau_{cf}=0.22f_{ct,sp}+217\rho_{sl}\frac{d}{a}+0.34\tau_{bf}F$, for $\frac{a}{d}<3$ $\tau_{cf}=0.19f_{ct,sp}+93\rho_{sl}\frac{d}{a}+0.34\tau_{bf}F$, for $\frac{a}{d}\geq 3$
Padmarajaiah; Ramaswamy (2001)	$\tau_{cf}=\zeta\left[e\left\{0.32\left(\frac{\sqrt{f_{cu}}}{3}+1.918\frac{L_f}{D_f}V_f\right)+75\rho_{sl}\frac{d}{a}\right\}+\bar{g}0.645\tau_{bf}F\right]$ with $\zeta=\frac{1}{\sqrt{1+d/(25d_a)}}$ *e* = 1 for *a/d* > 2,8 or *e* = 2.8 *d/a* for *a/d between* 1.0 and 2.8 or *e* = 1.5 for *a/d* ≤ 1; $\bar{g}$ = 1 for *a/d* > 2.8 or $\bar{g}$ = 1.3 for *a/d* ≤ 2,8.
Kwak et al. (2002)	$\tau_{cf}=3.7ef_{ct,sp}^{\;2/3}\left(\rho_{sl}\frac{d}{a}\right)^{1/3}+0.33\tau_{bf}F$ e = 1 for *a/d* > 3.4 or e = 3.4 *d/a* for *a/d* ≤ 3.4
Iman et al. (1994)	$\tau_{cf}=0.7\zeta\sqrt[3]{\rho}\left[f_c^{0.44}\left(1+F^{0.33}\right)+870\sqrt{\frac{\rho_{sl}}{(a/d)^5}}\right]$, with $\zeta=\frac{1}{\sqrt{1+d/(25d_a)}}$
Oh et al. (1999)	$\tau_{cf}=0.5\tau_{bf}V_f\frac{L_f}{D_f}$
Mansur et al. (1986)	$\tau_{cf}=\left(0.16\sqrt{f_c}+17.2\frac{\rho_{sl}Vd}{M_u}\right)+0.41\left(\tau_{bf}V_f\frac{L_f}{D_f}\right)$
Slater et al. (2012)	$\tau_{cf}=1+\frac{9}{100}f_c+\frac{3}{2}f_c\rho_{sl}-7.4\times10^{-4}f_c\frac{L_f}{D_f}-136\rho_{sl}\frac{a}{d}+\frac{11}{5}\rho_{sl}\frac{L_f}{D_f}+2F$ for $\frac{a}{d}<3$ and f_c > 50 MPa

Note: $F=(L_f/D_f)V_fd_f$ is the fiber factor; d_f is the fiber adherence factor, normally equal to 1 for fibers with hooks; *a/d* is a shear span-to-depth ratio; ρ is the longitudinal reinforcement rate; *d* is the beam height; d_a is the maximum dimension of the aggregate, in mm; V/M_u is the ratio between the shear load and the resistant bending moment in the considered section; τ_{bf} is the average interfacial bond stress of fiber-matrix, generally equal to 4.15 MPa; f_{cu} is the cubic compressive strength of the concrete.

The main aim of this study was to analyze the influence of great amount of steel fibers, that is, 1 and 2%, on shear strength of concrete beams with a/d < 3. Also, this study applies several previous empirical models and a project recommendation to determine which is capable to predict the shear strength of concrete beams with a high content of steel fibers.

Material and methods

Direct shear tests

Eight direct shear tests were conducted. The prismatic specimens used in these tests were 150 mm wide x 150 mm high x 600 mm long. These specimens were based on previous studies reported in literature (MIRSAYAH; BANTHIA, 2002). The purpose of this test was to determine the shear stress-slip relationship of steel fiber-reinforced concrete and thus verify the influence of fibers on shear strength and ductility of concrete. The specimens were then sawed at 15 mm along the transverse section perimeter in the region where a shear crack was expected, and the vertical displacement of the central region between the grooves was measured with a linear position transducer. Hence, the dimension of the shear strength section was 120 x 120 mm, as shown in Figure 1. Two identical specimens were made for tests with fiber-reinforced concrete, and four specimens were made for tests without fibers.

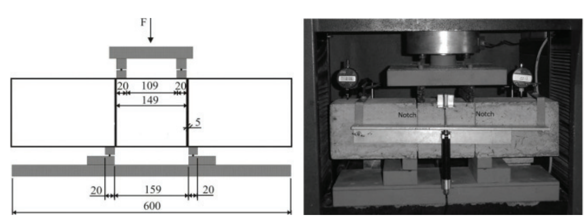


Figure 1. Diagram of a direct shear test on prismatic specimens (all dimensions are in mm).

The mix proportions per cubic meter of concrete in the specimens are given in Table 2. Dramix® RC 65/35 BN steel fibers with a tensile strength of 1150 MPa were added to the matrix. The volumetric fractions of steel fibers used were 1.0 and 2.0%, equal to 79 and 157 kg m^{-3}, respectively. The steel fibers had hooked-ends, were 35 mm long and aspect ratio of 65. A 2.5% content in concrete volume of Wollastonite mineral was also added to the matrix. These mineral additions in the form of fibers act as reinforcement during the first steps of matrix cracking.

Table 2. Mix proportions of concrete (quantities per m³).

Mixture⁽¹⁾	Cement (kg)	Silica Fume (kg)	Natural Sand (kg)	Coarse Aggregate (kg)⁽²⁾	Wollastonite mineral addition (kg)	Water	High-range water-reducing admixture (kg)⁽³⁾
1	440	35	817	817	72.5 (2.5%)	198	5.86 (1.2%)
2	360	29.5	836	836	72.5 (2.5%)	200	4.80 (1.2%)

(1)Mixture 1 was used in the direct shear tests; mixture 2 was used in the beams tested for shear; (2)The maximum size of the coarse aggregate was 12.5 mm; (3)The percentage in mass of the concrete plus the silica fume.

Beams subjected to shear loading

Six reinforced concrete beams, with and without steel fibers, were subjected to shear loading and tested until failure. In all of the beams, a longitudinal 3.05% reinforcement ratio (ρ_{sl}) was used. The variables analyzed were the volume of fibers added to the matrix (1.0 and 2.0%) and the presence of shear reinforcement, taken at a 0.21% ratio (ρ_{sw}). The main characteristics of the tested beams are shown in Table 3.

Table 3. Properties of beams tested.

Beam	Fiber volume fraction, V_f (%)	Transverse reinforcement ratio, ρ_{sw} (%)
V-0-0.21	0.0	0.21
V-1-0.21	1.0	0.21
V-2-0.21	2.0	0.21
V-0-0	0.0	0.0
V-1-0	1.0	0.0
V-2-0	2.0	0.0

The mix proportions of the concrete used in beam construction are listed in Table 2. The longitudinal and vertical reinforcement used in beam construction showed a yield strength of 570 MPa and 661 MPa and an elasticity modulus of 218 GPa and 200 GPa, respectively, for 20.0 and 6.3 mm bars.

To perform the tests, the beams were simply supported and subjected to four loading points. The distance between the loading points was kept constant and equal to 500 mm, resulting in a shear span-to-depth ratio (*a/d*) of 2.5. The load on beams was applied by means of a hydraulic activator with power control in pre-set load steps of 5 kN for beams without fibers and 10 kN for beams with fibers. Beams were instrumented to measure crack width in the shear span on each load step. For this purpose, three linear vertical displacement transducers (LVDT) were used on each shear span, positioned at 45° to the beam longitudinal axis and oriented in the direction of the principal tensile stress, as shown in Figure 2.

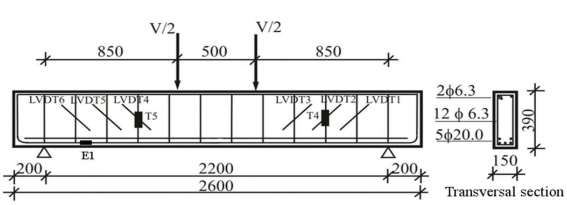


Figure 2. Geometric characteristics and loading diagram of tested beams (all dimensions are in mm).

Results and discussion

Mechanical properties of concrete

Mechanical properties of the concrete used for beam construction are presented in Table 4. It was noted that 1.0 and 2.0% of steel fibers provided a significant increase in the splitting tensile strength of concrete, on average 70 and 99% greater than in concrete without fibers, respectively. This tensile strength represents the maximum strength of steel fiber-reinforced concrete and not only the strength to matrix cracking.

Table 4. Average values of concrete mechanical properties.

Beam	f_c⁽¹⁾ (MPa)	$f_{ct,sp}$⁽¹⁾ (MPa)	$f_{ct,f}$⁽²⁾ (MPa)	E_c (MPa)	ν	TR⁽³⁾	FT⁽⁴⁾ (MPa)
V-0-0	46.30	4.35	-	27730	0.23	0.231	-
V-0-0.21	47.23	3.73	-	27200	0.20	0.204	-
V-1-0	58.87	6.78	8.26	31780	0.21	0.456	6.16
V-1-0.21	52.89	6.98	7.72	29170	0.18	0.400	5.81
V-2-0	51.67	7.17	12.51	39100	0.22	0.592	10.75
V-2-0.21	57.89	8.89	12.25	33330	0.16	0.537	10.01

(1)Obtained from the test on cylindrical specimens of 150 x 300 mm; (2)Obtained from the test on prismatic specimens of 100 x 100 x 400 mm; (3)Obtained from the test of compression with displacement control; (4)Flexural toughness obtained from the four-point bending test on prismatic specimens of 100 x 100 mm x 400 mm and calculated according to the procedure presented by JSCE (1984).

The contribution from steel fibers to increase ductility of concrete may be noted through the results of a compression test on a cylindrical specimen with displacement control (Figure 3a). For this purpose, the toughness ratio (*TR*) concept was used, defined as the ratio between energy consumed by the specimen during testing and energy considering a rigid plastic behavior. In this case, the axial deformation was limited to 1.5%. Table 4 shows an average increase of roughly 97 and 160% in the toughness ratio due to an addition of 1.0 and 2.0% in steel fibers, respectively, which indicates the positive influence of steel fibers to increase ductility of concrete under compression.

In turn, flexural strength ($f_{ct,f}$) was increased on average 55% when the volume of steel fibers increased from 1.0 to 2.0% (Figure 3b). In the same way, flexural toughness (*FT*) was increased on average 78% when the volume of steel fibers increased from 1.0 to 2.0%. Such results indicated the positive contribution from the amount of steel fibers in increasing both tensile strength and toughness of fiber-reinforced concrete.

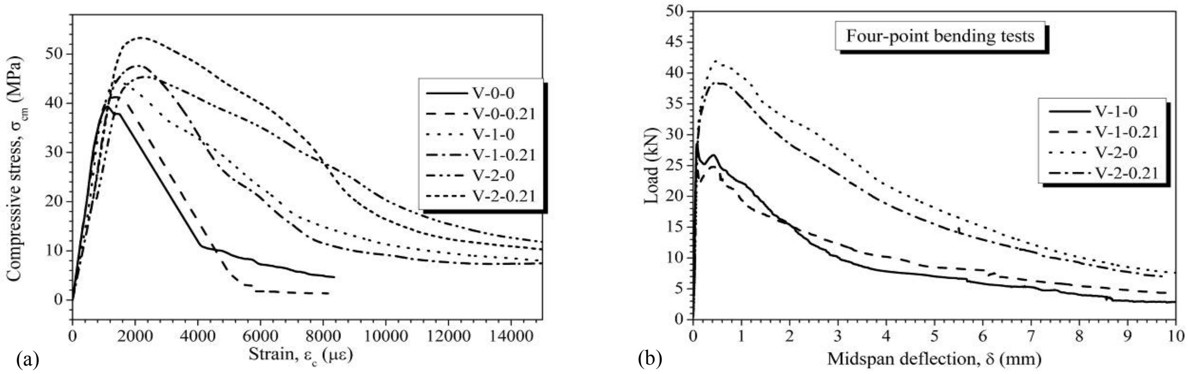


Figure 3. Average (a) compression test curve with displacement control and (b) four-point bending test on prismatic specimens.

Direct shear tests

Table 5 shows the maximum load (F_{max}) and maximum shear strength (τ_{cf}) for the specimens. Those values were divided by two, i.e., by the number of shear planes. On average, the addition of 1.0 and 2.0% in fiber content increased shear strength by approximately 110 and 133%, respectively, compared to concrete without fibers.

Table 5. Maximum load and shear strength on a shear plane.

Specimen number	V_f (%)	f_c (MPa)	F_{max} (kN)	τ_{cf} (MPa)	k
CP1	0.0	51.77	119.93	8.13	1.13
CP2			85.53	5.79	
CP3			122.96	8.33	
CP4			152.06	10.39	
CP1	1.0	65.27	268.92	18.22	2.12
CP2			232.58	16.02	
CP1	2.0	71.30	278.76	18.73	2.25
CP2			281.92	19.26	

This table also shows the correlation constant (k) between average shear strength and concrete average compressive strength as shown in the equation (1).

$$\tau_{cf} = k\sqrt{f_c} \tag{1}$$

An increase was found in the constant k value with the increase of fiber content and concrete compressive strength.

The influence from the fibers can also be noted from the load-versus-slip curves for the specimens showed in Figure 4. In the specimens without fibers, a softening curve was not seen after reaching shear strength. Shear strength increased expressively with an increase in fiber content. Moreover, when the peak load was reached, a notable softening response by the specimen was seen due to the fibers. There was also an abrupt decrease of around 60% in shear strength after the peak load in the mixture with 1.0% fiber content, whereas this abrupt decrease was around 50% in the mixture with 2.0% fiber content.

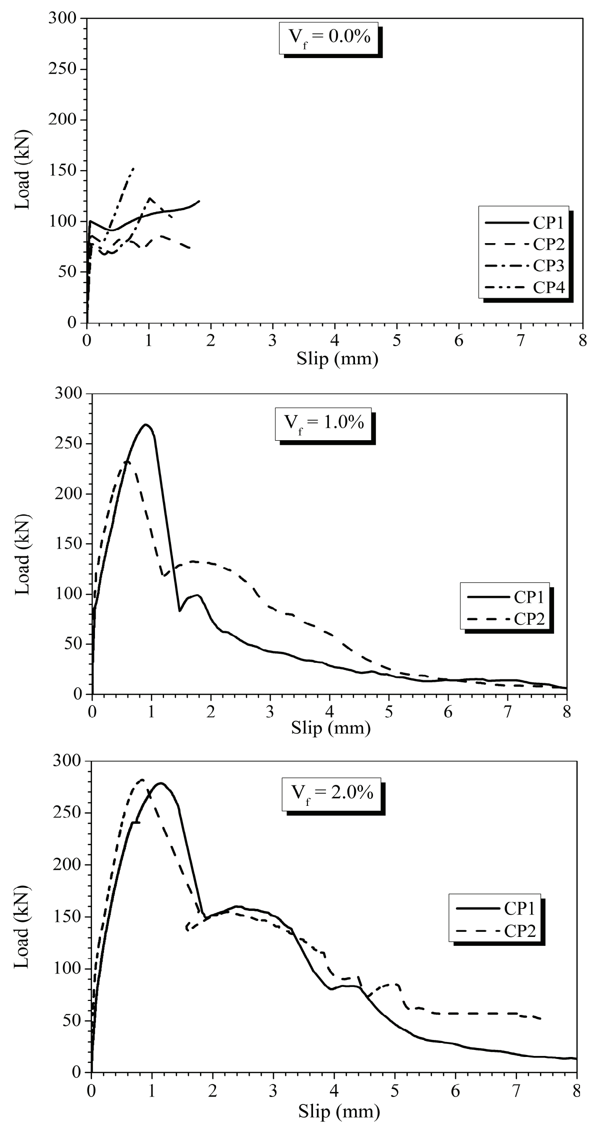


Figure 4. Load-slip curves of direct shear tests on prismatic specimens.

From the curves illustrated in Figure 4, the energy dissipated for a specific slip value can be determined for each test. Computing the energy for different slip values in the shear plan allowed the design of the curves shown in Figure 5.

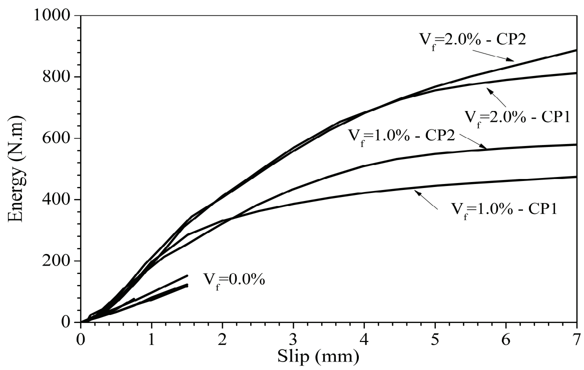


Figure 5. Energy vs. slip curve of direct shear tests on prismatic specimens.

These curves represent the energy dissipated by the shear plan for each specific slip. The toughness of the material is greater as much as more energy is dissipated. This figure shows that at a fixed slip value of 1.5 mm, the addition of 1.0 and 2.0% amount of fiber increased the energy in 106 and 151%, respectively, when compared to the mixture without fibers. After this slip value, only the specimens with fibers could bear load. For the slip of 7 mm, the mixture with 2.0% fiber amount dissipated 61% more energy than the mixture with 1.0% fiber amount.

Beams subjected to shear loading

The ultimate shear strength of beams tested for shear is shown in Table 6.

Table 6. Shear strength of beams tested.

Beam	V_u[1] (kN)	Failure[2]	Maximum crack width [3](mm)	Shear load at first shear crack (kN)
V-0-0	172.5	S	2.37 (1)	75
V-0-0.21	228.5	S	1.62 (3)	95
V-1-0	260.0	S	1.55 (2)	120
V-1-0.21	275.5	S	1.37 (1)	110
V-2-0	290.5	S	1.85 (4)	125
V-2-0.21	360.0	B	0.90	190

[1]Shear strength ($V_u = V/2$), where V was the maximum load measured by the load cell; [2]Type of failure: S = shear; B = bending; [3]Greater crack width measured by the transducer a step before the failure. The number in parenthesis shows the quantity of cracks formed between the attachment points of the transducer.

Figure 6 illustrates the load-deflection response of beams tested for shear. Failure of the plain concrete beam without stirrups (V-0-0) was characteristic of a shear failure by compression strut; i.e., a single diagonal crack was formed in the web that spread to the upper beam. After cracking, the beam could still support load increase due to the longitudinal reinforcement dowel action. The beam failure occurred through loss of the longitudinal reinforcement bond due to spreading of the diagonal crack to the support.

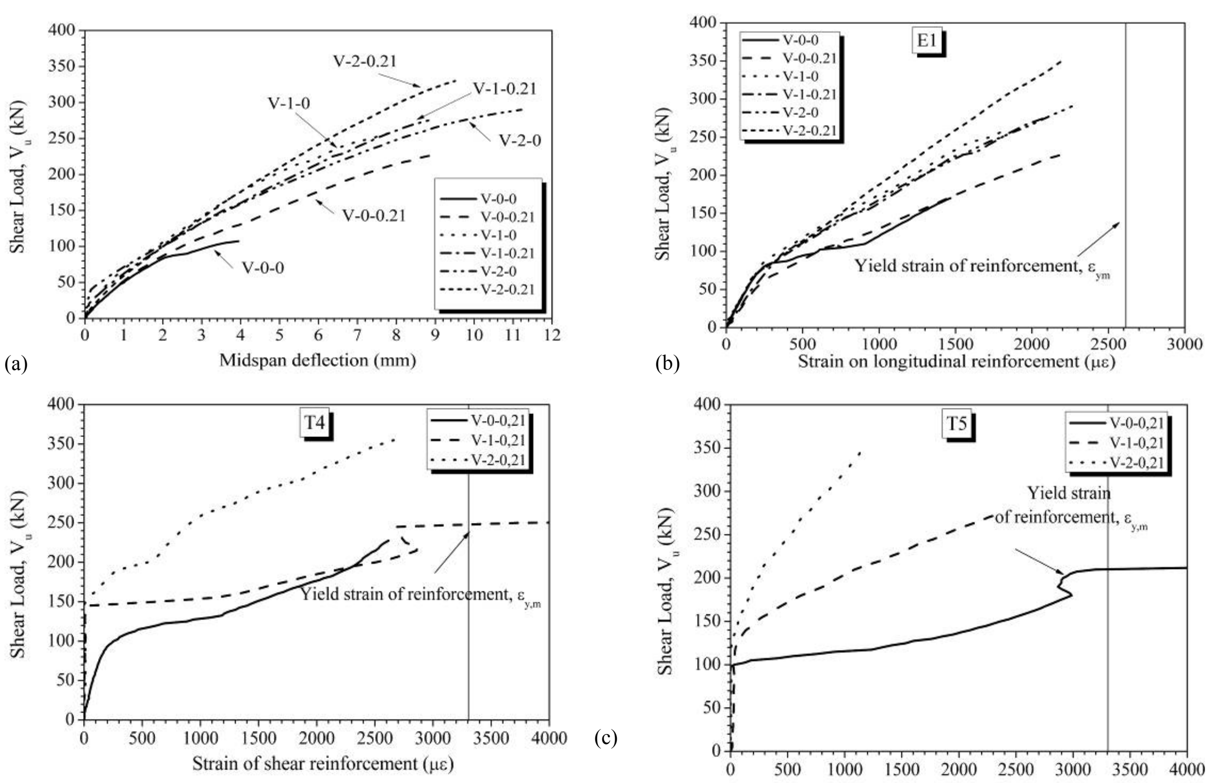


Figure 6. Load-deflection response from beams tested in shear: (a) Vertical deflection in the middle of the span, (b) Strain of longitudinal reinforcement intercepted by a critical shear crack, (c) Strain of shear reinforcement intercepted by a critical shear crack.

Failure of the plain concrete beam with stirrups, but without fibers (V-0-0.21) also occurred by compression strut, though with the concrete crushed above the neutral axis. Compared to the plain concrete beam without stirrups, inclusion of stirrups gave way to more cracking and increased shear strength of around 32%.

Failure of SFRC beams with 1.0 and 2.0% in fiber content and without stirrups occurred through shearing. Numerous diagonal cracks were found in the shear span, evidencing the positive influence of fibers when multiple cracks were caused after forming the first diagonal shear crack. Failure of these beams occurred when the main diagonal shear crack extended from the upper beam to the support. The SFRC beams with 1.0 and 2.0% in fiber content and without stirrups exhibited ultimate shear strength 51 and 68% greater than that of the plain concrete beam without stirrups. The SFRC beam with 1.0% in fiber content and stirrups also failed through shearing with strength 32% greater than that of the plain concrete beam with stirrups. The use of 2.0% in fiber content combined with stirrups in the reinforced concrete beam (V-2-0.21) caused a change in the failure mode, from shearing to bending (Figure 7). Such behavior was due to greater shear strength of SFRC with 2% in fiber content.

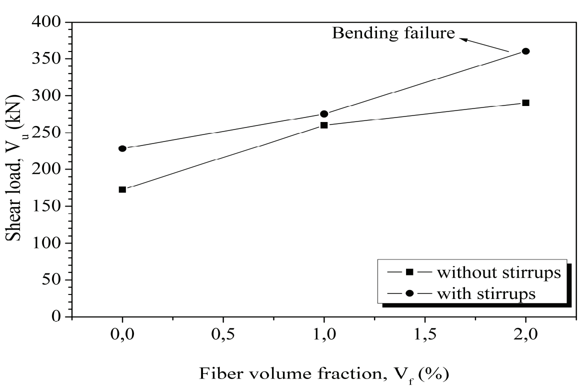
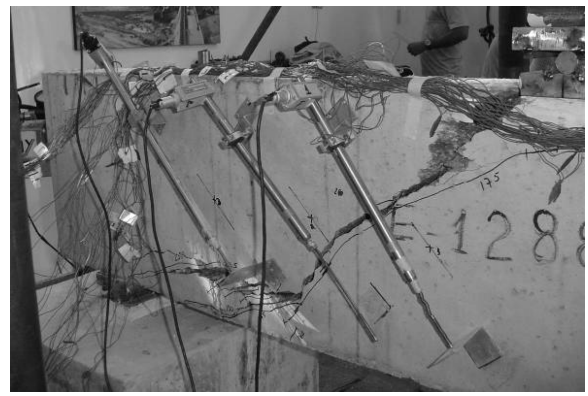


Figure 7. Influence of fibers and stirrups on ultimate shear load of beams.

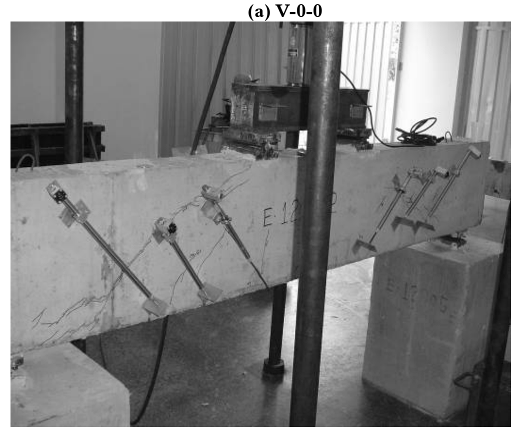
Table 6 shows that the average width of the diagonal crack in SFRC beams with 2.0% in fiber content and without stirrups at failure was reduced due to the addition of fibers (Figure 8). With 1.0% in fiber content, the width of diagonal cracks in SFRC beams without stirrups at failure was compatible with that noted for a plain concrete beam with stirrups. This demonstrates the efficiency of fibers as discontinued reinforcement, intercepting the crack in the concrete matrix, even in the absence of stirrups.

Table 6 also lists the influence of steel fibers in the increase of the first crack by shearing.

With the addition of 1.0% in fiber content, there was a 16% increase in the value of that strength, while when adding 2.0% in fiber content, this strength was increased by 100%. Such values refer to plain concrete beam with stirrups.



(a) V-0-0



(b) V-2-0

Figure 8. Detail of diagonal cracking of beams in the shear span and of transducers used to measure crack width.

Considering the plain concrete beam with stirrups (V-0-0.21), the estimated service shear load for this beam was 171 kN. Table 7 shows the crack widths recorded by six transducers for that shear load. The crack width in the beams with fibers was considerably lower than in the plain concrete beam. The largest width recorded in the plain concrete beam with stirrups (V-0-0.21) was 3.8 times greater than recorded in the SFRC beam with 1.0% in fiber content without stirrups (V-1-0). According to the above, fibers in this case are more efficient than stirrups in controlling cracking of concrete subjected to shear loading.

Table 7. Crack widths in beams considering a service shear load of 171 kN.

Beam	LVDT1	LVDT2	LVDT3	LVDT4	LVDT5	LVDT6
V-0-0	1.98	1.54	1.29	2.37	1.80	-
V-0-0.21	0.37	0.87	0.79	0.42	0.71	0.49
V-1-0	0.13	0.00	0.06	0.23	0.02	0.19
V-1-0.21	0.18	0.14	0.00	0.00	0.05	0.05
V-2-0	0.00	0.00	0.00	0.29	0.26	0.17
V-2-0.21	0.19	0.07	0.09	0.28	0.02	0.04

Comparison with empirical equations

Shear strength of beams subjected to shear loading was compared with project recommendations for SFRC structures proposed by RILEM TC TDF-162 (2003). This recommendation was chosen because it does not require previous knowledge of the tensile stress-strain relationship of steel fiber reinforced concrete. In this recommendation, shear strength provided by concrete and fibers (V_c and V_{cf}) was calculated separately. The contribution from shear reinforcement (V_{sw}) is calculated from the Simplified Truss Analogy. In this comparison, the angle of the main diagonal crack measured in the test was used. Final shear strength is the sum of the three parts, which are concrete, fibers and stirrups, as shown in equation (2).

$$V_{u,calc} = V_c + V_{cf} + V_{sw} \tag{2}$$

Shear strength provided by fibers is obtained from the recommendation of RILEM TC TDF-162 (2003) by means of equation (3).

$$V_{cf} = 0.7\, k_f\, k\, \tau_{cf}\, b\, d \text{, in N} \tag{3}$$

where k_f is the factor that considers the flange contribution in section T, and k is the coefficient that depends on beam height. Shear strength provided by steel fibers (τ_{cf}) is written as a function of residual tensile stress obtained from the three-point bending loading test with a notch in the middle of the span for displacement of 3.0 mm ($f_{R,4}$), which is calculated as:

$$\tau_{cf} = 0.12\, f_{R,4} \text{, in MPa} \tag{4}$$

In this study, residual tensile stress ($f_{R,4}$) was calculated with the load versus displacement curve obtained from the four-point bending tests of unnotched specimen, as shown in Figure 3b. This approximation was valid because, although there is no notch in the specimens, the failure occurred through the formation of a single crack, that is, no multiple cracks were found in the SFRC.

Shear strength and comparison with the beams subjected to shear loading are shown in Table 8. For the plain concrete beam without stirrups, shear strength evaluated was about 43% of shear strength obtained during testing. This can be explained by means of the strong influence of the dowel action due to high ratio of longitudinal reinforcement. After the diagonal shear crack for a 75 kN load, the beam supported an increased load and failed with a 172.4 kN load. Comparing the shear load that caused the first diagonal crack in that beam with shear strength evaluated from EUROCODE 2 (73.84 kN), they are practically the same (CEN, 2004).

For steel fiber-reinforced concrete beams, shear strength was lower than found in the beam test. For beams without stirrups, shear strength evaluated was not higher than 54% of shear strength obtained during testing. For beam with stirrups, this difference was smaller (around 79%) due to the shear strength provided by stirrups. It is important to mention that this smaller difference was observed in beam with 1% of steel fibers, once the beam with 2% of steel fibers and stirrups failed by bending. Therefore, it is possible to conclude that this project recommendation is conservative to predict the shear strength of concrete with higher amount of steel fibers. This is because of the tensile hardening observed in steel fiber-reinforced concrete used on these beams.

Shear strength of beams tested is also compared to shear strength evaluated through other empirical equations in Table 1. The results are described in Table 9. The following assumptions were adopted:

- The contribution by shear reinforcement strength was calculated according to the Simplified Truss Analogy, considering the compression strut angle equal to the critical diagonal crack angle measured at the end of the test (Table 8). This assumption was needed for the empirical equations and could be applied to the beams with stirrups, as these equations only provide the shear strength of concrete beams without stirrups.
- To obtain shear strength of steel fiber-reinforced concrete, a fiber adherence factor (d_f) was adopted 1.0 (IMAN et al., 1995) and for the average interfacial fiber-matrix bond stress (τ_{bf}) was adopted 4.15 MPa, according to Swamy et al. (1974).

Table 9 shows that the shear strength obtained through empirical equations suggested by Narayanan and Darwish (1987) and Ashour et al. (1992) were the closest among the beams tested and showed a $V_{u,exp}\, V_{u,calc}^{-1}$ ratio greater than 0.90. Then, these equations may be used to obtain the fiber-reinforced shear strength of SFRC beams with and without stirrups subjected to shear loading similar to the beams tested herein. The results of the 2.0% SFRC beam were not used in this analysis because of the failure owing to bending and not to shear.

Table 8. Comparison between experimental shear strength and values obtained from project recommendations of RILEM TC TDF-162 (2003).

Beam	$V_{u,exp}$ (kN)	Critical diagonal crack (θ)	V_c [(1)] (kN)	V_{sw} [(2)] (kN)	$f_{R,4}$ (MPa)	V_{cf} (kN)	$V_{u,calc}$ [(3)] (kN)	$V_{u,exp} / V_{u,calc}$
V-0-0	172.5	42°	73.84	-	-	-	73.84	2.34
V-0-0.21	228.5	30°	74.33	109.47	-	-	183.80	1.24
V-1-0	260.0	27°	79.99	-	4.36	33.24	113.29	2.29
V-1-0.21	275.5	33°	77.19	97.32	5.48	41.78	216.29	1.27
V-2-0	290.5	25°	76.59	-	12.23	93.13	169.73	1.71
V-2-0.21	360.0	40°	79.55	75.32	9.10	69.33	224.20	>1.61

[(1)]Obtained from recommendation of EUROCODE 2 (CEN, 2004): $V_c = \left[0.18k(100\rho f_c)^{1/3} + 0.15\sigma_{cp}\right]bd$ and $k = 1 + \sqrt{\frac{200}{d}} \leq 2$; d is the beam height; [(2)] $V_{sw} = \frac{A_{sw}}{s} 0.9 d f_y \cot\theta$; [(3)]$V_{u,calc} = V_c + V_{cf} + V_{sw}$.

Table 9. Comparison between experimental shear strength and values obtained from other empirical equations.

Investigator	$V_{u,exp}\,V_{u,calc}^{-1}$					Mean	CV(%)
	V-0-0	V-0-0.21	V-1-0	V-1-0.21	V-2-0		
Narayanan; Darwish (1987)	1.46	1.05	1.25	0.90	1.08	1.15	18.97
Sharma (1986)	1.45	1.08	1.40	0.96	1.48	1.28	18.81
Ashour et al. (1992)	1.57	1.04	1.21	0.90	0.97	1.14	23.67
Li et al. (1992)	0.93	0.77	1.07	0.84	0.91	0.90	12.54
Khuntia et al. (1999)	2.93	1.35	2.01	1.25	1.62	1.83	37.09
Iman et al. (1995)	1.53	1.02	1.02	0.79	0.81	1.04	28.55
Swamy et al. (1993)	2.95	1.36	2.24	1.31	1.79	1.93	35.42
Shin et al. (1994)	0.92	0.79	1.00	0.77	0.93	0.88	11.35
Padmarajaiah and Ramaswamy (2001)	2.48	1.27	1.55	1.05	1.12	1.49	39.15
Kwak et al. (2002)	1.08	0.90	1.00	0.76	0.93	0.93	12.99
Iman et al. (1994)	1.05	0.83	1.25	0.91	1.36	1.08	20.41
Oh et al. (1999)	1.85	1.12	1.80	1.15	1.48	1.48	23.33
Mansur et al. (1986)	3.08	1.38	2.19	1.29	1.70	1.93	38.05
Slater et al. (2012)	-	-	3.44	1.89	2.69	2.67	29.00

CV is the coefficient of variation defined by dividing the standard deviation by the mean.

Conclusion

The direct shear tests indicated that steel fibers are effective in increasing shear strength of concrete. Adding 1% in fiber content increased the ultimate shear strength of concrete by 87%, while adding 2% in fiber content increased it by 99%.

The fibers caused a decrease in shear crack width and in a beam with 2% of steel fibers and low stirrup ratio they were able to modify the failure form, from shear to bending.

The project recommendation proposed by RILEM TC TDF-162 (2003) is conservative to evaluate shear strength of SFRC beams with higher amount of fibers when hardening on tensile stress-strain curve was observed.

It was observed significant differences between shear strength evaluated by empirical equations. Equations proposed by Narayanan and Darwish (1987) and Ashour et al. (1992) are more adequate to evaluate shear strength of SFRC beams tested.

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