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Fernandes Marçal, Emerson; Hadad Junio, Eli
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Is It Possible to Beat the Random Walk Model in Exchange Rate Forecasting? More Evidence for Brazilian Case

(É Possível Bater o Passeio Aleatório na Previsão da Taxa de Câmbio? Mais Evidência para o Caso Brasileiro)

Emerson Fernandes Marçal*
Eli Hadad Junior**

Abstract The seminal study of Meese and Rogoff on exchange rate forecastability had a great impact on the international finance literature. The authors showed that exchange rate forecasts based on structural models are worse than a naive random walk. This result is known as the Meese–Rogoff (MR) puzzle. Although the validity of this result has been checked for many currencies, studies for the Brazilian currency are not common. In 1999, Brazil adopted the dirty floating exchange rate regime. Our goal is to run a "pseudo real-time experiment" to investigate whether forecasts based on econometric models that use the fundamentals suggested by the exchange rate monetary theory of the 80s can beat the random model for the case of the Brazilian currency. Our work has three main differences with respect to Rossi (2013). We use a bias correction technique and forecast combination in an attempt to improve the forecast accuracy of our projections. We also combine the random walk projections with the projections of the structural models to investigate if it is possible to further improve the accuracy of the random walk forecasts. However, our results are quite in line with her results. We show that it is not difficult to beat the forecasts generated by the random walk with drift using Brazilian data, but that it is quite difficult to beat the random walk without drift. Our results suggest that it is advisable to use the random walk without drift, not only the random walk with drift, as a benchmark in exercises that claim the MR result is not valid.
***Keywords*:** Meese-Rogoff puzzle, forecasting, exchange rate.
JEL Codes: F31, F32, F41, C51.

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*Head of Center for Applied Macroeconomic Research at Sao Paulo School of Economics and CSSA-Mackenzie. E-mail: emerson.marcal@fgv.br

**Mackenzie Presbyterian University E-mail: hadad.junior@gmail.com

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Resumo

O trabalho seminal de Meese e Rogoff sobre previsibilidade da taxa de câmbio teve grande impacto na literatura de finanças internacionais. Os autores mostraram que previsões baseadas em modelos econômicos estruturais tinham um desempenho pior que um passeio aleatório ingênuo. Este resultado é conhecido na literatura como o quebra-cabeça de Meese-Rogoff. Ainda que a validade deste resultado tenha sido checado para um número grande de moedas, estudos para a moeda brasileira ainda não são tão comuns pois o Brasil adotou o regime de câmbio flexível apenas a partir de 1999. Rossi (2013) realizou um estudo amplo do quebra-cabeça proposto pelos autores mas não fez a análise dos dados brasileiros. O objetivo deste trabalho é simular um exercício de tempo real para investigar se as previsões basedas em modelos econômicos de determinação de taxa de câmbio que usam os fundamentos dos modelos desenvolvidos nos anos oitenta tem desempenho melhor que o modelo de passeio aleatório. O trabalho tem três diferenças principais em relação ao feito por ela. Utiliza-se a técnica de correção de viés e de combinação de previsões na tentativa de melhorar a precisão das previsões. Também combina-se as previsões do passeio aleatório com as dos modelos estruturais. Entretanto os resultados obtidos continuam em linha com da autora. O presente trabalho mostra que não é difícil gerar previsões com melhor desempenho que um passeio aleatório com tendência (drift) mas é extremamente difícil bater o desempenho do passeio aleatório ingênuo (sem tendência). O trabalho sugere que é fortemente recomendado utilizar o passeio aleatório sem tendência em exercícios que visem avaliar o quebra-cabeça de Meese e Rogoff.
***Palavras-chave*:** Meese-Rogoff puzzle, previsão, taxa de câmbio.

1. Introduction

The seminal study of Meese & Rogoff (1983) on exchange rate forecastability had a great impact on international finance literature. The authors compared exchange rate projections obtained from structural models against a naive random walk. They used structural monetary models of the 80s.[1] Their main result showed that it is not easy to outperform forecasts of a naive random walk model. Subsequently, an extensive literature emerged, but the result of Meese & Rogoff (1983) still holds. This is the so-called Meese–Rogoff (MR) puzzle.

In a recent paper, Rossi (2013) reviewed the literature that followed the work of Meese and Rogoff, aiming to confirm and explain their result. Rossi (2013) showed that it is still difficult to beat the random walk, particularly in an out-of-sample exercise. She ran a comprehensive exercise with

[1]See, for example, Frenkel (1976), Dornbusch (1976), Frankel (1979), and Hooper & Morton (1982).

different sets of fundamentals, econometric model specifications, samples, and countries. She showed that the MR puzzle still holds, particularly in an out-of-sample exercise. However, she did not include Brazil in her research.

The purpose of our paper is to run an exercise similar to Rossi (2013) using Brazilian data. We focus our analysis on multivariate econometric models with monetary fundamentals. In addition, we opt to run a forecast exercise using bias correction and forecasting combination techniques. We combine the forecasts of the models among themselves and with the random walk. We perform a pseudo real-time exercise to replicate, as closely as possible, the forecast that one could have carried out at a particular time in the past. We use the Model Confidence Set (MCS) algorithm developed by Hansen *et al.* (2011) to evaluate the predictive equivalence of the forecasts.

Our results suggest that the MR puzzle holds for Brazilian data. It is hard to beat the random walk without drift for almost all analysed horizons from one up to six quarters. Moreover, it is much easier to beat the random walk with drift than without drift.

The paper is divided into six sections. The first section is this introduction. The second section discusses the strategy for constructing forecasts using the fundamentals suggested by the model of the 80s. In the third section, the MCS algorithm is described. In the fourth section, we briefly discuss the results of Rossi (2013) and some key references regarding the MR puzzle. The fifth section presents the results of our empirical exercise and compares them with the literature. Finally, some concluding remarks are drawn.

2. Constructing a strategy to forecast exchange rate

In this section, we briefly describe the equation used to construct forecasts based on the monetary exchange rate models of the 80s as well as on econometric models.

2.1 The random walk model

In this study, the goal is to compare forecasts obtained from the random walk models with and without drift against a wide array of econometric models. The random walk model with drift is given by

$$y_t = y_{t-1} + a + \varepsilon_t \quad (1)$$

where ε_t is a random variable with zero mean that is independent over time. The model without drift can be obtained by assuming that a=0.

The k steps ahead forecast is given by

$$E_t[y_{t+k}] \quad = \quad y_t + a * k \tag{2}$$

2.2 The structural models of the 80s

In addition to the aforementioned random walk models, this study uses vector autoregressive models with and without an error correction mechanism in order to construct forecasts.[2] The choice of which explanatory variables to include in the models is made based on the economic models of the 80s and 90s that served as a basis for the article of Meese and Rogoff. Some key references are Frenkel (1976), Bilson (1978), Dornbusch (1976), and Frankel (1979).

These models link the exchange rate to a set of fundamentals. The model of the 80s implies an equation similar to (3), with different restrictions imposed on the coefficients according to variants of the basic model:

$$e_t=\beta_0+\beta_1(y_t-y_t^*)+\beta_2(i_t-i_t^*)+\beta_3(m_t-m_t^*)+\beta_4(\pi_t-\pi_t^*)+\beta_5(p_t-p_t^*)+v_t \tag{3}$$

where e_t denotes an exchange rate between countries i and j, $y_t - y_t^*$ the difference in the real income, $m_t - m_t^*$ the difference in monetary aggregate, and $\pi_t - \pi_t^*$ the difference in inflation rates. v_t is a random variable with zero mean.

2.3 Single-equation models

The first step in constructing a forecast based on (3) is to estimate the parameters using some econometric technique. Ordinary least square is one common choice in the literature, but others techniques can be used as well.

We calculate the expectations based on the information available at time t-1.

$$E_{t-1}(e_t)=\beta_0+E_{t-1}[\beta_1(y_t-y_t^*)+\beta_2(i_t-i_t^*)+\beta_3(m_t-m_t^*)+\beta_4(\pi_t-\pi_t^*)+\beta_5(p_t-p_t^*)] \tag{4}$$

Assuming that it is not possible to predict any change in the fundamental using the information available until t-1, the forecast for the exchange rate in t based on information t-1 is given by (5):

[2] One can see Enders (2008) for a textbook explanation.

$$E_{t-1}(e_t) = \beta_0 + \beta_1(y_{t-1} - y^*_{t-1}) + \beta_2(i_{t-1} - i^*_{t-1}) + \beta_3(m_{t-1} - m^*_{t-1}) + \beta_4(\pi_{t-1} - \pi^*_{t-1}) + \beta_5(p_{t-1} - p^*_{t-1})] \quad (5)$$

The forecasts are constructed using (5). It is also possible to predict a change in fundamentals using past information. If this is the case, an econometric model can be formulated, which leads us to the multivariate equation approach.

2.4 Multiple-equations models

Two different econometrics models are used in this paper. The first is the vector autoregressive (VAR) model, and the second is the vector error correction (VEC) model.

2.4.1 VAR model

One possible way of modelling the exchange rate and the fundamental is to use a VAR model:

$$Y_t = \Pi_1 Y_{t-1} + ... + \Pi_{k-1} Y_{t-k+1} + \tau + \varepsilon_t \quad (6)$$

where ε_t are random normal and uncorrelated errors, Ω denotes the variance and covariance matrix of the errors that do not vary with time, and $\theta = [\Pi_1, ..., \Pi_k, \tau]$ contains the parameters of the model. The vector Y_t contains the exchange rate and set of fundamentals chosen by the analyst.

2.4.2 VEC model

We assume that the local data generation process for the exchange rate and a set of fundamentals is given by the following VAR model:

$$\Delta Y_t = \Gamma_1 \Delta Y_{t-1} + ... + \Gamma_{k-1} \Delta Y_{t-k+1} + \alpha\beta' Y_{t-1} + \mu + \varepsilon_t \quad (7)$$

where ε_t are random normal and uncorrelated errors, Ω denotes the variance and covariance matrix of the errors that do not vary with time, and $\theta = [\Gamma_1, ...\Gamma_{k-1}, \alpha, \beta, \mu]$ contains the parameters of the model. The vector Y_t contains the exchange rate and set of fundamentals chosen by the analyst. Δdenotes the first difference.

2.5 Bias correction approach

One way to improve the forecast performance of a particular model is the bias correction approach. If one model systematically forecasts in one wrong direction, the analyst can, ideally, correct the forecast by adding a term to avoid the bias.

Our approach is inspired by the paper of Issler & Lima (2009). Suppose that we want to forecast the exchange for t+1 with information available until t. We compute forecasts for a window of length τ from $t - \tau$ to t and collect all errors of these forecasts. Using an average of these errors ($\hat{bc}$) and under certain conditions, this simple average will provide a consistent estimate of the bias.

Our bias-corrected forecast is calculated by the following formula:

$$ {}_tF^{BC}_{t+h} = {}_t F_{t+h} - \hat{bc} \tag{8} $$

where h>0 denotes the horizon of the forecast.

2.6 Combined forecast techniques

Granger & Ramanathan (1984) and Bates & Granger (1969) suggested that a combination of two forecasts can generate more precise forecasts. There is extensive literature discussing alternative methods for combining forecasts. In this paper, we opt to use a simple combination technique. We combine each pair of forecasts using a simple average. We aim to evaluate whether this simple technique pays off. In her empirical exercise, Rossi (2013) did not use any forecast combination; nor did the seminal paper of Meese & Rogoff (1983).

We explore two types of combinations. The first is a combination of all possible pairs of structural model forecasts. The second combines the random walk forecast with each structural model forecast. If any structural model contains relevant information regarding the future, it may not be able to beat the random walk; however, combined with it, the projection may outperform the random walk. We aim to investigate if it is possible to further improve the predictive power of the random walk.

3. How to choose among different forecast models

In this section, we discuss two criteria used to compare the predictive forecasts of different models. The first is the classical Diebold-Mariano test Diebold & Mariano (1995). The second is the model confidence set developed by Hansen *et al.* (2011). The latter can be seen as a refinement of the former test.

3.1 Classical Diebold–Mariano test

In empirical applications, it is often the case that two or more time series models are available for forecasting a variable:

Define $\Theta = \{y_\tau;\ \tau = 1, 2, ..., k\}$ as the set with the actual values of a variable and $\Theta_1 = \{y^1_\tau;\ \tau = 1, 2, ..., k\}$, $\Theta_2 = \{y^2_\tau;\ \tau = 1, 2, ..., k\}$ as the set of predictions of models 1 and 2, respectively.

Define the forecast error for model i as:

$$e^i_\tau \quad = \quad y_\tau - y^i_\tau \tag{9}$$

Then, choose some loss function $g(e^i_\tau)$, with the difference given by:

$$d^{i,j}_\tau \quad = g(e^i_\tau) - g(e^j_\tau) \tag{10}$$

Let us state that the two models will have equal forecast accuracy if and only if the loss function has an expected value of zero for all τ.

Diebold and Mariano formulated the following null hypothesis:

$$H^0 : E(d^{i,j}_\tau) = 0\ for\ all\ \tau \tag{11}$$

against the alternative hypothesis that the models do not have the same level of accuracy:

$$H^1 : E(d^{i,j}_\tau) \neq 0 \tag{12}$$

Now consider the following quantity:

$$\bar{d}_{i,j} \quad = \frac{\sum_{\tau=1}^{M} d^{i,j}_\tau}{M} \tag{13}$$

Using a robust estimate of the variance of $\bar{d}_{i,j}$ denoted by $V\hat{A}R(\bar{d}_{i,j})$, and providing that certain regularity conditions hold, the following statistic is proposed to test the null:

$$DM = \frac{\bar{d}_{i,j}}{\sqrt{V\hat{A}R(\bar{d}_{i,j})}} \overset{a}{\sim} N(0,1) \tag{14}$$

One serious limitation of the Diebold-Mariano framework is that it is not designed to deal with many different competing models simultaneously. If there is a benchmark, all remaining models could be compared against the benchmark. However, if the analyst wants to rank the models and has no particular interest in choosing a benchmark, another framework should be tried. Hansen *et al.* (2011) tries to fill this gap.

3.2 The model confidence set

The model confidence set (MCS) is a model selection technique developed by Hansen, Lunde, and Nason (2011). It consists of an algorithm that ranks the forecasts from models. M^* contains the best model(s) chosen from a collection of models, M^0, in which the "best model" is defined using criteria related to prediction quality.

Definition 1: The set of superior objects is defined by:

$M^* \equiv \{i \,\epsilon\, M^0 : E(d_\tau^{i,j}) \leq 0 \; for \; all \; j \; \epsilon \, M^0\}$

In the following, we let $M^\dagger$ denote the complement to M^*. That is, $M^\dagger \equiv \{i \,\epsilon\, M^0 : E(d_\tau^{i,j}) > 0 \; for \; all \; j \; \epsilon \, M^0\}$

The MCS selects a model using an equivalence test, δ_M, and an elimination rule, ϱ_M. The equivalence test is applied to the set $M = M^0$. If the equivalence hypothesis is rejected, then there is evidence that there is a set of inferior models in terms of forecast accuracy. Therefore, the rule ϱ_M is used to eliminate the models with poor predictive quality. The procedure is repeated until the equivalence test, δ_M, is accepted. Then, the model $(\widehat{M_F^*})$ is selected to be the set of the best final models.

The null hypothesis of the test is:

$$H_M^0 : E(d_\tau^{i,j}) = 0 \; for \, all \; i,j \; \epsilon \; M \tag{15}$$

where $M \subset M^0$.

The alternative hypothesis is:

$$H_M^1 : E(d_\tau^{i,j}) \neq 0 \; for \; some \; i,j \; \epsilon \; M \tag{16}$$

Note that there might be better models outside of the set of "candidate models", M^0. The goal of the MCS is to determine M^*.

The null hypothesis can be tested using the following statistic:[3]

$$T_D = \sum_{i \,\epsilon\, M} t_i^2 \tag{17}$$

where $t_i = \frac{\bar{d}_i}{\sqrt{V\hat{A}R(\bar{d}_i)}}$ and $\bar{d}_i \equiv M^{-1} \sum_{j \,\epsilon\, M} \bar{d}_{ij}$

The statistic given by (17) has a non-standard distribution that can be simulated using bootstrap techniques.

The elimination rule is:

$$\varrho_M = arg\, max_i(t_i) \tag{18}$$

3.2.1 The algorithm

The MCS algorithm takes the following steps:

(i) Initially set $M = M^0$;

(ii) Test H_M^0 using δ_M at level α;

(iii) If H_M^0 is not rejected then the procedure ends and the final set is $\hat{M}_{1-\alpha}^* = M$, otherwise we use ϱ_M to eliminate an object from M and repeat step (i).

The authors show that the MCS has the following statistical properties:

(i) $lim_{n\to\infty} P(M^* \subset \hat{M}_{1-\alpha}^*) > 1 - \alpha$ and

(ii) $lim_{n\to\infty} P(i^\dagger \,\epsilon\, \subset \hat{M}_{1-\alpha}^*) = 0$ for all $i^\dagger \,\epsilon\, \subset M^\dagger$

3.2.2 Ranking the models: MCS p-values

The elimination rule, ϱ_M, defines a sequence of random sets, $M_0 = M_1 \supset M_2 \supset ... \supset M_{m0}$, where $M_i = \{\varrho_i, ..., \varrho_{m0}\}$ and m_0 are the number of elements in M_0. ϱ_{M0} is the first to be eliminated, ϱ_{M1} is the second to be eliminated, and so forth. At the end, only one model survives. We set the p-value of this model as 1. We collect the p-value of the eliminated model if it is higher than the p-value of the previously eliminated model. If it is not, we opt to maintain the p-values of the previous rejections.

The MCS p-values are convenient because they make it easier for the analyst to determine whether a particular object is in $\hat{M}_{1-\alpha}^*$.

[3]There are others possible choices.

3.3 Pseudo real-time exercise

The data gathered for the countries is used to create many variants of the structural models in order to forecast the exchange rate. The sample is split into two parts. The first half of the sample is used to estimate the models, and the second half is used to evaluate the forecast performance of the models in various horizons. In the exercise, we attempt to simulate a real-time operation. We use an information set that reflects, as closely as possible, the one available to agents at the time of the forecast. In other words, the models are re-estimated at each point in order to incorporate the new information arriving at each instant of time. For each model, forecasts are generated for up to six quarters (a year and a half). All of the projections performed by the models are grouped according to time horizon. Thus, there are six groups — one for each horizon.

Some of the data we collected are revised from the initial publication in their original sources. The values we use to run our projections are not exactly the same as those available to agents at that time. We run projections in our pseudo real-time experiment with a slightly better information set. This may result in better forecast accuracy compared to the projections generated in real time. Because a dataset with original published data is not available, this is the best we can do to simulate a real exercise.

4. Meese–Rogoff Puzzle

A comprehensive survey on the literature that followed the study of Meese & Rogoff (1983) was conducted by Rossi (2013). Her main conclusions are as follows:

1. There is a consensus in the literature that models based on the Taylor rule and that use the net foreign assets position produce better forecasts outside the sample than do other traditional fundamentals, such as interest rates, inflation, gross domestic product, and differentials between monetary aggregates. The monetary fundamentals in long horizons and the interest rate differentials in short horizons have predictive power in some studies, but not in others. However, there are differences of opinion regarding whether monetary fundamentals are useful, as suggested by Meese and Rogoff.

2. Among all the classes of model, those with the best performance are linear and error-correction models. For single-equations models,

explanatory variables are more relevant than lagged, contemporary, or historical data are.

3. Data transformations, such as seasonal adjustments, lags, de-trending, and differentiations can substantially affect the predictive power of the model, and can explain why there are differences in results between studies. For example, consider the forecasting ability of the monetary model in long horizons. For some fundamentals, the forecasting ability changes significantly when historical data is replaced with real-time data. For some models, the forecasting ability also appears to depend on the chosen country. With few exceptions, the frequency of data and whether they are historical or projected appear not to affect the forecasting ability of the model.

4. The choices of benchmark, projection time horizon, data sample, and projection method are very important. The random walk without drift is the most difficult benchmark to beat.

5. On one hand, the empirical analysis confirms most of the foreign exchange studies. Due to instability in the parameters of the models used, some variables have forecasting ability within the sample but do not have projection relevance outside of the sample. Moreover, the predictive power of models varies between countries, models and variables used, period of time analysed, and data sample. Although the Taylor rule and the net foreign assets variable have forecasting ability for short periods of time, and other models based on monetary fundamentals (error-correction models) have forecasting ability for longer time horizons, none of them appears to discard the conclusions of Meese and Rogoff.

Another important point to be addressed comes from the papers of Engel & West (2005) and Engel *et al.* (2007). They showed that under reasonable parameters configuration, the exchange rate models reflect dynamics that are similar to those a near random walk. If they were correct, one implication of the monetary exchange rate approach is poor out-of-sample predictive performance. They suggested that the MR puzzle should not be seen as evidence against these models.

5. Results

5.1 Database description

The study aims to analyse the forecast performance of a set of models to predict the nominal exchange rates between the Real and a set of currencies: Real–Dollar, Real–Yen, and Real–Pound pairs are analysed. The datasets for the analysis were collected from the DATASTREAM data system (Thomson-Reuters) and the International Monetary Fund's (IMF's) International Financial Statistics (IFS) database.

The frequency of the data is quarterly. The period covers the years from 1995 to 2013. The fundamentals are gross domestic product (GDP), monetary aggregates (M1 and M2), consumer price index (CPI, and net foreign asset position as a share of GDP. The data of the net foreign asset position are gathered from Lane & Milesi-Ferretti (2001) and updated based on the IFS database. All of the models are estimated using the STATA-12 program. The analysis of the results and the choice of the best models are performed via the MCS Hansen *et al.* (2011) and implemented in the Oxmetrics 6 program through the code made available by the authors via their webpage.

5.2 Results and Discussion

5.2.1 The U.S. Dollar–Brazilian Real case

We estimated a VAR model and a VEC model with and without seasonal dummies for 18 different combinations of the fundamentals. This resulted in a total of 72 models. We then performed the bias correction procedure for all of these models, bringing the total to 144 models. Next, we combined all of the models in pairs. We also combined the random walk forecast with and without drift with each previous model. This yielded a total of 10,730 model forecasts. The forecasts were ranked by their mean forecast squared error (MFSE). The random walk with and without drift were also included. We collected the forecasts for all horizons from one to six quarters ahead. All of these models were re-estimated at each point in time in our pseudo real-time data experiment.

Because we had too many forecasts to compare, we opted to run a preliminary selection round using the MFSE. The forecasts were ranked by their MFSE in ascending order. The first 250 best models were selected and classified for the second round. In the second round, we ran the MCS algorithm to define the best forecasts. If the random models with or without drift were not classified in the first round, we opted to award them a wildcard. They were always included in the MCS round.

Multivariate models versus random walk The results of this pseudo real-time experiment can be seen in Table 1. The MCS algorithm classified the random walk without drift as the best forecast for all horizons. For shorter horizons, there were other forecasts that could be seen as at least as good as the random walk, but with higher MFSE. As for the forecasts from one to three quarters, some models were considered equivalent to the random walk. However, for horizons from four to six quarters, the random walk was the best model to forecast the exchange rate. All other models were eliminated from the final set. The set of fundamentals that seemed to generate some forecastability contained monetary aggregates, gross domestic product, and net foreign asset position. Finally, the random walk with drift was eliminated from all final sets. This last result is in line with Rossi (2013). She claimed that the random walk without drift is the hardest benchmark to beat.

Full set of forecasts versus random walk The results of the completed pseudo real-time experiment can be seen in Table 2. The MCS algorithm classifies the random walk without drift as the best forecast for all horizons. Contrary to the previous case, there are other forecasts that can be seen as at least as good as the random walk, but with higher MFSE, up to five quarters. For the six-quarters-ahead forecast, the random walk is the only model in the MCS final set. The random walk with drift was eliminated from all final sets. This last result is line with Rossi (2013). The bias correction procedure did not seem to add relevant forecastability to the models. None of them were selected in all horizons. Some combined forecast models are in the final set. The finalists include pairs of multivariate models as well as pairs that combine a multivariate model with a random walk without drift. Nonetheless, even in this broader experiment, it is not possible to beat the random walk if the mean squared error is the metric.

5.3 Robustness check

Almost all transactions that involved the Brazilian currency were performed against the Dollar. However, we ran a similar exercise, looking to the bilateral exchange rate of the Real against the British Pound and the Japanese Yen.

Table 1
MCS results for the case of multivariate models versus random walk — Brazil and the United States.

Countries:	Brazil and United States	Full Sample: 1995q1 to 2013q4	1T	2T	3T	4T	5T	6T
All Models	**BEST MODEL**	**RW without drift**	✓	✓	✓	✓	✓	✓
		RW with drift						
		VAR or VECM						
		VAR or VECM with bias correction						
		Variables in the model *	rw	rw	rw	rw	rw	rw
	FINAL SET	**VAR or VECM**	✓	✓	✓			
		VAR or VECM with bias correction						
		RW with drift						
		RW without drift	✓	✓	✓	✓	✓	✓
		Variables in the models *	rw, 8,9,10,12, 13,14,16	rw, 12,16	rw, 16	rw	rw	rw
		Total of model in final set	12	3	2	1	1	1

* Legend of variables in the model

rw - random walk without drift	6 - m2, m2*	13 - (m1-m1*), (ir - ir*), nfa, nfa*
rwd - random walk with drift	7 - m2, m2*, gdp, gdp*	14 - m2, m2*, nfa, nfa*
1 - m1, m1*	8 - (m2-m2*), (gdp-gdp*)	15 - m2, m2*, gdp, gdp*, nfa, nfa*
2 - m1, m1*, gdp, gdp*	9 - m1, m1*, nfa, nfa*	16 - (m2-m2*), (gdp-gdp*), nfa, nfa*
3 - m1, m1*, ir, ir*	10 - m1, m1*, gdp, gdp*, nfa, nfa*	17 - cpi, cpi*
4 - (m1-m1*), (gdp-gdp*)	11 - m1, m1*, ir, ir*, nfa, nfa*	18 - cpi, cpi*, nfa, nfa*
5 - (m1-m1*), (ir-ir*)	12 - (m1-m1*), (gdp-gdp*), nfa, nfa*	

Table 2
MCS results for the case of the full set of forecasts versus random walk — Brazil and the United States.

Countries:	Brazil and United States	Full Sample: 1995q1 to 2013q4	1T	2T	3T	4T	5T	6T
All Models	BEST MODEL	**RW without drift**	✓	✓	✓	✓	✓	✓
		RW with drift						
		BIAS						
		Combined VAR or VECM with RW						
		Combined VAR or VECM not with RW						
		Combined VAR or VECM both with bias correction						
		Combined VAR or VECM with RW and bias correction						
		VAR or VECM						
		Variables in the model *	rw	rw	rw	rw	rw	rw
	FINAL SET	**VAR or VECM**	✓					
		VAR or VECM with bias correction						
		Combined VAR or VECM	✓	✓				
		Combined VAR or VECM both with bias correction						
		Combined VAR or VECM with bias correction						
		Combined VAR or VECM with a drift RW						
		Combined VAR or VECM with RW	✓	✓	✓	✓	✓	
		Combined VAR or VECM with bias correction and RW						
		Combined VAR or VECM with bias correction and a drift RW						
		RW with drift						
		RW without drift	✓	✓	✓	✓	✓	✓
		Variables in the model *						
		Total of model in final set	14	5	3	5	3	1

* Legend of variables in the model

rw - random walk without drift
rwd - random walk with drift
1 - m1, m1*
2 - m1, m1*, gdp, gdp*
3 - m1, m1*, ir, ir*
4 - (m1-m1*), (gdp-gdp*)
5 - (m1-m1*), (ir-ir*)
6 - m2, m2*
7 - m2, m2*, gdp, gdp*
8 - (m2-m2*), (gdp-gdp*)
9 - m1, m1*, nfa, nfa*
10 - m1, m1*, gdp, gdp*, nfa, nfa*
11 - m1, m1*, ir, ir*, nfa, nfa*
12 - (m1-m1*), (gdp-gdp*), nfa, nfa*
13 - (m1-m1*), (ir - ir*), nfa, nfa*
14 - m2, m2*, nfa, nfa*
15 - m2, m2*, gdp, gdp*, nfa, nfa*
16 - (m2-m2*), (gdp-gdp*), nfa, nfa*
17 - cpi, cpi*
18 - cpi, cpi*, nfa, nfa*

5.3.1 The Japanese Yen–Brazilian Real case

Multivariate models versus random walk Table 3 shows the results of the pseudo real-time experiment regarding the exchange rate of Brazil and Japan. For all horizons, the random walk is not the best model when the metric is the MFSE. For one, two, and five quarters ahead, the random walk has the lowest mean squared error. For the other periods, the random walk is not the best model. Looking at the MCS results, the random walk is part of the final set at all horizons. The models that beat the random walk in terms of mean squared error contain the following fundamentals: monetary aggregates, net foreign asset position, and interest rate differential. The list of fundamentals that generates models with predictive power includes monetary aggregates and net foreign asset position.

Full set of forecasts versus random walk Table 4 shows the results of the exercise using all of the forecasts. Here, the random walk is no longer the best model. For the horizons one, two, three, and five quarters ahead, the best model using the MFSE criteria is a model that combines the random walk and some structural model. For the four quarters ahead forecast, the best model is a combination of two structural models with bias correction. Finally, for six quarters ahead, the best model is a structural model. The final set contains many models at all horizons, but random walk is among them. These results contrast with the Brazil–United States case analysed previously. However, we must stress that Japan and Brazil are not engaged in major trade and financial relationships with each other.

5.3.2 The British Pound–Brazilian Real case

Multivariate models versus random walk Table 5 shows the results for Brazil and the United Kingdom. The random walk model is the best model when the criteria is the MFSE. At shorter horizons, from one up to three quarters ahead, the final set contains not only the random walk but some others models as well. At the horizon from four to six quarters ahead, the random walk is the only element in the final set. The fundamentals that may help to forecast the exchange rate are monetary aggregates, net foreign asset position, real GDP, and interest rate.

Full set of forecasts versus random walk Table 6 shows the results for Brazil and the United Kingdom using all of the models. The random walk model is the best model when the criteria is the MFSE for all horizons. The

Table 3
MCS results for the case of multivariate models versus random walk — Brazil and Japan.

Countries:	Brazil and Japan	Full Sample: 1995q1 to 2013q4	1T	2T	3T	4T	5T	6T
All Models	BEST MODEL	**RW without drift**	✓	✓			✓	
		RW with drift						
		VAR or VECM						
		VAR or VECM with bias correction			✓	✓		✓
		Variables in the model *	rw	rw	14	16	rw	6
	FINAL SET	**VAR or VECM**						
		VAR or VECM with bias correction		✓	✓	✓		✓
		RW with drift						
		RW without drift	✓	✓	✓	✓	✓	✓
		Variables in the models *	rw	rw,9	rw,14,16	rw,16	rw	rw,6
		Total of model in final set	1	2	3	2	1	2

* Legend of variables in the model

rw - random walk without drift
rwd - random walk with drift
1 - m1, m1*
2 - m1, m1*, gdp, gdp*
3 - m1, m1*, ir, ir*
4 - (m1-m1*), (gdp-gdp*)
5 - (m1-m1*), (ir-ir*)
6 - m2, m2*
7 - m2, m2*, gdp, gdp*
8 - (m2-m2*), (gdp-gdp*)
9 - m1, m1*, nfa, nfa*
10 - m1, m1*, gdp, gdp*, nfa, nfa*
11 - m1, m1*, ir, ir*, nfa, nfa*
12 - (m1-m1*), (gdp-gdp*), nfa, nfa*
13 - (m1-m1*), (ir - ir*), nfa, nfa*
14 - m2, m2*, nfa, nfa*
15 - m2, m2*, gdp, gdp*, nfa, nfa*
16 - (m2-m2*), (gdp-gdp*), nfa, nfa*
17 - cpi, cpi*
18 - cpi, cpi*, nfa, nfa*

Table 4
MCS results for the case of the full set of forecasts versus random walk — Brazil and Japan.

Countries:	Brazil and Japan	Sample: 1995q1 to 2013q4	1T	2T	3T	4T	5T	6T
All Models	BEST MODEL	**RW / DRIFT**						
		BIAS						
		Combined VAR or VECM with RW						
		Combined VAR or VECM not with RW						
		Combined VAR or VECM both with bias correction				✓		✓
		Combined VAR or VECM with RW and bias correction	✓	✓	✓		✓	
		VAR or VECM						
		Variables in the model *	rw, 18	rw, 12	rw, 16	7, 16	rw, 12	6, 7
	FINAL SET	**VAR or VECM**		✓				
		VAR or VECM with bias correction						✓
		Combined VAR or VECM	✓			✓		
		Combined VAR or VECM both with bias correction	✓	✓	✓	✓	✓	✓
		Combined VAR or VECM with bias correction		✓	✓	✓	✓	✓
		Combined VAR or VECM with a drift RW						
		Combined VAR or VECM with RW	✓	✓		✓		
		Combined VAR or VECM with bias correction and RW	✓	✓	✓	✓	✓	✓
		Combined VAR or VECM with bias correction and a drift RW						
		RW with drift						
		RW without drift	✓					
		Total of model in final set	57	115	19	136	67	104

* Legend of variables in the model

rw - random walk without drift
rwd - random walk with drift
1 - m1, m1*
2 - m1, m1*, gdp, gdp*
3 - m1, m1*, ir, ir*
4 - (m1-m1*), (gdp-gdp*)
5 - (m1-m1*), (ir-ir*)
6 - m2, m2*
7 - m2, m2*, gdp, gdp*
8 - (m2-m2*), (gdp-gdp*)
9 - m1, m1*, nfa, nfa*
10 - m1, m1*, gdp, gdp*, nfa, nfa*
11 - m1, m1*, ir, ir*, nfa, nfa*
12 - (m1-m1*), (gdp-gdp*), nfa, nfa*
13 - (m1-m1*), (ir - ir*), nfa, nfa*
14 - m2, m2*, nfa, nfa*
15 - m2, m2*, gdp, gdp*, nfa, nfa*
16 - (m2-m2*), (gdp-gdp*), nfa, nfa*
17 - cpi, cpi*
18 - cpi, cpi*, nfa, nfa*

final set includes combined models that have a random walk without drift in the pair. None of the models with bias correction appears in the final set. At six quarters ahead, the random walk is the lone element in the final set. In the horizons from one to five quarters ahead, the random walk is not the only model in the final set.

5.4 Limitations, comparison with other studies, and paths for future research

Some studies have investigated the MR puzzle using Brazilian data. Perdomo & Botelho (2007) tested the random walk hypothesis for the Brazilian case by comparing the error of exchange rate projections performed by banks, consulting firms, and financial institutions. They used forecast data collected by the Brazilian Central Bank (FOCUS) from the top-5 forecasters. Their study included projections of a random walk model for three forecast horizons. The authors concluded that the random walk is more accurate than the models used by financial institutions.

Moura *et al.* (2008) reported the results of an out-of-sample exercise to predict the Brazilian exchange rate using the fundamentals and techniques employed for this paper. They also investigated Taylor rule models,[4] inspired by the papers of Engel, Mark, and West (2007), Mark (2007), Clarida and Waldman (2007), and Molodtsova and Papell (2007). They ran the DM test to compare the forecasts of the model with the benchmark of a random walk with drift. They reported that some models beat the benchmark for horizons up to 12 months. However, they did not report an exercise that uses the random walk without drift as a benchmark. Based on our results, their benchmark may not be the hardest one to beat. It is possible that that their results may have been different if they opted to use the random walk without drift as a benchmark. In our exercise, we show that the random walk with drift is an easy benchmark to beat for Brazil.

Galimberti & Moura (2013) also reported results for Brazil. They analysed a group of emerging market countries using a panel data technique. They showed evidence in favour of Taylor rule models and against the random walk with drift for Brazil and others countries. However, they too did not report results for the toughest benchmark (i.e., the random walk without drift).

Our paper focuses on models based on purchasing power parity, a monetary approach to the exchange rate. However, in future research, we can

[4] Taylor (1993)

Table 5
MCS results for the case of multivariate models versus random walk — Brazil and the United Kingdom.

Countries:	Brazil and United Kingdom	Full Sample: 1995q1 to 2013q4	1T	2T	3T	4T	5T	6T
All Models	BEST MODEL	**RW without drift**	✓	✓	✓	✓	✓	✓
		RW with drift						
		VAR or VECM						
		VAR or VECM with bias correction						
		Variables in the model *	rw	rw	rw	rw	rw	rw
	FINAL SET	**VAR or VECM**	✓	✓	✓			
		VAR or VECM with bias correction						
		RW with drift						
		RW without drift	✓	✓	✓	✓	✓	✓
		Variables in the models *	rw, 8,9,10,12, 13,14,16	rw,12,16	rw,16	rw	rw	rw
		Total of model in final set	12	3	2	1	1	1

* Legend of variables in the model

rw - random walk without drift
rwd - random walk with drift
1 - m1, m1*
2 - m1, m1*, gdp, gdp*
3 - m1, m1*, ir, ir*
4 - (m1-m1*), (gdp-gdp*)
5 - (m1-m1*), (ir-ir*)
6 - m2, m2*
7 - m2, m2*, gdp, gdp*
8 - (m2-m2*), (gdp-gdp*)
9 - m1, m1*, nfa, nfa*
10 - m1, m1*, gdp, gdp*, nfa, nfa*
11 - m1, m1*, ir, ir*, nfa, nfa*
12 - (m1-m1*), (gdp-gdp*), nfa, nfa*
13 - (m1-m1*), (ir - ir*), nfa, nfa*
14 - m2, m2*, nfa, nfa*
15 - m2, m2*, gdp, gdp*, nfa, nfa*
16 - (m2-m2*), (gdp-gdp*), nfa, nfa*
17 - cpi, cpi*
18 - cpi, cpi*, nfa, nfa*

Table 6
MCS results for Full set of forecasts versus Random Walk case - Brazil and the United Kingdom.

Countries:	Brazil and United Kingdom	Sample: 1995q1 to 2013q4	1T	2T	3T	4T	5T	6T
All Models	BEST MODEL	RW without drift	✓	✓	✓	✓	✓	✓
		RW with drift						
		BIAS						
		Combined VAR or VECM with RW						
		Combined VAR or VECM not with RW						
		Combined VAR or VECM both with bias correction						
		Combined VAR or VECM with RW and bias correction						
		VAR or VECM						
		Variables in the model *	rw	rw	rw	rw	rw	rw
	FINAL SET	VAR or VECM	✓					
		VAR or VECM with bias correction						
		Combined VAR or VECM	✓	✓				
		Combined VAR or VECM both with bias correction						
		Combined VAR or VECM with bias correction						
		Combined VAR or VECM with a drift RW						
		Combined VAR or VECM with RW	✓	✓	✓	✓	✓	
		Combined VAR or VECM with bias correction and RW						
		Combined VAR or VECM with bias correction and a drift RW						
		RW with drift						
		RW without drift	✓	✓	✓	✓	✓	✓
		Total of model in final set	14	5	3	5	3	1

* Legend of variables in the model

rw - random walk without drift
rwd - random walk with drift
1 - m1, m1*
2 - m1, m1*, gdp, gdp*
3 - m1, m1*, ir, ir*
4 - (m1-m1*), (gdp-gdp*)
5 - (m1-m1*), (ir-ir*)
6 - m2, m2*
7 - m2, m2*, gdp, gdp*
8 - (m2-m2*), (gdp-gdp*)
9 - m1, m1*, nfa, nfa*
10 - m1, m1*, gdp, gdp*, nfa, nfa*
11 - m1, m1*, ir, ir*, nfa, nfa*
12 - (m1-m1*), (gdp-gdp*), nfa, nfa*
13 - (m1-m1*), (ir - ir*), nfa, nfa*
14 - m2, m2*, nfa, nfa*
15 - m2, m2*, gdp, gdp*, nfa, nfa*
16 - (m2-m2*), (gdp-gdp*), nfa, nfa*
17 - cpi, cpi*
18 - cpi, cpi*, nfa, nfa*

investigate whether Taylor rule–based models can help to predict the exchange rate in Brazil and whether they can outperform the random walk models both with and without drift.

One source of forecast failure in economics is a not-modelled change in the mean of the data generation process (DGP). An automatic model selection algorithm, such as Autometrics[5] can be helpful in improving forecast accuracy. Castle *et al.* (2014) discussed how to increase the robustness of a forecast obtained from a VEC model with a change in mean. This approach can be tested to investigate whether forecasts from a VEC model with monetary exchange rate fundamentals can be improved using their procedure.

6. Final Remarks

The main goal of this paper was to investigate whether the models of the 80s can outperform the predictions from the random walk model for Brazil. The main conclusion of our paper is that the random walk model without drift is the most difficult benchmark to be beat. In our exercise, we were able to outperform the random walk with drift, but not the random walk without drift.

Our results are in line with Rossi (2013), but our work differs from hers in the following aspects: (a) we used the MCS algorithm to investigate the forecast equivalence among the models; (b) we implemented a bias forecast correction in order to improve the forecasts; and (c) we also attempted a simple forecast combination technique. The random walk puzzle seems to hold for Brazilian data. The investigation of the predictive power of Taylor rule models is left for future research.

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