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Modeling and Forecasting the Volatility of Gas Futures Prices

(Modelagem e Previsão da Volatilidade dos Preços Futuros de Gás)

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Abstract

We examine the ability of three different GARCH-class models, with four innovation distributions, to capture the volatility properties of natural gas futures contracts traded on the New York Mercantile Exchange. We jointly estimate the long-memory processes for conditional return and variance investigating the long-memory and persistence of long and short maturities contracts. We examine the ability of these models and distributions to forecast the conditional variance. We find that AR(FI)MA-FIAPARCH model is a better fit for short- and long-term contracts. However, there is not a single innovation distribution that provides a better fit for all of the data examined. The out-of-sample forecast of variance also provides mixed results concerning the best innovation distribution. Further, the persistence decreases as the maturity of contracts increases.

Keywords: Natural gas markets, long-memory, volatility forecasting, GARCH class models, shale gas.

JEL: Codes C1, C4, C5, Q4.

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**In memoriam

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Resumo

Examinamos a capacidade de três modelos diferentes da classe GARCH, com quatro distribuições de inovações, para capturar as propriedades de volatilidade dos contratos futuros de gás natural negociados na New York Mercantile Exchange. Nós estimamos conjuntamente os processos de memória longa para retorno condicional e variância investigando a longa memória e persistência de contratos de vencimentos longos e curtos. Examinamos a capacidade desses modelos e distribuições para prever a variância condicional. Nós achamos que o modelo AR(FI)MA-FIAPARCH é um ajuste melhor para contratos de curto e longo prazo. No entanto, não há uma única distribuição de inovação que fornece um melhor ajuste para todos os dados examinados. A previsão de variância fora da amostra também fornece resultados mistos sobre a melhor distribuição de inovação. Além disso, a persistência diminui à medida que o vencimento dos contratos aumenta.

Palavras-chave: Mercados de gás natural, memória longa, previsão de volatilidade, modelos de classe GARCH, gás de xisto.

1. Introduction

The behaviour analysis of energy future prices and volatility is of great interest in terms of trading physical products, government planning, capital budget decisions and portfolio management. It is a well-known fact that commodity markets products exhibit high price volatilities. Also, the natural gas and electricity markets are even more volatile than other energy commodities like crude oil and oil-refined products. The consequences of such high volatility for the agents are enormous. In these volatile markets consumers and producers often hedge their positions through financial instruments. For example, Taylor (2001) Taylor (2001) examines the consequences of a long-memory process in volatility for option pricing. Further, the strategy in the financial-derivatives markets is linked directly to the volatility levels of the underlying asset prices. The short-term hedging strategy can also be done through inventories. Pindyck (2004) Pindyck (2004) explains the mechanism that volatility plays in the short-run dynamics of prices and inventories. Besides, the volatility behaviour can also be considered in pricing formulas; for example in Chorro, Guégan and Ielpo (2012) Chorro et al. (2012).

In the last decade, the energy commodity markets exhibited more

liquidity where agents used even more long-term contracts to hedge their positions. This can be interpreted that long-term investment decisions and financial positions on real assets have recently become more important for the agents. Hence understanding volatility in the commodity markets is of crucial importance for investment decisions and planning by policymakers; not only for the short-term, but also for the long-term. It is worth mentioning that the recent importance of gas in the energy markets has increased due to the abundant supply of shale gas in the U.S. market, which has worldwide consequences. The recent Energy Information Administration (EIA) Annual Energy Outlook 2015 report U.S.-EIA (2015) shows that natural gas production will continue to increase. The U.S. transitions from a net importer of 1.3 Tcf (trillion cubic feet) in 2013 to net exporter in 2017. As a result, the investments in long-term projects in the gas sector are even more important today.

It is a well-documented fact that the behaviour of volatility is a long-memory process; see, for example, Baillie (1996) Baillie (1996), Baillie, Bollerslev and Mikkelsen (1996) Baillie et al. (1996) and Bollerslev and Mikkelsen (1996) Bollerslev and Ole Mikkelsen (1996). The presence of long-memory on a series means that shocks decay hyperbolically (or slowly). In the energy sector, many studies have investigated the long-memory process in return series. In the oil industry, we particularly mention the works of Elder and Serletis (2008) Elder and Serletis (2008), Tabak and Cajueiro (2007) Tabak and Cajueiro (2007) and Choi and Hammoudeh (2009) Choi and Hammoudeh (2009). Further evidence of long-memory in commodity future returns can be found in Power and Turvey (2010) Power and Turvey (2010) who study energy and agricultural commodities. Chang, McAleer and Tansuchat (2012) Chang et al. (2012) also focus on the agricultural industry when examining volatility, including different futures markets. In addition, Weron (2002) Weron (2002) studies long-range dependencies in the electricity markets.

Many studies have also investigated the long-memory process of volatility in the oil industry using GARCH-class models: Wei, Wang and Huang (2010) Wei et al. (2010); Kang, Kang and Yoon (2009) Kang et al. (2009); Kang and Yoon (2013) Kang and Yoon (2013) and

Cheong (2009) Cheong (2009), among others. Most energy market studies focus on the crude oil markets (Brent or West Texas Intermediate [WTI] crude oil types) and refined products (gasoline and heating oil). Less attention has been devoted to the natural gas futures market with this type of analysis. Sadorsky (2006) Sadorsky (2006) models volatility in oil the industry (including natural gas) with GARCH-class models but he does not use the long-memory property. Pindyck (2003) Pindyck (2003) analyzes the volatility behavior of crude oil and natural gas, without accounting for any long-memory properties.

In the same context as this study, in terms of dual long-memory processes, we can mention Mensi, Hammoudeh and Yoon (2013) Mensi et al. (2013) who investigate the behaviour of the foreign exchange markets of oil exporters and Arouri, Hammoudeh, Lahiani and Nguyen (2012) Arouri et al. (2012) who analyzed the dynamics of precious metals.

In this study, we explore natural gas futures price series, due their growing importance in recent years. This is related to the presence of shale gas. The abundance of this unconventional energy source has imposed a low-price regime ranging from US\$2/MM Btu to US\$3/MM Btu recently. We investigate the use of long-memory to model returns and volatility jointly. Our analysis involves short- and long-term futures contracts using the AR(FI)MA-GARCH, AR(FI)MA-IGARCH and AR(FI)MA-FIAPARCH models considering four different innovation distributions. Furthermore, we explore the out-of-sample forecasting based on multiple horizons (1-, 5- and 20- day ahead horizons) for each time series. We find that the AR(FI)MA-FIAPARCH model is a better fit for all contracts. Nevertheless, there is no single innovation distribution that outperforms the others when considering in-sample and out-of-sample analysis.

The remainder of this paper is organized as follows. Section 2 specifies the models used in the analysis; Section 3 presents the data used in the analysis; Section 4 includes the long-memory tests, the estimation results and the out-of-sample forecasts; and Section 5 concludes.

2. Model set up

In a general framework, a time series y_t is defined as

$$y_t = E(y_t | \Omega_{t-1}) + \epsilon_t, \quad (1)$$

where Ω_{t-1} is the information available up to time $t - 1$, $E(\cdot|\cdot)$ denotes the conditional expectation operator and ϵ_t is the innovation process at time t . We define the return series as the log-return; that is $y_t = \log P_t - \log P_{t-1}$, where P_t is the future price at time t . We follow Sadorsky (2006) Sadorsky (2006); Kang, Kang and Yoon (2009) Kang et al. (2009); Wei, Wang and Huang (2010) Wei et al. (2010) and Kang and Yoon (2013) Kang and Yoon (2013) by defining the actual variance as the square of the return series; that is, $\sigma_t^2 = y_t^2$.

To jointly describe the conditional mean and conditional volatility through a long-memory process, we simultaneously use ARFIMA-class and fractional integrated GARCH-class models, respectively. In this way, the conditional mean series y_t is specified through ARFIMA (r, ζ , s) as

$$\Phi(L)(1-L)^\zeta z_t = \Theta(L)\epsilon_t, \quad (2)$$

where L is the lag operator in the AR and MA polynomials $\Phi(L) = 1 - \phi_1 L - \dots - \phi_r L^r$ and $\Theta(L) = 1 + \theta_1 L + \dots + \theta_s L^s$, respectively. $\epsilon_t = \sigma_t \eta_t$, where η_t is the independently distributed innovation modeled through a normal, generalized error distribution (GED), as well as Student and skewed-Student distributions which are conditional distributions with zero mean and variance one. The conditional variance σ_t^2 is modeled according to each GARCH-class model presented in equation (5) and in subsequent equations. z_t is the unseasoned conditional mean series defined by

$$z_t = y_t - \sum_{i=1}^4 S_i D_i, \quad (3)$$

where D_i is a dummy variable defined as winter ($i = 1$), spring ($i = 2$), summer ($i = 3$), autumn ($i = 4$) and S_i is the correspondent coefficient. ζ is the fractional exponent and the fractional difference operator is defined by the following binomial expansion

$$(1-L)^\zeta = \sum_{j=0}^{\infty} \vartheta_j L^j = \sum_{j=0}^{\infty} \binom{\zeta}{j} (-L)^j. \quad (4)$$

The ARFIMA(r, ζ, s) process for z_t in equation (2) is covariance-stationary if $-0.5 < \zeta < 0.5$, see Hosking (1981) Hosking (1981). In case $0 < \zeta < 0.5$ the process is called long-memory. For the case $-0.5 < \zeta < 0$ the process is called anti-persistent or exhibiting intermediate memory. In the particular case where $\zeta = 0$ the process is an ARMA(r, s) and the shocks of ϵ_t decay exponentially. When $\zeta = 1$, we have an ARIMA ($r, 1, s$) process. The conditional variance σ_t^2 in this study includes the following GARCH-class models.

The GARCH(p, q) of Bollerslev (1986) Bollerslev (1986) is given by

$$\sigma_t^2 = \omega + \alpha(L) \epsilon_t^2 + \beta(L) \sigma_t^2, \tag{5}$$

where $\omega > 0$, $\alpha(L) = \alpha_1 L + \dots + \alpha_q L^q$ and $\beta(L) = \beta_1 L + \dots + \beta_p L^p$. Assuming that all the roots of $1 - \beta(L)$ are outside the unity circle, the model can be rewritten as an infinite ARCH process. In other words, the GARCH (p, q) can be interpreted as an ARMA(m, p) for ϵ_t^2 , where $m = \max(p, q)$, or else

$$\epsilon_t^2 = \omega + \sum_{i=1}^m (\alpha_i + \beta_i) \epsilon_{t-i}^2 - \sum_{i=1}^p \beta_i v_{t-i}, \tag{6}$$

where $v_t = \epsilon_t^2 - \sigma_t^2$ represents innovations with zero mean and is serially uncorrelated. In an abbreviate form equation (6) is given by

$$\varphi(L) \epsilon_t^2 = \omega + [1 - \beta(L)] v_t, \tag{7}$$

where $\varphi(L) = 1 - \alpha(L) - \beta(L)$. In the case where the roots of $\varphi(L)$ are outside the unity circle, the GARCH(p, q) process is covariance-stationary. This is the case where a shock on volatility decays exponentially and is said to have a short-memory. A particular interest is the case where $\varphi(L)$ has a unit root. This is the integrated GARCH process (IGARCH). The IGARCH process of Engle and Bollerslev (1986) Engle and Bollerslev (1986) is given by

$$\varphi(L) (1 - L) \epsilon_t^2 = \omega + [1 - \beta(L)] v_t \tag{8}$$

The IGARCH process is not covariance-stationary as a shock on conditional volatility never dies out and is said to have an infinite memory.

The models where the conditional volatility is modeled by ϵ_t^2 belong to the symmetric-class models. It is a well-known fact in empirical finance that the shocks have an asymmetric effect on conditional volatility. That is, the volatility at time t can be higher for a negative precedent shock than for a positive shock of the same magnitude. The extensions of linear GARCH-class models were developed to capture this asymmetric effect and they are called non-linear GARCH-class models.

The Asymmetric Power GARCH (APARCH) process encompasses many asymmetric processes. Ding, Granger and Engle (1993) Ding et al. (1993) imposed a Box-Cox transformation on the conditional standard deviation process and on the absolute residuals defining the APARCH(p,q) model as

$$\sigma_t^\delta = \omega + \sum_{i=1}^q \alpha_i (|\epsilon_{t-i}| - \gamma_i \epsilon_{t-i})^\delta + \sum_{j=1}^p \beta_j \sigma_{t-j}^\delta, \quad (9)$$

where δ is a positive parameter that represents a transformation of the volatility σ_t , γ_i is designed to capture the asymmetric effect such that $|\gamma_i| < 1$ for $i = 1, \dots, q$ and $\omega > 0$. If this is the case and $\sum_{i=1}^q E(|\eta| - \gamma_i \eta)^\delta + \sum_{j=1}^p \beta_j < 1$, the process is stationary. When $\gamma_i > 0$, negative shocks give rise to higher volatility than positive shocks of the same magnitude, and vice-versa.

The fractionally integrated GARCH process (FIGARCH) was developed to fill the gap between the integrated (non-stationary) GARCH process in equation (8) and the classical stationary GARCH process in equation (5). The FIGARCH(p,d,q) of Baillie, Bollerslev and Mikkelsen (1996) Baillie et al. (1996) is given by

$$\varphi(L) (1-L)^d \epsilon_t^2 = \omega + [1 - \beta(L)] v_t, \quad (10)$$

where $0 \leq d \leq 1$. The cases $d = 0$ and $d = 1$ are the GARCH(p,q) (equation (7)) and IGARCH(p,q) (equation (8)), respectively. The parameter d measures the magnitude of the persistence of the shock on the conditional volatility. Using the definition of v_t in equation (10) one can write the FIGARCH(p,d,q) for σ_t^2 as

$$\sigma_t^2 = \omega^* + \lambda(L) \epsilon_t^2, \quad (11)$$

where $\omega^* = \omega [1 - \beta(L)]^{-1}$ and $\lambda(L) = 1 - [1 - \beta(L)]^{-1} \varphi(L) (1 - L)^d$. For practical purposes, $\lambda(L)$ is an infinite summation that needs to be truncated. Baillie, Bollerslev and Mikkelsen (1996) Baillie et al. (1996) suggest the truncation at 1000 lags. Chung (1999) Chung (1999) uses another parametrisation and proposes the truncation at $T - 1$, where T is the size of the sample.

To account for the asymmetric effect on conditional volatility and simultaneously for the long-memory, Tse (1998) Tse (1998) proposes the fractionally integrated APARCH (FIAPARCH) process. The FI-APARCH(p,d,q) is written as

$$\sigma_t^\delta = \omega^* + \lambda(L) (|\epsilon_t| - \gamma\epsilon_t)^\delta, \tag{12}$$

where $\lambda(L)$ is truncated at lag 1000 and all other parameters have been defined previously. The FIAPARCH reduces to FIGARCH when $\delta = 2$ and $\gamma = 0$.

The ARFIMA-GARCH-class models can be estimated using optimization techniques to maximize the likelihood function for the particular innovation distribution. The normal distribution is the classical case. Others distributions are better suited to capture the asymmetry and fat tails that are present in financial returns. Besides the normal, we use the GED, Student and skewed-Student distributions to model the innovation process η_t , as defined in equation (2). These are the most common distributions in the literature in this type of analysis. Nelson (1991) Nelson (1991) proposes the GED. A random variable η is said to be a GED with zero mean and unit variance if

$$f(\eta|\psi) = \frac{\nu e^{-\frac{1}{2}|\frac{\eta}{\psi}|^\nu}}{\psi 2^{\frac{\nu+1}{\nu}} \Gamma\left(\frac{1}{\nu}\right)}, \tag{13}$$

where the parameter ν describes the fat tails of the distribution and ψ is given by

$$\psi = \left[\frac{2^{-\frac{2}{\nu}} \Gamma\left(\frac{1}{\nu}\right)}{\Gamma\left(\frac{3}{\nu}\right)} \right]^{\frac{1}{2}}.$$

Fernández and Steel (1998) Fernández and Steel (1998) propose the skewed-Student distribution. Following Lambert and Laurent (2001)

Lambert and Laurent (2001), a random variable is said to be a standardized (zero mean and unit variance) skewed-Student distribution if

$$f(\eta|\nu, \nu) = \begin{cases} \frac{2}{\iota + \frac{1}{\iota}} \kappa g[\iota(\kappa\eta_t + m) | \nu] & \text{if } \eta_t < -\frac{m}{\kappa} \\ \frac{2}{\iota + \frac{1}{\iota}} \kappa g\left[\frac{\kappa\eta_t + m}{\iota} | \nu\right] & \text{if } \eta_t \geq -\frac{m}{\kappa}, \end{cases} \quad (14)$$

where $g(\cdot|\nu)$ is the Student density with unit variance, ν degrees of freedom and ι is the asymmetry parameter. The parameters m and κ^2 are the mean and variance, respectively, of the non-standard skewed-Student, and are given by

$$m = \frac{\Gamma\left(\frac{\nu-1}{2}\right) \sqrt{\nu-2}}{\sqrt{\pi} \Gamma\left(\frac{\nu}{2}\right)} \left(\iota - \frac{1}{\iota}\right),$$

$$\kappa^2 = \left(\iota^2 + \frac{1}{\iota^2} - 1\right) - m^2.$$

Once each model has been estimated, we proceed with the forecasting analysis. The out-of-sample errors were evaluated through the mean-squared error (MSE) and mean-absolute error (MAE) following the formulas

$$MSE = E\left[\left(V_t - \hat{V}_t\right)^2\right], \quad (15)$$

$$MAE = E\left[|V_t - \hat{V}_t|\right], \quad (16)$$

where \hat{V}_t and V_t are the forecast and actual variables, respectively. When predicting the conditional mean and variance, V_t represents y_t and σ_t^2 , respectively.

To compare the predictive accuracy of the out-of-sample forecast, we use the Diebold-Mariano (1995) Diebold and Mariano (1995) test (DM test). As mentioned in Diebold (2012) Diebold (2012), the DM test was intended to compare forecasts, not to compare models. Consider we have two different forecasts of size n and time horizon h . The forecast error for model 1 can be written as $e_{t+h|t}^1 = V_{t+h} - \hat{V}_{t+h|t}^1$. The same approach applies to model 2, where $e_{t+h|t}^2 = V_{t+h} - \hat{V}_{t+h|t}^2$. The accuracy of each forecast is measured by a particular loss function $\mathcal{L}\left(e_{t+h|t}^i\right)$, $i = 1, 2$. We apply the loss functions on equations (15) and (16). The null hypothesis of equal predictive accuracy can

be expressed by $E[\mathcal{L}(e_{t+h|t}^1)] = E[\mathcal{L}(e_{t+h|t}^2)]$. Or else, defining $g_t = \mathcal{L}(e_{t+h|t}^1) - \mathcal{L}(e_{t+h|t}^2)$, the null hypothesis is $E(g_t) = 0$. The mean of the difference between the loss functions for $e_{t+h|t}^i$ is given by $\bar{g} = \frac{1}{n} \sum_{t=1}^n g_t$ and the long-run variance of \bar{g} is given by $Var(\bar{g}) = \frac{1}{n} [\xi_0 + 2 \sum_{j=1}^{\infty} \xi_j]$, where $\xi_j = Cov(g_t, g_{t-j})$. The DM test statistic is given by

$$DM = \frac{\bar{g}}{Var(\bar{g})}. \tag{17}$$

Under the null hypothesis of equal predictive accuracy, Diebold and Mariano (1995) Diebold and Mariano (1995) show that DM has an asymptotic standard normal distribution; that is, $DM \xrightarrow{A} N(0, 1)$. Harvey, Leybourne and Newbold (1997) Harvey et al. (1997) propose a small-sample modification of the DM test. The modification changes the variance of \bar{g} asymptotically when forecast accuracy is measured in terms of the mean-squared error and also when h -step ahead forecast errors are assumed to have zero autocorrelations at order h and beyond. An approximately unbiased estimator of the variance of \bar{g} leads to a modified DM test written as

$$DM^* = DM \left[\frac{T + 1 - 2h + h(h - 1) T^{-1}}{T} \right]^{\frac{1}{2}}. \tag{18}$$

In this work the Diebold-Mariano statistic will be referred as DM^* .

3. Data

The data used consists of three time series of futures contract prices traded on the NYMEX. We use the first (F1), second (F2) and fourteenth (F14) contracts, where Fi means the futures prices of a contract maturing i-months ahead. Hence F1 is the first nearby contract, F2 is the second nearby contract, and so on. We sampled these three series from 03 January 1995 through 03 May 2013. To avoid the natural turbulence that prices are subjected close to maturity, we rolled over the series three days before the expiration of each contract. The short-terms contracts, represented by F1 and F2, are the most liquid, while the long-term contract is analyzed through the F14 series which has also liquidity in NYMEX. This last series is

the longest maturity contract encompassing the starting and ending dates without missing values. Hence, the data collected represent complete panel series. Prices are daily closing prices expressed in US\$/MM Btu and total 4596 data entries. We left 252 data (one trading year) for the forecasting analysis.

Table 1 shows the main statistics of the return series. The values in brackets are the p-values associated with the statistics. In this ta-

Table 1

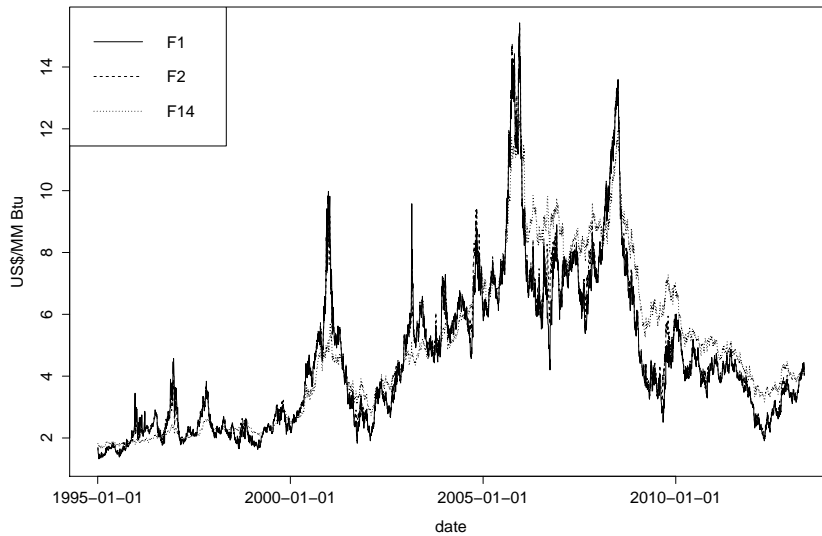
Main statistics of the return series: 03 January 1995 to 03 May 2013

	F1	F2	F14
Mean	0.000191	0.000194	0.000180
Std. dev.	0.037013	0.032623	0.015750
Maximum	0.324354	0.232446	0.084729
Minimum	-0.375749	-0.233242	-0.154328
Skewness	0.085650	0.209544	-0.549459
Kurtosis	10.58128	7.380673	11.04147
Jarque-Bera	11009	3708	12612
	[0.0000]	[0.0000]	[0.0000]
$Q_1(30)$	43.23	42.89	42.05
	[0.0560]	[0.0600]	[0.0710]
$Q_2(30)$	606.15	542.70	175.33
	[0.0000]	[0.0000]	[0.0000]
LM ARCH (2)	77.67	90.46	13.93
	[0.0000]	[0.0000]	[0.0009]
LM ARCH(20)	302.47	280.81	108.79
	[0.0000]	[0.0000]	[0.0000]

ble, we see that the means of the return series are small. However, the standard deviation in the F1 and F2 series is high. Compared to the short-term contracts, the F14 series has a much lower standard deviation. Translating to historical annualized volatilities these statistics means approximately 59%, 52% and 25% for F1, F2 and F14, respectively. The historical volatilities decrease according to contract maturity. This is a well-known fact in empirical finance and it is called the “Samuelson hypothesis”. The Jarque-Bera test statistic and kurtosis provide a strong evidence of non-normality in the three series. The Ljung-Box statistics $Q_1(M)$ for serial correlation up to lag M are not rejected at the 5% significance level and, apparently, there are no strong serial correlations within each return series. On the other

hand, the Ljung-Box statistics $Q_2(M)$ for the squared-return series rejects the null hypothesis of no serial correlation, meaning that there is a strong dependence within each squared-return series. The Lagrangian multiplier ARCH (LM ARCH) tests for lags 2 and 20 reject the null hypothesis of no ARCH effect in all of the series. Figure 1 shows the price series where one can observe that the F14 series has much less extreme values than the short-term series; thus, confirming a lower volatility.

Figure 1
Natural gas future prices for F1, F2 and F14 contracts traded on the NYMEX



4. Empirical results

4.1 Long-memory estimation

To investigate the evidence of long-memory in these time series, we use the classical Geweke and Porter-Hudak (1983) Geweke and Porter-Hudak (1983) (GPH) test and the Sowell (1992) Sowell (1992) exact maximum likelihood (EML) test. The GPH is a semi-parametric test to estimate ζ in the frequency domain. The test is a regression of log spectral density on a trigonometric function of

frequencies represented by

$$\log [I(\omega_j)] = \beta_1 + \beta_2 \log \left[4 \sin^2 \left(\frac{\omega_j}{2} \right) \right] + u_j, \quad (19)$$

where $\omega_j = \frac{2\pi j}{T}$, $j = 1, 2, \dots, n$; ω_j is the Fourier frequency, and we provide the analysis for $n = \frac{T}{2}$, $n = T^{0.5}$, $n = T^{0.6}$ and $n = T^{0.8}$ (see Diebold and Rudebusch (1989) Diebold and Rudebusch (1989)). u_j is the residual term, which is assumed to be iid with zero mean and variance $\frac{\pi}{6}$. The estimator of the fractional differencing parameter is $\hat{\zeta} = -\hat{\beta}_2$. The term $I(\omega_j)$ is the sample periodogram defined as

$$I(\omega_j) = \frac{1}{2\pi T} \left| \sum_{t=1}^T \epsilon_t \exp(-\omega_j t) \right|^2. \quad (20)$$

The EML method estimates the long-memory parameter ζ in the ARFIMA(r, ζ , s) model based on the maximization of the likelihood function $\ell(\varepsilon; \mathbf{\Lambda})$ written as

$$\log \ell(\varepsilon; \mathbf{\Lambda}) = -\frac{T}{2} \log(2\pi) - \frac{1}{2} \log |\mathbf{\Sigma}| - \frac{1}{2} \varepsilon' \mathbf{\Sigma}^{-1} \varepsilon, \quad (21)$$

where ε is the vector of ϵ_t , $\mathbf{\Sigma}$ is the variance-covariance matrix and $\mathbf{\Lambda}$ is the vector of parameters to be estimated. This problem means that $\hat{\mathbf{\Lambda}} = \arg \max_{\mathbf{\Lambda}} \ell(\varepsilon; \mathbf{\Lambda})$.

Both methods were applied to the return series (y_t) and to the variance series as defined in the beginning of section 2 ($\sigma_t^2 = y_t^2$). Table 2 shows the results for the GPH and EML tests. In this table, we see that the F1 contract indicates a long-memory for the variance series, but the tests are inconclusive for the return series. The results obtained for F1 contract are in line with Elder and Serletis (2008) Elder and Serletis (2008) who found that the shortest contract with an estimation window of $T^{0.5}$ exhibits long-memory with anti-persistence ($-0.5 < \zeta < 0$). The F2 contract exhibits long-memory for the variance series while the results for the return series are also inconclusive. The F14 contract does not exhibit long-memory for the return series, while the results for the variance series indicate a long-memory process (except for $n = \frac{T}{2}$ for the GPH test). Based on these results, an adjustment was made using ARFIMA for the conditional mean jointly

Table 2
GPH and EML estimation (values in brackets are p-values)

3-5 Method		Contract		
		F1	F2	F14
GPH $n = \frac{T}{2}$	return	-0.0475 [0.0004]	-0.0370 [0.0083]	-0.0259 [0.0646]
	variance	0.0741 [0.0000]	0.0902 [0.0000]	-0.0194 [0.1760]
GPH $n = T^{0.5}$	return	-0.2342 [0.0108]	-0.1848 [0.0928]	-0.0071 [0.9370]
	variance	0.1873 [0.0496]	0.2350 [0.0170]	0.2920 [0.0132]
GPH $n = T^{0.6}$	return	-0.0872 [0.1380]	-0.0471 [0.4337]	0.0702 [0.2460]
	variance	0.2429 [0.0000]	0.2792 [0.0000]	0.2715 [0.0000]
GPH $n = T^{0.8}$	return	-0.0406 [0.0739]	-0.0160 [0.5140]	-0.0239 [0.3350]
	variance	0.1686 [0.0000]	0.1679 [0.0000]	0.0678 [0.0089]
EML	return	-0.0412 [0.0000]	-0.0280 [0.0150]	-0.0011 [0.9250]
	variance	0.1144 [0.0000]	0.1156 [0.0000]	0.0600 [0.0000]

with the GARCH-class models, including the long-memory type for the short-term contracts (F1 and F2). For the long-term contract (F14) we proceed in the same manner, but consider an ARMA model for the conditional mean.

4.2 In-sample analysis

We jointly estimate the conditional mean and conditional variance processes. In terms of the conditional mean for the short-term contracts, we select the ARFIMA(r, ζ, s) order based on the minimisation of the Akaike criteria (AIC) for all combinations of values $r = s = 0, 1, 2$. The same was approach is used for the long-term contract in the selection of the ARMA(r, s) order. For the conditional volatility series, we chose the classical GARCH(1,1). In order to capture the properties of long-memory, we estimate the IGARCH(1,1). And to capture the long-memory and the asymmetry of shocks, we

estimate the FIAPARCH(1,d,1), which includes many other GARCH-class models as a special case. The fractional integrated models for volatility are estimated using 1,000 lags for truncation. We estimate three contracts (three time series), with three different models and each one with four distributions. This means that we have 36 estimations. To save space we included in the paper only the most relevant tables in the Appendix. All results are available under request. For each estimation result we have the diagnostic statistics. We use the following three tests: (i) the Ljung-Box $Q_1(M)$ and $Q_2(M)$ to test the serial correlations of the standard residuals and the squared-standardized residuals up to lag M , respectively; (ii) the LM ARCH(M) test to verify the presence of ARCH effects up to lag M and (iii) the Residual-based Diagnostic, (RDB(M)) test, for conditional heteroscedasticity up to lag M (see Tse (2002) Tse (2002)). We also present four information criteria: Akaike (AIC), Bayesian Information Criteria (BIC), Shibata and Hannan-Quinn (HQ). All diagnostic results are also available under request.

Analyzing all of the results we notice that the lowest information criteria for the F1 contract is obtained with the ARFIMA-FIAPARCH (skewed-Student distribution) model (Table 3). The F2 contract has similar results, that is, ARFIMA-FIAPARCH (skewed-Student distribution) gives the lowest information criteria. The same result is found for the F14 contract but with the Student distribution. Observing the diagnostic tests for these selected models and based on the LM-ARCH test, we do not reject the null hypothesis of no ARCH effect at the 5% significance level. Similar results can be observed with the RDB test.

Observing the fractional parameters d of all contracts, we note that they are highly significant; thus confirming the results of long-memory behaviour in Table 2. Hence, the short- and long-term contracts are long-memory processes. We also note that the F1 contract has the highest persistence among the three series and that this property decays with maturity. The fractional parameters ζ of the conditional mean are not significant at the 5% level for both the F1 and F2 contracts. These results are not different from those of the GPH method in Table 2, where their significance was inconclusive. The asymmetry

parameter γ is significant at the 5% level for the F1 contract, but not for F2 and F14 contracts. Furthermore, this parameter is negative, which means that a positive shock has a greater impact on volatility than a negative shock of the same magnitude. This is contrary to the results in the equity markets. The parameter δ is significant for all contracts. The values of δ close to unity suggest that modeling the conditional standard deviation is more relevant than modeling the variance (see Taylor (1986) Taylor (1986) and Schwert (1990) Schwert (1990) who modeled standard deviation instead of variance). The parameter ν (the tail parameter of the skewed-Student) is highly significant for the F1 and F2 contracts. It is also significant for the F14 contract (Student distribution). The ι parameter (the asymmetry of the skewed-Student distribution) was found to be significant for both short-term contracts. Since $\log(\iota) > 0$, the skewness is positive and the density is skewed to the right. The seasonal effect is significant at the 1% level and positive for spring for all contracts, making the returns higher and prices lower on average for this season.

4.3 Out-of-sample analysis

The out-of-sample forecast encompasses the period from 04 May 2012 through 03 May 2013. We compute the MSE and MAE using equations (15) and (16), respectively. This forecast exercise covers 1-, 5- and 20-days ahead corresponding to 1-day, 1-week and 1-month trading periods, respectively. Table 4 shows the results for MAE variance forecast (a similar table with the MSE results is available under request). The ARFIMA-FIAPARCH (skewed-Student) was found to be the best model to fit the data for the F1 and F2 contracts. However, according to Table 4 the ARFIMA-FIAPARCH (Student) model has lower MAE variance forecasts for the F1 contract, while the F2 contract favors the GED distribution. For the F14 contract, the ARFIMA-FIAPARCH (Student) model is the best fit and it also has the lowest error when forecasting variance (the results are almost the same for MSE).

We compute the DM^* test statistics for the variance forecasts of the most relevant cases. We test for the null hypothesis of no difference in forecast accuracy. For the F1 contract, the DM^* test rejects the null hypothesis that the forecast errors of the ARFIMA-FIAPARCH (Stu-

dent) model have the same accuracy as the ARFIMA-FIAPARCH (GED) model at the 5% significance level (except for the 20-day horizon, which has a dubious result). The rejection is strongest for the 1-day horizon and becomes weaker as the horizon increases. We also test the ARFIMA-FIAPARCH (Student) model against the ARFIMA-GARCH (Student) and ARFIMA-GARCH (skewed-Student) models. In both cases, the DM^* test rejects the null hypothesis of the same accuracy.

For the F2 contract, we test the ARFIMA-FIAPARCH (GED) model against the ARFIMA-FIAPARCH (Student), ARFIMA-GARCH (Student) and ARFIMA-GARCH (skewed Student) models. The results show that there is no difference in accuracy between the GED and Student distributions in the ARFIMA-FIAPARCH model. When compared to the ARFIMA-GARCH (Student and skewed-Student) models, the DM^* test rejects, for 1- and 5-day horizons, the same accuracy, but the result is not conclusive for 20-day horizon.

For the F14 contract, we test the ARMA-FIAPARCH (Student) model against the ARMA-FIAPARCH (skewed-Student) model and find that the results are mixed, depending on the choice of loss function. We find the same result when we test against the ARMA-GARCH (GED) model. When compared with ARMA-FIAPARCH (GED) model the DM^* test does not reject the same accuracy between both models.

Summarizing the out-of-sample results: (i) for the F1 contract ARFIMA-FIAPARCH (Student) model outperforms all of the other competing models; (ii) for the F2 contract, the ARFIMA-FIAPARCH (GED) model does not outperform the same model with Student distribution and the comparisons with other models are mixed; (iii) for the F14 contract, the ARMA-FIAPARCH (Student) model does not exhibit any clear evidence that it outperforms the alternative models.

4.4 The shale gas effect

To get insight on the effect of shale gas we split the three time series in the beginning of the low-price regime, that is May 2009. We repeat the process of estimation we did before fixing the innovations

for F1 and F2 as skewed Student distribution, and for F14 as Student distribution.

Before the low-price regime the best fitted model for F1 contract was FIAPARCH(1,d,1), for F2 and F14 contracts the best fitted model was FIGARCH(1,d,1). During the low-price regime the best fit model was the classical GARCH(1,1) in the three cases. We computed the average volatility before and during the low-price regime using the volatility estimated from fitted models. We found that for F1 contract the average volatility decreased from 59.4% to 51.4% a year. For F2 contract it decreased from 56.1% to 46.5% a year. Finally, for F14 contract it was practically the same, 24.6% to 24.4% a year.

The abundance of gas due to the shale phenomenon reduced the uncertainty the agents were facing regarding the availability of gas. And the short-term contracts reflected this exhibiting lower volatility. The long-term contract had almost no change since the uncertainty is less affected in the long future.

5. Conclusions

The measurement of volatility is of enormous importance, not only for financial agents and firms investing in real projects in long-term, but also for government regulators. The U.S. gas market has changed in many different aspects due to the abundance of shale gas. More specifically, this unconventional source of energy is attracting many firms who deal with long-term investments and face uncertainties in their cash-flows. Recently, the supply of gas from shale deposits imposed a low-price regime for all contract maturities. Moreover, shale gas has implications not only for the internal U.S. market but also for foreign markets. This is why it is important to analyse the volatility of short- and long-term contracts in the gas sector. Hence, modeling volatility related to the term structure of gas prices has policy relevance for the different classes of agents and this is the main reason motivating our analysis.

Our analysis emphasizes the volatility of short- and long-term maturity contracts for natural gas traded on the NYMEX. We jointly estimate the long-memory processes for the conditional mean (re-

turn) and variance using different types of innovations. Considering the selected models, we found that the fractional parameter for the conditional mean is not significant for short-term contracts. On the other hand, the fractional parameter for the conditional variance is significant for short- and long- maturity contracts. Furthermore, this parameter decreases according to the maturity of the contract. This means that the long-term investors are likely to suffer less due to price shocks, since such shocks are less persistent in the long-term. Our in-sample analysis suggests that the ARFIMA-FIAPARCH model with a skewed-Student distribution is the best model to use for short-term contracts, while the ARMA-FIAPARCH model with a Student distribution is better in defining the long-term contracts. The asymmetry of shocks is found to be significant only for the shortest contracts and it is negative, meaning higher volatility for positive shocks. This fact is in contrast to equity markets and should be subject to further investigation. The AR(FI)MA-FIAPARCH model also gave the lowest error for forecasting variance, but there was no single distribution that outperformed the others for all contracts. The clearest result for our out-of-sample analysis was that the ARFIMA-FIAPARCH model (contract F1) outperformed all of the other competing models. Moreover, the lowest errors for the short-term contracts were associated with the distributions other than those related to the in-sample analysis. Finally, we found that during the low-price regime (the shale gas phenomenon) the volatility of short-term contracts decreased when compared to the previous period.

This study can be extended to include the investigation of asymmetric effects of the shocks mentioned earlier and also to the direction of the structural breaks in the time series of natural gas contracts. In this case, one can use Markov switching-type volatility models or algorithms to map such breaks, and the results can be compared with our results in terms of the forecasting ability. To sum up, the findings in this study suggest that the agents dealing in the gas sector should be judicious in their choice of the model to describe the volatility of gas prices over time.

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6. Appendix

Table 3
Estimations for F1 and F2 with skewed-Student and for F14 with Student distributions (p-values in brackets)

Series	Contract F1			Contract F2			Contract F14		
	ARFIMA GARCH	ARFIMA IGARCH	ARFIMA FIAPARCH	ARFIMA GARCH	ARFIMA IGARCH	ARFIMA FIAPARCH	ARMA GARCH	ARMA IGARCH	ARMA FIAPARCH
Model	ARFIMA GARCH	ARFIMA IGARCH	ARFIMA FIAPARCH	ARFIMA GARCH	ARFIMA IGARCH	ARFIMA FIAPARCH	ARMA GARCH	ARMA IGARCH	ARMA FIAPARCH
S_1	-7.007E-4 [0.3961]	-5.830E-4 [0.4805]	-1.13E-4 [0.8941]	-2.54E-4 [0.7523]	-9.30E-5 [0.9066]	3.19E-4 [0.7984]	-4.330E-4 [0.2396]	-4.350E-3 [0.2359]	-4.110E-3 [0.2729]
S_2	1.657E-3 [0.0116]	1.648E-3 [0.0134]	2.055E-3 [0.0029]	1.599E-3 [0.0162]	1.645E-3 [0.0150]	1.778E-3 [0.0048]	1.059E-3 [0.0015]	1.057E-3 [0.0009]	1.063E-3 [0.0005]
S_3	-1.160E-3 [0.1883]	-1.068E-3 [0.2312]	-1.358E-3 [0.1371]	-5.51E-4 [0.5238]	-4.54E-4 [0.6047]	-1.058E-4 [0.1374]	-2.010E-4 [0.5329]	-1.920E-4 [0.5439]	-1.620E-4 [0.6127]
S_4	1.044E-3 [0.3443]	1.212E-3 [0.2634]	6.82E-4 [0.5236]	2.87E-4 [0.7707]	4.55E-4 [0.6400]	1.27E-4 [0.9095]	3.36E-4 [0.3309]	3.33E-4 [0.3254]	3.03E-4 [0.3770]
ϕ							0.7428 [0.0000]	0.7465 [0.0000]	0.0085 [0.9933]
θ							-0.7485 [0.0000]	-0.7525 [0.0000]	-0.0059 [0.9954]
ζ	-0.0220 [0.0567]	-0.0220 [0.0514]	-0.0188 [0.1001]	-0.0129 [0.2546]	-0.0130 [0.2401]	-0.0125 [0.0701]			2.518E-4 [0.7467]
ω	2.915E-5 [0.0000]	1.808E-5 [0.0000]	9.192E-4 [0.1532]	2.310E-5 [0.0000]	1.145E-5 [0.0000]	12.716E-4 [0.1520]	2.771E-7 [0.8503]	5.56E-8 [0.7421]	0.4921 [0.0038]
d			0.9510 [0.0000]			0.9296 [0.0000]			0.4921 [0.0038]
α	0.0803 [0.0000]	0.0969 [0.0000]	0.0000 [1.0000]	0.0644 [0.0000]	0.0785 [0.0000]	0.0000 [1.0000]	0.1800 [0.2575]	0.0189 [0.0003]	0.2280 [0.0049]
β	0.8996 [0.0000]	0.9031 [0.0000]	0.8892 [0.0000]	0.9149 [0.0000]	0.9215 [0.0000]	0.8926 [0.0000]	0.9800 [0.0000]	0.9809 [0.0000]	0.6689 [0.0000]
γ			-0.1895 [0.0130]			-0.1475 [0.0969]			-0.1358 [0.1180]
δ			1.0152 [0.0000]			0.8866 [0.0000]			1.3567 [0.0442]
ν	6.0772 [0.0000]	5.4238 [0.0000]	6.2168 [0.0000]	6.3344 [0.0000]	5.5788 [0.0000]	6.3832 [0.0000]	4.3237 [0.0000]	4.1280 [0.0000]	4.5775 [0.0000]
$\log(\iota)$	0.0591 [0.0052]	0.0622 [0.0042]	0.0618 [0.0039]	0.0651 [0.0029]	0.0699 [0.0018]	0.0677 [0.0023]			

Table 4
MAE variance forecast expressed in 10E-3 (lower errors in bold)

Model	Distribution	Contract F1			Contract F2			Contract F14		
		1-day	5-day	20-day	1-day	5-day	20-day	1-day	5-day	20-day
AR(FI)MA-GARCH	Normal	1.4271	1.4210	1.1689	1.0353	0.9903	0.8067	0.2297	0.2122	0.1873
	GED	1.2054	1.1904	0.9764	0.9755	0.9309	0.7629	0.2161	0.1975	0.1765
	Student	1.1710	1.1564	0.9512	0.9728	0.9284	0.7606	0.2779	0.2641	0.2355
AR(FI)MA-IGARCH	Skewed Student	1.1708	1.1564	0.9512	0.9728	0.9284	0.7604	0.2660	0.2518	0.2235
	Normal	3.2765	3.2590	3.2267	2.2426	2.1933	2.0868	0.4176	0.3951	0.3721
	GED	3.0203	2.9946	2.9469	2.1614	2.1148	1.9867	0.2473	0.2317	0.2054
AR(FI)MA-FIAPARCH	Student	3.3076	3.2919	3.2590	2.4078	2.3527	2.2586	0.2313	0.2151	0.1902
	Skewed Student	3.2527	3.2352	3.1989	2.3650	2.3114	2.2113	0.2309	0.2145	0.1897
	Normal	1.4751	1.4707	1.2319	1.2385	1.2080	1.0033	0.3479	0.3360	0.3115
AR(FI)MA-FIAPARCH	GED	1.0712	1.0484	0.8681	0.9423	0.8918	0.7545	0.2041	0.1847	0.1684
	Student	1.0647	1.0403	0.8614	0.9447	0.8933	0.7581	0.1983	0.1736	0.1620
	Skewed Student	1.2326	1.2139	1.0142	1.0504	1.0026	0.8365	0.1970	0.1774	0.1636