# On the Determination of a Scatter Factor for Fatigue Lives Based on the Lead Crack Concept

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**ABSTRACT:** The lead crack concept is adopted as the basis for a new fatigue lifeing method using the safe life philosophy. As part of its management strategy, a full-scale fatigue test is conducted to identify high-risk locations in the airframe, and for each item the time for retirement or structural repair is dictated by the safe life limit. A scatter factor is defined to account for the scatter in the material fatigue performance. The fatigue life variation of any given region of aircraft metallic structures is assumed to primarily correlate to the distribution of the equivalent pre-crack size of the fatigue crack initiators. By assuming that the sizes of these crack initiators are independent from each other, the present paper estimates the scatter factor by calculating safe life limit based on known growth characteristics of critical cracks.

**KEYWORDS:** Fatigue crack propagation, Aluminum alloys, Scatter factor, Airframe.

## INTRODUCTION

This paper describes a new fatigue lifeing method, which uses the lead crack concept (Molent, Barter and Wanhill, 2011) within the safe life (SL) philosophy framework. The lead crack initiates upon entry into service and leads to failure first in any given airframe location. As part of the SL management strategy, a full-scale fatigue test (FSFT) is conducted under representative loading in order to identify high-risk locations in the airframe, and for each item a safety-by-inspection schedule is defined or the time for retirement or structural repair is dictated by the SL limit (SLL). A scatter factor (SF) is established to account for the scatter in the material fatigue performance and the specific acceptable cumulative probability of failure. The FSFT is normally conducted for a period representing the desired service life times the SF. However, it is known that the current SF determination methods have an empirical basis. On the other hand, the fatigue life variation of any given region of aircraft metallic structures appears to correlate primarily to the distribution of what is known as the equivalent pre-crack size (EPS) of the fatigue crack initiators (for a certain material, spectrum, and stress level). The EPS is a measure of the crack-like effectiveness of the initiating discontinuity. Only one example of a production induced initial discontinuity was considered, namely pits induced through chemical etching, which is a common production material degreasing process. By assuming that the sizes of these crack initiators are independent from each other, the present paper intended to estimate the "true" SF of a critical location by calculating the SL based on known growth characteristics of critical cracks.

# THE LEAD CRACK CONCEPT

A fatigue lifeing framework using a lead crack concept has been developed by the Defence Science and Technology Organisation (DSTO) for metallic primary airframe components (Molent, Barter and Wanhill, 2011). Many observations at the DSTO and by other researchers have shown that approximately exponential fatigue crack growth (FCG) is a common occurrence for naturally-initiating lead cracks (i.e. those leading to first failure) in test specimens, components, and airframe structures subjected to variable amplitude load histories.

## LEAD CRACK CHARACTERISTICS

The lead cracks start to grow shortly after testing begins or the aircraft has been introduced into service. They are subject to several conditions and grow approximately exponentially with time, i.e. FCG may be represented by an equation of the form $a = a_0 e^{\lambda N}$, where $a$ is the crack size at time $N$, $a_0$ is the initial crack size (or EPS), and $\lambda$ is a growth rate parameter that includes the geometrical factor $\beta$. The conditions are the following:

- Little error is made when assessing the EPS of the fatigue-initiating discontinuity. An underestimate or overestimate of the discontinuity effectiveness will cause a small temporary departure from an exponential trend near the commencement of FCG.
- The crack does not grow into an area with significant thickness change, particularly if the crack length/depth is small compared to the specimen or component thicknesses or widths.
- The crack is not unloaded, either by the cracked area losing stiffness and shedding load to other sites of the specimen, component or structure; or because it grows primarily under bending loads and it approaches a neutral axis or develops away from an externally induced stress concentration, such as a pin-loaded hole in a multi-pin loaded joint.
- The crack does not find a significantly changing stress field by growing into or away from an area containing residual stresses.
- FCG is not retarded by infrequent very high loads (usually in excess of 1.2 times the peak load in the load history).
- The small fraction of FCG life influenced by quasi-static fracture close to final failure is ignored.

Within the bounds of these conditions, observations from the DSTO about the formation, growth, and failure of lead cracks have led to various deductions (Molent, Barter and Wanhill, 2011), including:

- For a given material, spectrum and item, the $\lambda$ parameter of the exponential equation, i.e. the slope of the crack growth curve shown in Fig. 1, is approximately a constant.
- Typical initial discontinuity sizes of AA7050-T7451 are approximately equivalent to a 0.01 mm deep fatigue crack (Molent, Sun and Green, 2006; Molent and Barter, 2007). In other words, this is a good starting point for FCG assessment (Fig. 1). This value is below the smallest initial flaw/crack size — the equivalent initial flaw size (EIFS) — usually assumed in the damage tolerant method (USAF, 1974).
- The metallic materials used in highly stressed areas of a high-performance aircraft have typical critical crack depths of about 10 mm (Molent, Sun and Green, 2006; Molent and Barter, 2007).

## EQUIVALENT FULL LIFE

As mentioned, it is usual to conduct a FSFT for a specified period representing a multiple of the required service life and then to perform a residual strength test – RST (Molent *et al.*, 2009a; 2009b). If the test article survives the residual strength loading, typically the equivalent service hours (ESH) achieved at the end of the test will be divided by a SF to result in the SL.

However, this may be a lower bound SL, as in many cases the FSFT may have been capable of sustaining further fatigue cycling and would still have survived RST. The lead crack concept provides simple means of estimating the elapsed ESH when the test article would have just met the RST criterion (i.e. equivalent full life) as illustrated in Fig. 1. For illustration, assume that the FCG data shown in Fig. 1 represent the termination of a FSFT, when a nondestructive inspection (NDI) detectable crack ($a_{NDI}$) was found in a principle component at approximately 5,000 ESH. The critical crack size under the RST conditions can be determined through conventional fracture mechanics ($a_{RST}$). The lead crack concept can then be used to extrapolate from $a_{NDI}$ to $a_{RST}$, estimating an equivalent full life of approximately 7,000 ESH, as seen in Fig. 1. Thus, by using the lead crack concept, an optimal SL can be achieved, even if limited FSFT cycling has occurred. This technique has already been used at the DSTO (see for example Molent *et al.*, 2009b).

## EQUIVALENT PRE-CRACK SIZE

Production aircraft components and structures often have many sources of surface or near-surface discontinuities capable of initiating fatigue cracking. These include various forms of machining damage (e.g. scratches, grooves, burrs, small tears, and nicks), etch pits from surface treatments (e.g. pickling, pre-anodising), porosity (especially in thick aluminum alloy plate and castings), and in the case of aluminum and steels, constituent particles that may themselves be cracked. The procedure for predicting the lead crack depth at any given life and location in the airframe and subsequently calculating its SL depends on an accurate measure of the crack-like effectiveness of the initial discontinuities for the locations and areas to be assessed, as what is known as the EPS or $a_0$. The EPS is a recent development, and its values for some defect types are approximations to a physical measurement of the initiating flaw or discontinuity derived via back projection to time zero from experimental FCG data. Therefore, it should not be compared to the more traditional EIFS concept (Molent, Sun and Green, 2006). The fatigue life variation of any given region of metallic structure appears to correlate primarily to the distribution of the EPS of these fatigue initiators for a given material, spectrum, and stress level (Pell, Molent and Green, 2004).

A previous investigation into fatigue crack defect types and EPS in commercial 7050-series aluminum alloy subjected to various F/A-18 aircraft spectra loading considered over 360 cracks in full-scale structural components and coupon test specimens. Note that this is the EPS distribution for the discontinuities present at lead cracks, i.e. not the EPS distribution for say, all etch pits initiated cracks in one specimen. The defect descriptions and EPS results were obtained from quantitative fractography (QF) data. Comparisons between the coupon specimen and the test article specimen from EPS distributions with same surface conditions verified that the former are representative of typical flaw sizes in service aircraft for each surface finish considered. It was also noted that the log-normal distribution approximates well the EPS population, i.e. the $a_0$ values (Molent, Sun and Green, 2006). Taking into account the lead crack exponential growth behavior and assuming that the $a_0$ population follows the log-normal distribution, then the life to grow a crack to a given size, $N$, must follow the normal distribution. This is not in accordance with the hypothesis assumed in a recent work (Underhill and DuQesnay, 2008), which $N$ follows the log-normal distribution. However, this apparent inconsistency can be explained by the fact that when
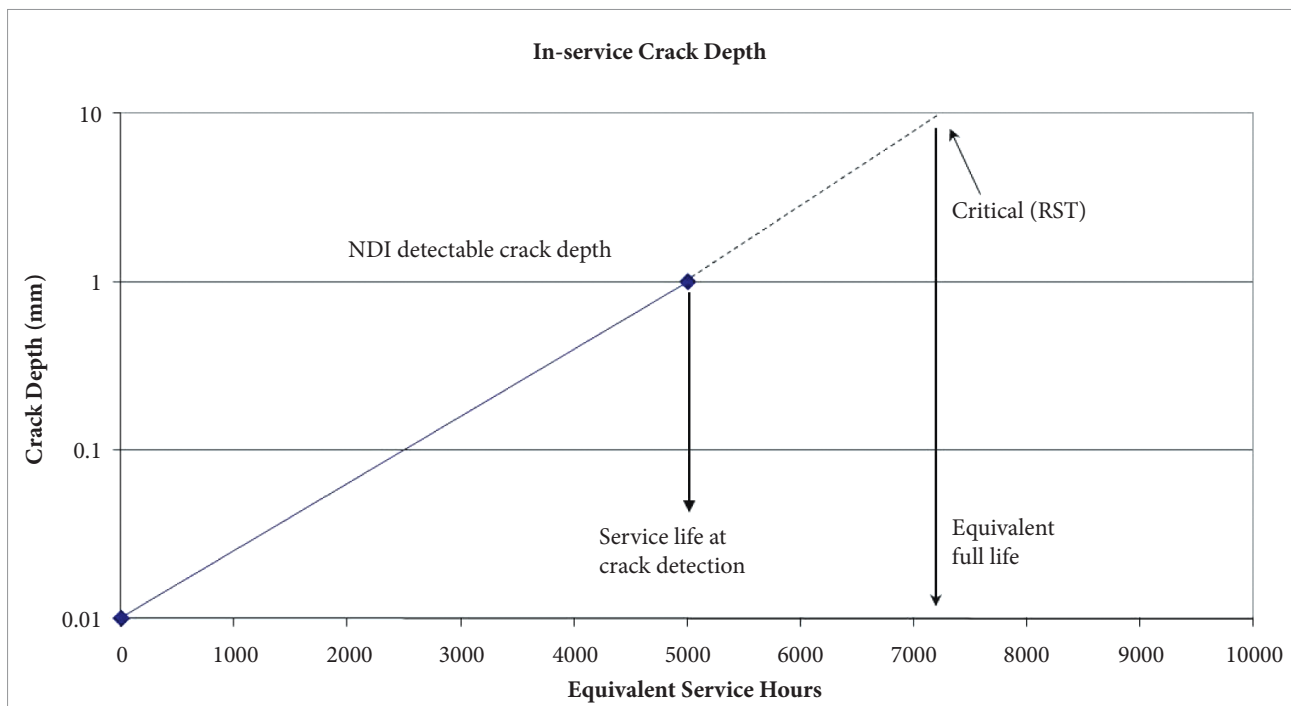


**Figure 1.** Schematic of the lead crack growth showing the crack depth *versus* time history for blocks of loading, the limit of crack detection (nondestructive inspection), and the critical crack depth for the required residual strength – RST (critical RST).

a normal distribution (μ, σ) has an arithmetic mean μ that is much higher than its arithmetic standard deviation σ, which is the case of the typical airframe fatigue lifetimes involved, the distribution tends to look log-normal.

## BACKGROUND ON SCATTER FACTORS

The SLL corresponds to the maximum life of each location, while maintaining an acceptable safety level. For example, DEFSTAN 00-970 defines the SLL of an item as the life at which the weakest example only retains the required standard of strength, deformation, stiffness, or mechanical function under 80% of the design ultimate load (British Ministry of Defence, 1983).

A SF is defined to account for scatter in the material fatigue performance and build-quality variations (British Ministry of Defence, 1983). The SF is a factor that the test demonstrated fatigue life must be reduced by to ensure that the failure probability does not exceed a specified level. A large number of fatigue scatter estimates for aluminium alloy aircraft structures is available in literature (Hoffman and Hoffman, 2001; Forgues, 1996; Parish, 1965; Stagg, 1969; Benoy, 1981; Cardrick and Mew, 1999; Impellizzeri, 1966; Young and Ekvall, 1981; Cardrick, 2008). It was verified that some of the differences found in the fatigue scatter estimates may be related to the varied structural details tested, the different surface treatments of batches of material that were tested, the introduction of artificial crack starters, surface residual stresses and whether the cracks were surface or sub-surface initiated. As many of the results quoted are total lives (FCG was not investigated), therefore the individual effects of these influencing factors are masked or unclear.

Moreover, care needs to be exercised with the figures presented in the literature to determine whether the scatter estimate relates to individual items or whole components. An important consideration here is the area effect. Some results determined through small coupon testing may not reveal the lead crack, because the probability of having a discontinuity at the optimum locations to grow a lead crack is much reduced compared to the whole component. Therefore, the current SF determination methods appear to have a weak basis. The present paper intended to contribute to the estimation

of a "true" SF by calculating SLL based on known growth characteristics of critical cracks.

## DEVELOPMENT

In this work, it is assumed that the major contributor to the scatter in the fatigue performance of monolithic metallic structure like airframes is governed by variability in the metal fatigue resistance and manufacturing quality. These features can be quantified through the variability in: the initial discontinuity that leads to fatigue cracking; time to crack initiation; variations in local stress concentration due to manufacturing tolerances; fit-up or residual stresses; the fracture toughness, and the FCG rate.

Except for the latter, these items define the aircraft build quality from a fatigue perspective. Indeed, specifically for monolithic structures, the effects of fit-up or residual stresses are minimal. It is known (e.g. from White, Molent and Barter, 2005) that the most significant variable for a given loading is the initiating discontinuity. Furthermore, for lead cracks, the initiation periods are negligible and conservatively ignored in the following analysis (note that fretting induced cracking is an exception to this). Also, modern aircraft like the F/A-18 are built to exacting standards and close tolerance so that for a specific location the variability of a local stress concentration between aircraft is negligible. It was also shown by White, Molent and Barter (2005) that variation in fracture toughness had little effect on the probability of failure.

In this paper, the EPS relevant to etched aluminium alloy 7050-T7451 surfaces from F/A-18 Hornet test airframes (Molent, Sun and Green, 2006) were considered. Many aircraft components are chemically etched prior to the application of a corrosive preventative treatment (e.g. anodised or ion-vapour deposited pure aluminium). The etching produces pitting that readily initiates the growth of cracks.

The individual EPS values from the crack leading to failure in representative full-scale tests are determined through using back-extrapolation of the FCG data (i.e. crack depth *versus* load blocks or flight hours) obtained through QF. Generally, the extrapolation from the first identified fracture surface progression mark to the start of fatigue was less than 0.1 mm. For the initial discontinuity types considered by Molent, Sun and Green (2006), it was shown that the EPS distribution was

independent of spectra and stress levels. Only the etched-out pit EPS distribution from 97 cracks examined from full-scale test articles were considered in the present analysis.

Knowing the critical crack size $a_{cr}$ (e.g. from fracture mechanics considerations or test) and using the exponential growth equation, the fatigue lifetime Nv is determined, corresponding to the number of blocks/flight hours for a lead crack to grow from $a_0$ to $a_{cr}$. The SL is the fatigue lifetime divided by SF, i.e.: SL = Nv/SF. Generally, more than one fatigue prone area will exist and each will have its own SL, which can be estimated. For the purpose of this paper, the slope of the crack growth curve, λ, assumed to be a constant for a given part location/material. Therefore, the problem of fatigue scatter is reduced to the $a_0$ (EPS) distribution. The keystone is that the initial discontinuity size can be assumed as an independent random variable. A fundamental hypothesis of the present work is that the EPS is derived from small (sub-millimeter) discontinuities introduced randomly during the processes of material and component productions.

The histogram with normal natural logarithm distribution of the EPS from the 97 cracks in the test articles is shown in Fig. 2. These data were tested for the hypothesis that $a_0$ follows the log-normal distribution. The obtained results, shown in Fig. 3, present a p-value corresponding to 0.623, meaning that there is no evidence to refute this hypothesis based on the collected data. Therefore, it is assumed in this paper that $a_0$ follows the distribution given by Eq. 1:

$$a_0 \sim log\ normal\ (\mu_0, \sigma_0^2) = log\ normal\ (-4.428, 0.957^2) \quad . \quad (1)$$

The starting point for the method is that every crack nucleated under fatigue conditions shows the potential to be the lead one of a certain item, and only one is identified in post-mortem analysis.

Let us now assume that the number $M$ of nucleated cracks, which possibly develop into a lead crack in a given component, follows Poisson's distribution, i.e. $M$ is a random variable with a mean value equals to μ. The approximation of the number of initial cracks in a given component by a punctual Poisson's process also can be justified by the assumed hypothesis that the small initial discontinuities are randomly introduced in that location, and they are not due to a systematic factor.

By using the known EPS distribution, a critical value for $a_0$ is defined as the initial crack size having a determined probability of occurrence among the $M$ nucleated cracks, which possibly develop into a lead crack. Thus, the probability that none of the $M$ initial cracks in a certain location have an $a_0 > a_{0c}$ (where $a_{0c}$ is the critical value of $a_0$) is determined as follows: assuming that the initial cracks are independent from each other (hypothesis of the independence of $a_0$), and considering $M$ as a random variable modeled by Poisson's process, the required probability is calculated according to Gumbel (2004), conditioned to the occurrence of a specific $M$ value. This is shown by Eq. 2:

$$P(None\ a_0 > a_{0c}) = \sum_{i=0}^{\infty} P(None\ a_0 > a_{0c}/M=i) \cdot P(M=i) =$$
$$\sum_{i=0}^{\infty} \left( \exp(-ip) \left( \frac{\exp(-\mu)\mu^i}{i!} \right) \right) \quad (2)$$

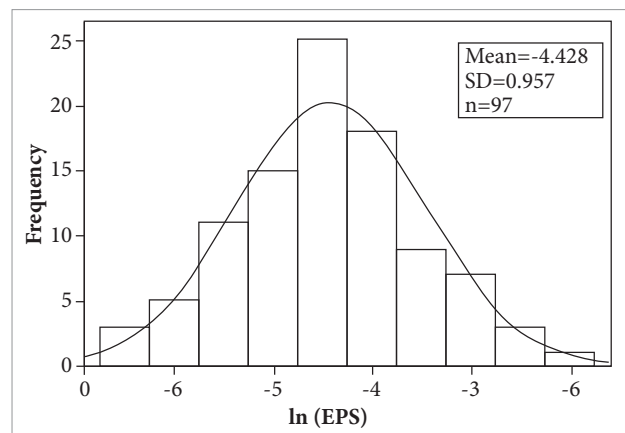where $p$ is the probability that one initial crack has $a_0 > a_{0c}$, and $p$ is calculated as shown.



**Figure 2.** Normal distribution of the ln (equivalent pre-crack size – EPS) of 97 cracks in test articles (Molent, Sun and Green, 2006).
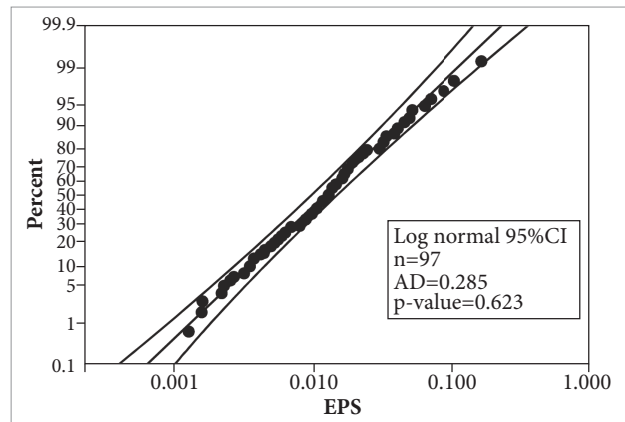


**Figure 3.** Probability plot of the equivalent pre-crack size (EPS) of the 97 cracks from test articles.

By developing Eq. 2, we obtain Eq. 3:

$$P(None\ a_0 > a_{0c}) = exp(-\mu) \sum_{i=0}^{\infty} \left( \frac{(exp(-p)\mu)^i}{i!} \right) = exp(exp(-p)\mu - \mu) \qquad (3)$$

Thus, from a preestablished level of confidence (for instance, 0.999), a critical value of $a_0$ can be determined (i.e. the $a_{0c}$ meaning that, in 0.1% of the situations, the $a_0$ of a crack initiated in a critical location will be higher than $a_{0c}$). Therefore, for the probability of none of the initial cracks have $a_0 > a_{0c}$ we choose the value P(None $a_0 > a_{0c}$) = 0.999. Then, from Eq. 3, we have ln(0.999) = exp(-p)μ-μ, or as in Eq. 4:

$$exp(-p) = \left( \frac{\mu + \ln(0.999)}{\mu} \right) . \qquad (4)$$

The expression for p calculus is from Eq. 5:

$$p = \ln \left( \frac{\mu}{\mu + \log(0.999)} \right) . \qquad (5)$$

With the calculated value for p and using the log-normal distribution shown in Fig. 3, the $a_{0c}$ value was determined. With this $a_{0c}$ and taking the appropriate λ for the considered critical location, we grow $a_{0c}$ to $a_{crit}$ to provide $N_{ext}$, which is a SL estimation corresponding to this critical initial crack size. Thus, the "true" SF for the established probability is given by Eq. 6:

$$SF = \frac{Nv}{N_{ext}} . \qquad (6)$$

## NUMERICAL EXAMPLE

In order to run a numerical example, it must be recognized that critical locations (or parts) having the same EPS distribution can show distinct amounts of nucleated cracks. In other words, although the EPS distribution is location independent for a given material/manufacturing process, the mean number of nucleated cracks, μ, that possibly develop into a lead crack is assumed to be location dependant, due to the spectrum/stress/environmental history of each site.

Considering the EPS distribution of the 97 cracks from test articles (Fig. 3) and a 0.999 confidence level, the $a_{0c}$ value depends exclusively on μ. For instance, if the average number of nucleated cracks in a given critical location is μ = 100, we

have $a_{0c}$ = 0.707 mm. For the adopted confidence level, it is expected that, for each 1,000 parts presenting this behavior (i.e., having μ = 100), one, in average, will show $a_0 > a_{0c}$. This is illustrated in Fig. 4, which shows a simulation of the maximum $a_0$ values for 1,000 similar parts.

In the numerical example it will be considered the growth of a typical crack from a F/A-18 full scale fatigue test as shown by Molent and Barter (2010), with $a_0$ = 0.011 mm, λ = 0.0004 and equivalent RST = 16,000 flight hours. Table 1 shows the calculated $a_{0c}$, SL and "true" SF values corresponding to several μ values ranging from 1 to 1,000. Therefore, if the average number of nucleated cracks in the considered critical location is μ = 100,



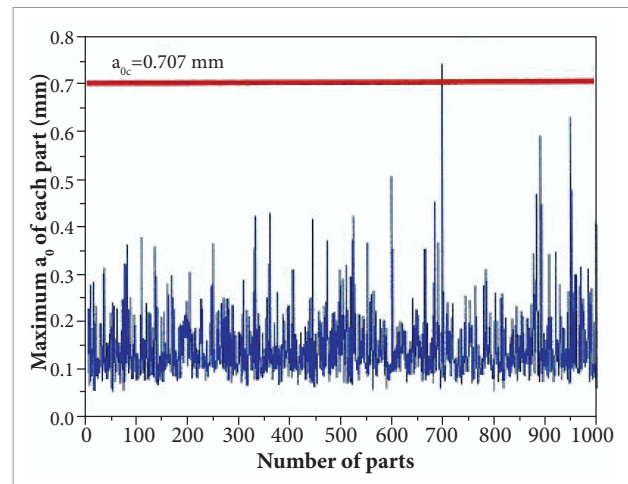**Figure 4.** A computer simulation of the maximum $a_0$ values for each of 1,000 similar parts.

**Table 1.** Calculated $a_{0c}$, safe life (SL), and "true" scatter factor (SF) values corresponding to several μ ones ranging from 1 to 10,000 for confidence level of 0.999

| μ | $a_{0c}$ (mm) | SL (Flight Hours) | SF |
|---|---|---|---|
| 1 | 0.2297 | 9,433.79 | 1.782 |
| 3 | 0.3099 | 8,685.38 | 1.936 |
| 5 | 0.3534 | 8,357.19 | 2.012 |
| 6 | 0.3699 | 8,242.74 | 2.040 |
| 10 | 0.4194 | 7,929.05 | 2.121 |
| 100 | 0.7071 | 6,622.98 | 2.539 |
| 1,000 | 1.1285 | 5,454.14 | 3.083 |
| 10,000 | 1.9534 | 4,387.27 | 3.832 |

then for a 0.999 confidence level the "true" SF to be applied to the equivalent RST is 2.539. It is interesting to observe that, by plotting the "true" SF against the natural logarithm of μ, a linear relationship is obtained, as observed in Fig. 5.

Another important point is that, because the way it was defined, the SF does not depend on the growth rate parameter λ. In other words, even distinct loading spectra or 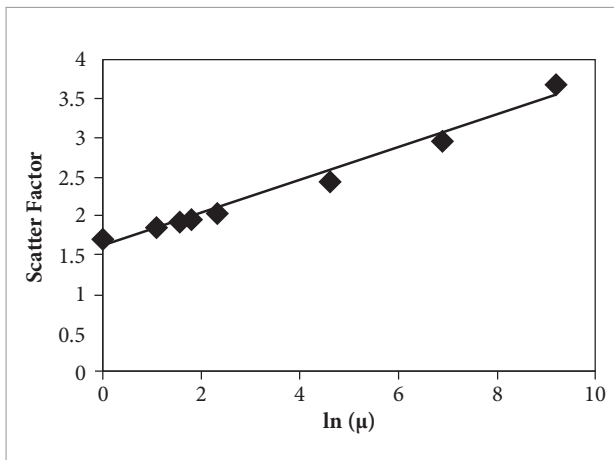crack geometry may eventually present the same "true" SF, if μ and EPS distributions are similar. This can be verified by the relationship given by Eq. 7:

$$SF = \frac{\frac{1}{\lambda}\ln\left(\frac{10}{a_0}\right)}{\frac{1}{\lambda}\ln\left(\frac{10}{a_{0c}}\right)} = \frac{\ln\left(\frac{10}{a_0}\right)}{\ln\left(\frac{10}{a_{0c}}\right)} \tag{7}$$



**Figure 5.** Plot of the "true" scatter factor estimation against the natural logarithm of μ showing a linear fashion.

## CONCLUSIONS

By assuming that the scatter in a material's fatigue performance is mainly governed by equivalent crack-like size distribution of initiating discontinuities, the present paper illustrated a method of estimating a "true" SF, with a confidence level and average number μ of nucleated cracks in an aluminium alloy 7050-T7451 critical airframe location.

The presented results illustrate how the developed method makes note that, even for similar parts, the average initial crack size for a given confidence level increases with the number of nucleated cracks, and thus so does the "true" SF. Therefore, the adoption of a SF for a given part should also take into account the average number of nucleated cracks.

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