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ANALYSIS OF PRECIPITATION TIME SERIES USING THE WAVELET TRANSFORM

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ABSTRACT

The water erosion in semiarid region of northeastern Brazil is mainly influenced by the intense rainfalls, which can occur during long periods of drought, when a large amount of sediment yield can be observed during such events. Thus, this paper aims to study the rainfall frequencies in such region through the wavelet transform.

An application of wavelet analysis is done with long time series of the total monthly rainfall amount of several places from Paraíba state in northeastern Brazil. The main frequency components in the time series are studied by the global wavelet spectrum and the modulation in separated periodicity bands were done in order to extract additional information, e.g., the 8 and 16 months band was examined by an average of all scales, giving a measure of the average annual variance versus time, where the periods with low or high variance could be identified. In Taperoá II and Bodocongó raingages, important reductions were observed in the average variance for the following periods: 1998 to 2000 and 1993 to 1997, which can be considered as dry periods. Although, the precipitation in Taperoá II and Bodocongó raingages showed a similar global wavelet spectrum, their wavelet spectrum revealed particular features. This study can be considered an important tool for time series analysis, which can help the studies concerning soil erosion and land degradation, mainly when they are applied together with runoff-erosion simulations.

Keywords: rainfall data, wavelet transform, water erosion

INTRODUCTION

An application of wavelet analysis is done with the total monthly rainfall amount of several places from Paraíba state in northeastern Brazil. The water erosion in semiarid region of northeastern Brazil is mainly influenced by the intense rainfalls, specially when they occur during long periods of drought because the soil moisture content is low and, then, the erosion within the basin tends to increase.

Wavelets are versatile tools for harmonic analysis. Due to their many uses, the word 'wavelet' comes with different connotations to users in different fields. The wavelet transform is a recent advance in signal processing that has attracted much attention since its theoretical development in 1984 by Grossman and Morlet (1984). Its use has increased rapidly as an alternative to the Fourier Transform (FT) in preserving local, non-periodic, multiscaled phenomena. It has advantage over classical spectral analysis, because it allows analyzing different scales of temporal variability and it does not need a stationary series. Thus, it is appropriated to analyze irregular distributed events and time series that contain nonstationary power at many different frequencies. Then, it is becoming a common tool for analyzing localized variations of power within a time series. Several applied fields are making use of wavelets such as astronomy, acoustics, data compression, nuclear engineering, sub-band coding, signal and image processing, neurophysiology, music, magnetic resonance imaging, speech discrimination, optics, fractals, radar, human vision, pure mathematics, and geophysics such as tropical convection, the El Niño-Southern Oscillation, atmospheric cold fronts, temperature variability, the dispersion of ocean waves, wave growth and breaking, structures in turbulent flows, and stream flow characterization (Santos *et al.,* 2001).

Meteorological forecasts with reasonable skill offer useful information of rainfall estimation with antecedence ranging from some days to one year. Thus, this study can be considered an important tool for time series analysis, especially when used in rainfall forecasts, which can help the studies concerning forecast of soil erosion and land degradation. The following sections describe the wavelet transform, the selected rainfall data of northeastern Brazil, and then the application of the wavelet transform.

WAVELET TRANSFORM

There are several mathematical transformations that can be applied, among which the Fourier transforms are probably by far the most popular. However, the wavelet transform can be used to analyze time series that contain non-stationary power at many different frequencies (Torrence & Compo, 1998), because the wavelet analysis maintains time and frequency localization in a signal analysis by decomposing or transforming a one-dimensional time series into a diffuse two-dimensional time-frequency image, simultaneously. Then, it is possible to get information on both the amplitude of any "periodic" signals within the series, and how this amplitude varies with time. Examples of basic waves or mother wavelets, as they are known in the literature, are shown in **Fig. 1**. These mother wavelets have the advantage of incorporating a wave of a certain period, as well as being finite in extent.

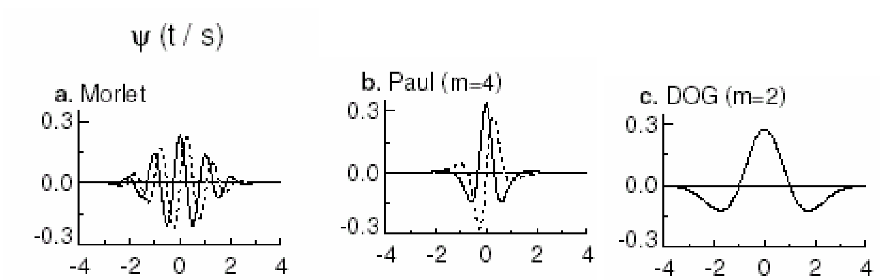

Fig. 1. Mother wavelets: (a) Morlet, real and imaginary parts in solid and dashed lines, respectively; (b) Paul, with *m=4*; (c) DOG – Derivative of a Gaussian, with *m=2*.

The wavelet analysis always uses a wavelet of the exact same shape, only the size scales is up or down with the size of the window. In addition to the amplitude of any periodic signals, it is worth to get information on the phase. In practice, the Morlet wavelet shown in **Fig. 1a** is defined as the product of a complex exponential wave and a Gaussian envelope:

$$\Psi_0(\eta) = \pi^{-1/4} e^{i\omega_0 \eta} e^{-\eta^2/2} \qquad (1)$$

where $\Psi_0(\eta)$ is the wavelet value at nondimensional time η and ω_0 is the nondimensional frequency, equal to 6 in this study in order to satisfy an admissibility condition; i.e., the function must have zero mean and be localized in both time and frequency space to be "admissible" as a wavelet. This is the basic wavelet function, but it will be now needed some way to change the overall size as well as slide the entire wavelet along in time. Thus, the "scaled wavelets" are defined as:

$$\Psi\left[\frac{(n'-n)\delta t}{s}\right] = \left(\frac{\delta t}{s}\right)^{1/2} \Psi_0\left[\frac{(n'-n)\delta t}{s}\right] \qquad (2)$$

where s is the "dilation" parameter used to change the scale, and n is the translation parameter used to slide in time. The factor of $s^{-1/2}$ is a normalization to keep the total energy of the scaled wavelet constant. We are given a time series X, with values of x_n, at time index n. Each value is separated in time by a constant time interval δt. The wavelet transform $W_n(s)$ is just the inner product (or convolution) of the wavelet function with the original time series:

$$W_n(s) = \sum_{n'=0}^{N-1} x_{n'} \Psi * \left[\frac{(n' - n)\delta t}{s} \right] \tag{3}$$

where the asterisk (*) denotes complex conjugate.

By sliding this wavelet along the time series, a new time series of the projection amplitude versus time can be constructed. Finally, the "scale" of the wavelet can be varied by changing its width. Technical details used in different mother wavelets can be seen in appropriate literature (e.g., Torrence & Compo, 1998).

RAINFALL DATA

The northeastern Brazil has an area of 1.552.619,2 km^2 (18.28% of the Brazilian territory and it is divided practically into three great zones: coast, rural and interior. The two last ones form the semiarid region, which is denominated as the "Polygon of Droughts" embracing 70% of the northeast (1.086.833,44 km^2 – 13% of Brazil). In climatic terms, northeastern Brazil can be considered as a complex area, not due to the variation in the temperatures, but for the variation in the rainfalls. The medium temperatures vary between 23 °C and 27 °C, with minimum temperatures during Winter (5 °C to 10 °C) and maximum ones during Summer (30 °C to 40 °C). The mean precipitation is normally between 500 and 600 mm/y, and just in the coastal area and in the west of Maranhão State, where the precipitation is above 1,000 mm/y. In the center area of the "Polygon of Droughts" the precipitation varies from 200–250 mm/y to 800–900 mm/y in the high lands, during a period of three to four months, then the dry season lasts from eight to nine months, in normal times.

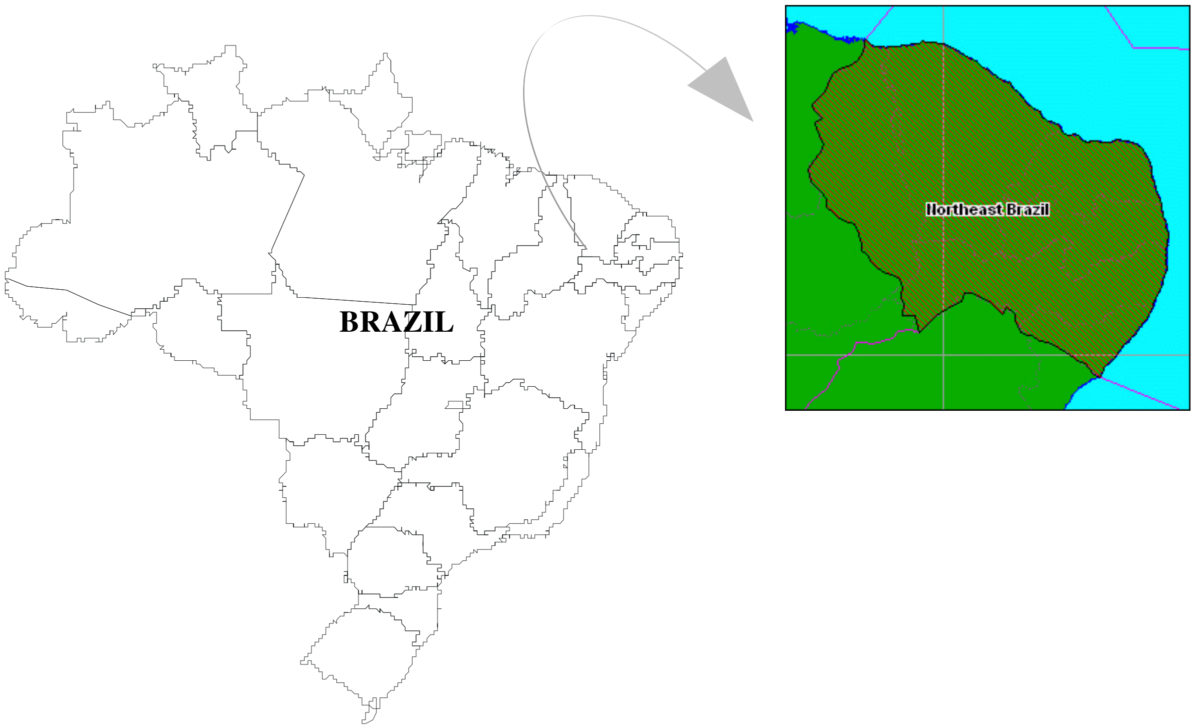

Figure 2. Location of the studied area in northeastern Brazil.

It was selected eleven raingages within the studied area: Antenor Navarro, Aparecida, Balanças, Barra do Juá, Belém do Brejo do Cruz, Brejo do Cruz, Bodocongó, Sousa, São Vicente, Taperoá II, Uiraúna within Paraíba state.

DATA ANALYSIS

Wavelet power spectrum

Since the present data are monthly distributed, the parameters for the wavelet analysis are set as δt = 1 month and s_0 = 2 months because $s = 2\delta t$, δj = 0.25 to do 4 sub-octaves per octave, and $j_1 = 7/\delta j$ in order to do 7 powers-of-two with δj sub-octaves each. **Fig. 3b** shows the power (absolute value squared) of the wavelet transform for the monthly rainfall in Taperoá II raingage presented in **Fig. 3a**. As stated before, the (absolute value)^2 gives information on the relative power at a certain scale and a certain time. This figure shows the actual oscillations of the individual wavelets, rather than just their magnitude. Observing **Fig. 3b**, it is clear that there is more concentration of power between the 8–16-month band, which shows that this time series has a strong annual signal. Classical statistical analysis applied by previous authors for this area has mentioned the existence of important low frequency peaks. Here, we show that such results are misleading as no significant peaks were attained for low frequency periods. However, wavelet power spectrum for precipitation episodes with characteristic scale of 8–16 months presents an important peak, almost significant at the 5% level.

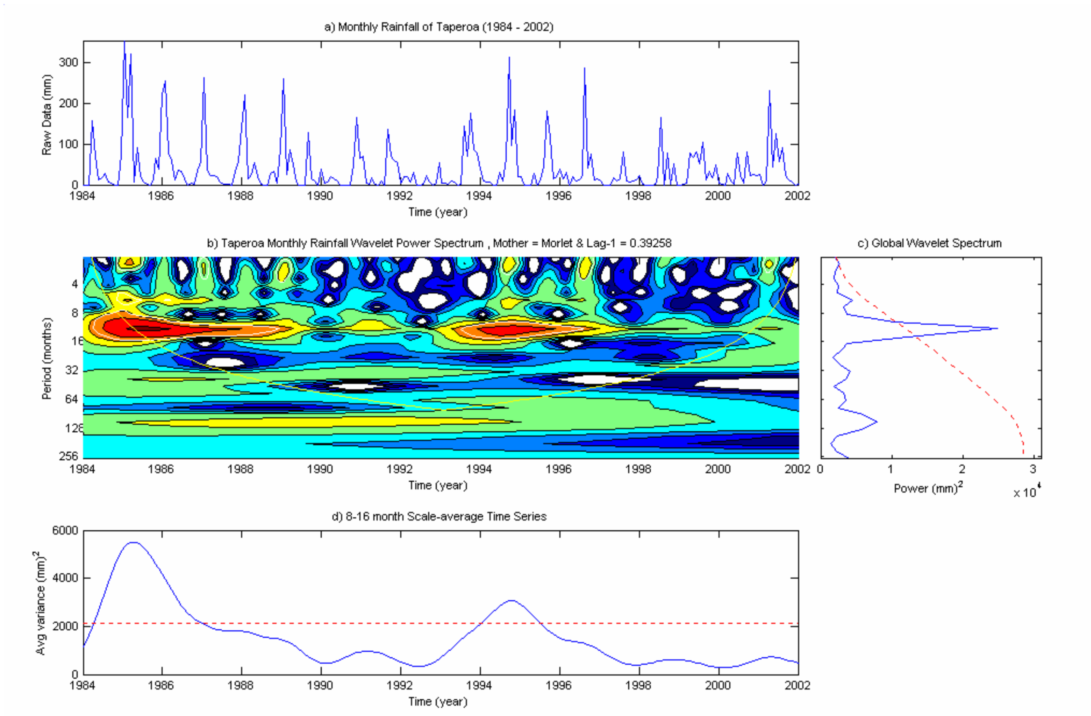

Figure 3. (a) Monthly rainfall in Taperoá II raingage for 1984-2002 period. (b) The wavelet power spectrum using Morlet mother wavelet. Cross-hatched region is the cone of influence, where zero padding has reduced the variance. (c) The global wavelet power spectrum. The dashed line is the 5% significance level for the global wavelet spectrum; and (d) Scale-average wavelet power over the 8-16 months band. The dashed line is the 95% confidence.

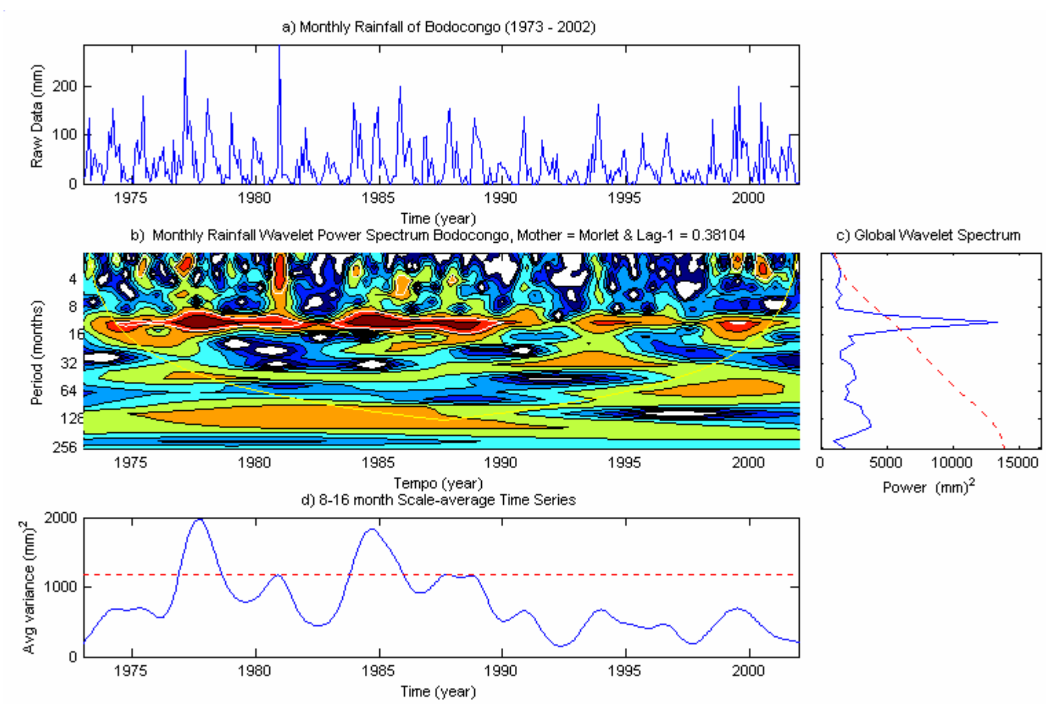

Figure 4. (a) Monthly rainfall in Bodocongo raingage for 1984-2002 period. (b) The wavelet power spectrum using Morlet mother wavelet. Cross-hatched region is the cone of influence, where zero padding has reduced the variance. (c) The global wavelet power spectrum. The dashed line is the 5% significance level for the global wavelet spectrum; and (d) Scale-average wavelet power over the 8-16 months band. The dashed line is the 95% confidence.

At the border of the wavelet power spectra exist a zone called the cone of influence, where zero padding has reduced the variance. Since we are dealing with finite-length time series, errors will occur at the beginning and end of the wavelet power spectrum (Santos *et al.*, 2001).

Global wavelet power spectrum

The annual frequency (periodicity at 12 months) of this time series is confirmed by an integration of power over time (**Figs 3c** and **4c**), which shows only one significant peak above the 95% confidence level for the global wavelet spectrum, assuming $\alpha = 0.39258$ represented by the dashed lines. However, **Figs 3c** and **4c** also present an almost significant peak (at the 5% level) centered in the 2–4-months band. In fact, most extreme monthly precipitation values for Taperoá II (values above 300 mm in **Fig. 3a**) correspond to pulses of highly significant power within the 8–16-months band (**Fig. 3b**). This global wavelet spectrum provides an unbiased and consistent estimation of the true power spectrum of the time series, and thus it is a simple and robust way to characterize the time series variability. Global wavelet spectra should be used to describe rainfall variability in non-stationary hyetographs. For regions that do not display long-term changes in hyetograph structures, global wavelet spectra are useful for summarizing a region's temporal variability and comparing it with rainfall in other regions. The global wavelet spectral shape is controlled primarily by the distribution of feature scales.

Scale-average time series

The scale-average wavelet power (**Figs 3d** and **4d**) is a time series of the average variance in a certain band, in this case 8–16-month band, used to examine modulation of one time series by another, or modulation of one frequency by another within the same time series. This figure is made by the average of **Figs 3b** and **4b** over all scales between 8 and 16 months, which gives a measure of the average year variance versus time. The variance plot shows distinct periods when monthly rainfall variance was low in Taperoá II (**Fig. 3d**), e.g., from 1998 to 2000, and an important peak in the scale-average time series can be identified for 1984 to 1986, clearly indicating a period wetter than normal years. **Figure 5** shows the scale-average wavelet power for the other raingages in Paraíba state, where high and low variance periods can be identified.

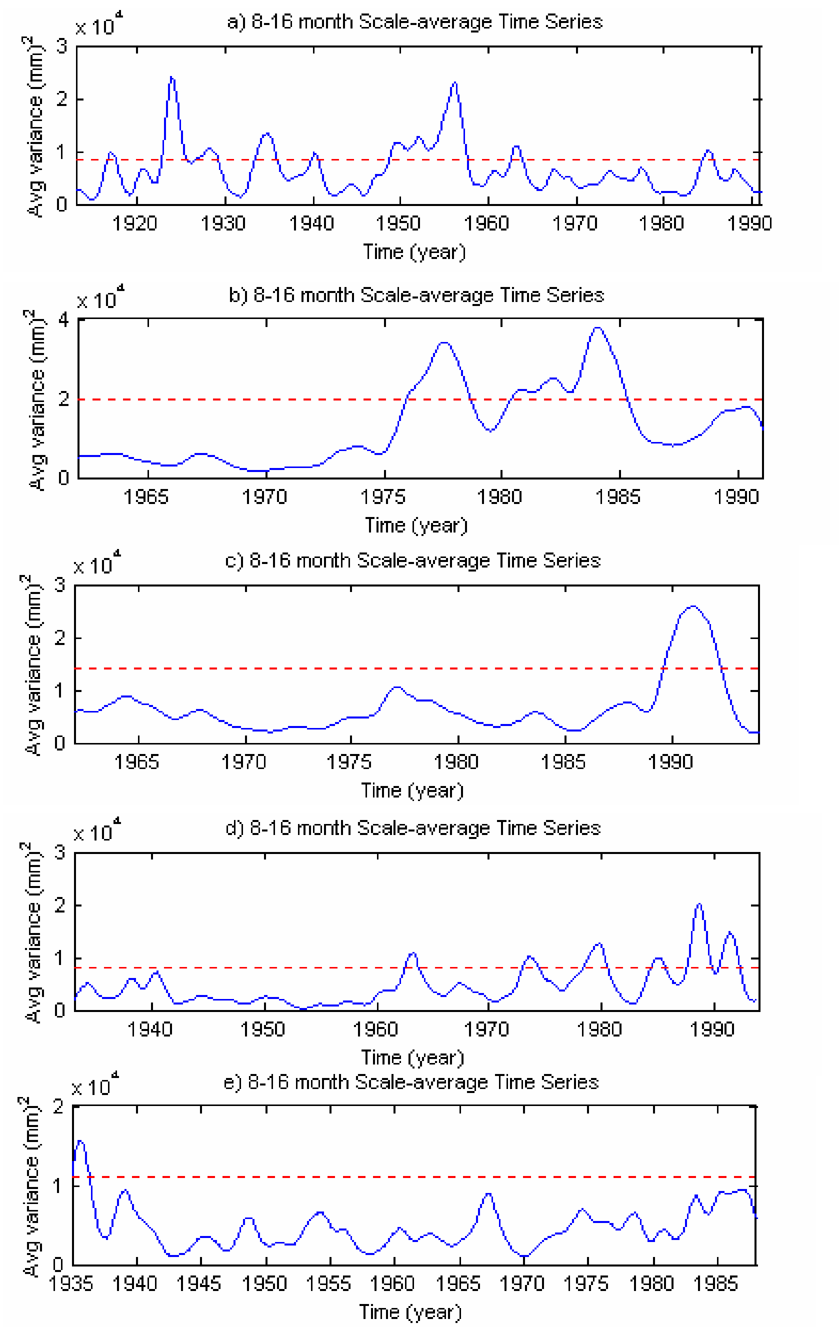

Figure 5. Scale-average wavelet power over the 8–16 month bands for (a) Antenor Navarro, (b)Aparecida, (c) Balanças , (d) Barra do Jua, (e) Belem do Brejo do Cruz. The dashed lines are the 95% confidence level.

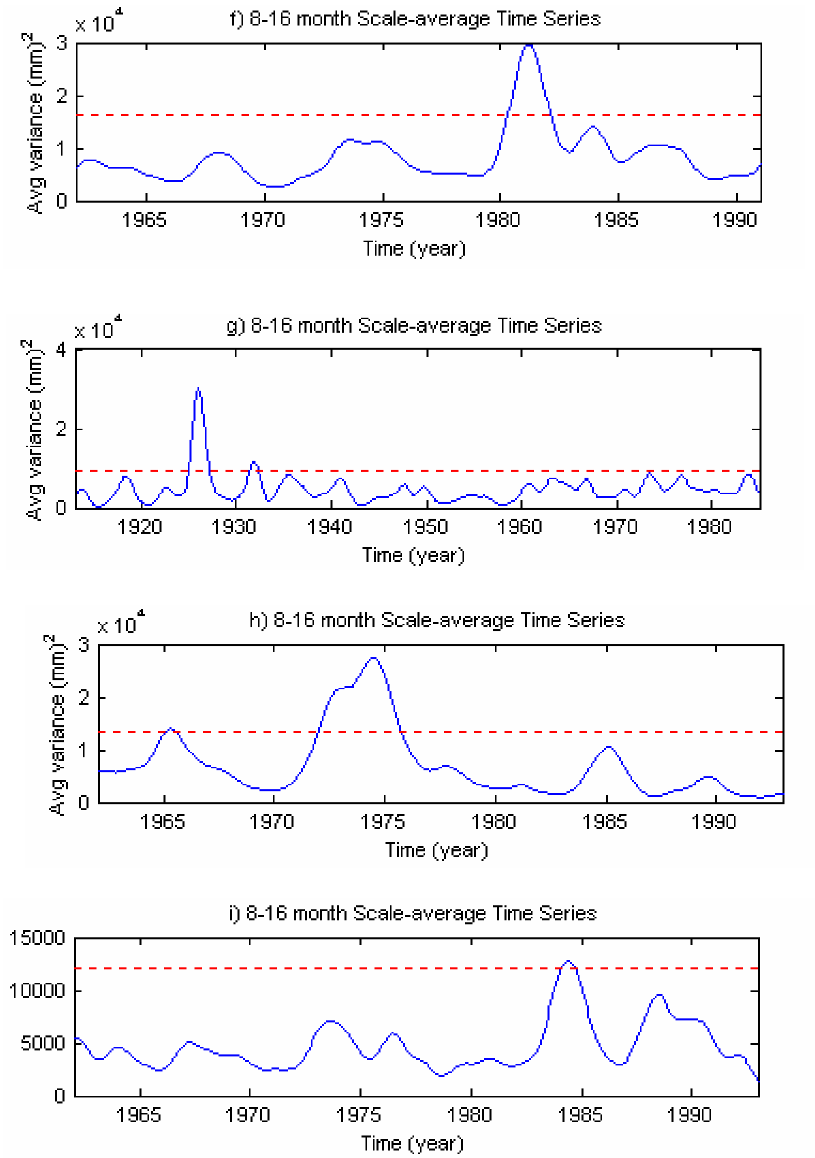

Figure 5 continuation. Scale-average wavelet power over the 8–16 month bands for (f) Brejo do Cruz , (g)Sousa , (h) São Vicente, and (i) Uirauna. The dashed lines are the 95% confidence level.

CONCLUSIONS

In order to study the variability of the monthly rainfall time series of several places in northeastern Brazil, wavelet analysis was applied. The wavelet power spectra showed a big power concentration between the 8–16-month band, revealing an annual periodicity of such events, which is confirmed by the peak of the integration of transform magnitude vectors over time that show again a strong annual signal. The periods with low variance in such a band could be identified by the average of the all scales between 8 and 16 months, which gave a measure of the average monthly variance versus time. These rainfall analyses using wavelet transform for Taperoá II, Bodocongó and several other places in northeastern Brazil can benefit erosion models and can help the studies concerning soil erosion and land degradation, mainly when they are applied together with runoff-erosion simulations.

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