



Anais da Academia Brasileira de Ciências

ISSN: 0001-3765

aabc@abc.org.br

Academia Brasileira de Ciências

Brasil

Cai, Guang-Hui

On the other law of the iterated logarithm for self-normalized sums

Anais da Academia Brasileira de Ciências, vol. 80, núm. 3, septiembre, 2008, pp. 411-418

Academia Brasileira de Ciências

Rio de Janeiro, Brasil

Available in: <http://www.redalyc.org/articulo.oa?id=32713466002>

- How to cite
- Complete issue
- More information about this article
- Journal's homepage in redalyc.org

redalyc.org

Scientific Information System

Network of Scientific Journals from Latin America, the Caribbean, Spain and Portugal

Non-profit academic project, developed under the open access initiative



## On the other law of the iterated logarithm for self-normalized sums

GUANG-HUI CAI

Department of Mathematics and Statistics, Zhejiang Gongshang University, Hangzhou 310035, P.R. China

*Manuscript received on March 17, 2008; accepted for publication on May 15, 2008;  
presented by ARON SIMIS*

### ABSTRACT

In this note, we obtain a Chung's integral test for self-normalized sums of i.i.d. random variables. Furthermore, we obtain a convergence rate of Chung law of the iterated logarithm for self-normalized sums.

**Key words:** Chung's integral test, self-normalized sums, convergence rate.

### 1 INTRODUCTION

Let  $X, X_1, X_2, \dots$  be i.i.d. random variables with mean zero and variance one, and set

$$S_n = \sum_{k=1}^n X_k, \quad M_n = \max_{1 \leq k \leq n} |S_k| \quad \text{and} \quad V_n^2 = \sum_{k=1}^n X_k^2, \quad n \geq 1.$$

Also let  $\log x = \ln(x \vee e)$ ,  $\log_2 x = \log(\log x)$ . Then by the so-called Chung's law of the iterated logarithm we have

$$\liminf_{n \rightarrow \infty} \sqrt{\log_2 n / n} M_n = \pi / \sqrt{8} \quad a.s. \quad (1.1)$$

This result was first proved by Chung (1948) under  $E|X|^3 < \infty$ , and by Jain and Pruitt (1975) under the sole assumption of a finite second moment. Einmahl (1989) obtained the Darling Erdős theorem for sums of i.i.d. random variables. Griffin and Kuelbs (1989) got Self-normalized laws of the iterated logarithm. Griffin and Kuelbs (1991) obtained some extensions of the laws of the iterated logarithm via self-normalized. Lin (1996) got a self-normalized Chung-type law of iterated logarithm. Einmahl (1993) obtained the following integral test refining (1.1) under the minimal conditions.

**THEOREM A.** *Let  $\{X, X_n; n \geq 1\}$  be a sequence of i.i.d. random variables with  $EX = 0$ ,  $EX^2 = 1$  and*

$$EX^2 I\{|X| \geq t\} = O((\log_2 t)^{-1}) \quad \text{as } t \rightarrow \infty. \quad (1.2)$$

Then for any eventually non-decreasing function  $\phi : [1, \infty) \rightarrow (0, \infty)$ ,

$$\begin{aligned} P(M_n \leq \sqrt{n}/\phi(n) \text{ i.o.}) &= 0 \text{ or } 1 \\ \text{according as } J(\phi) &:= \int_1^\infty \frac{\phi(t)^2}{t} \exp(-\pi^2 \phi(t)^2/8) dt < \infty \text{ or } = \infty. \end{aligned} \quad (1.3)$$

Einmahl (1993) showed that if (1.2) is not true, Theorem A is false. We thus see that condition (1.2) is sharp. However, if we use  $V_n$  to replace  $\sqrt{n}$ , we can eliminate the condition (1.2) in Theorem A. Explicitly, we get the following theorem.

**THEOREM 1.1.** *Let  $\{X, X_n; n \geq 1\}$  be a sequence of i.i.d. random variables with  $EX = 0$ ,  $EX^2 = 1$ . Then for any eventually non-decreasing function  $\phi : [1, \infty) \rightarrow (0, \infty)$ ,*

$$\begin{aligned} P(M_n \leq V_n/\phi(n) \text{ i.o.}) &= 0 \text{ or } 1 \\ \text{according as } J(\phi) &:= \int_1^\infty \frac{\phi(t)^2}{t} \exp(-\pi^2 \phi(t)^2/8) dt < \infty \text{ or } = \infty. \end{aligned} \quad (1.4)$$

Our next theorem gives a result on a convergence rate of (1.1).

**THEOREM 1.2.** *Let  $\{X, X_n; n \geq 1\}$  be a sequence of i.i.d. random variables with  $EX = 0$ ,  $EX^2 = 1$ . Then for any  $b > -1$ , we have*

$$\sum_{n=1}^\infty \frac{(\log_2 n)^b}{n \log n} P\left(M_n \leq \varepsilon \sqrt{\pi^2 V_n^2 / (8 \log_2 n)}\right) < \infty, \quad \forall \varepsilon > 0. \quad (1.5)$$

Throughout this note, let  $C$  denote a positive constant, whose values can differ in different places.

## 2 PROOF

**PROOF OF THEOREM 1.1.** It is enough to prove the result for eventually non-decreasing function  $\phi : [1, \infty) \rightarrow (0, \infty)$  satisfying

$$\frac{1}{2}(\log_2 t)^{1/2} \leq \phi(t) \leq (\log_2 t)^{1/2}, \quad t \geq 1 \quad (2.1)$$

(See Einmahl 1993). Let

$$\begin{aligned} X_{1j} &= X_j I \left\{ |X_j| \leq \sqrt{j}/(\log_2 j)^2 \right\}, \quad j \geq 1 \\ B_n^2 &= \sum_{i=1}^n EX_{1i}^2 \quad \text{and} \quad \Delta_n = \left| \frac{M_n}{B_n} - \frac{M_n}{V_n} \right|, \quad n \geq 1. \end{aligned}$$

Observe that by (2.1)

$$\begin{aligned} &P(M_n \leq V_n/\phi(n) \text{ i.o.}) \\ &\leq P(M_n \leq V_n/\phi(n), \Delta_n \geq (\log_2 n)^{-3/2} \text{ i.o.}) + P(M_n \leq V_n/\phi(n), \Delta_n \leq (\log_2 n)^{-3/2} \text{ i.o.}) \\ &\leq P(M_n \leq V_n/\phi(n), \Delta_n \geq (\log_2 n)^{-3/2} \text{ i.o.}) + P(M_n/B_n \leq \phi(n)^{-1} + (\log_2 n)^{-3/2} \text{ i.o.}) \\ &\leq P(M_n \leq V_n/\phi(n), \Delta_n \geq (\log_2 n)^{-3/2} \text{ i.o.}) + P(M_n \leq B_n/\Psi(n) \text{ i.o.}), \end{aligned} \quad (2.2)$$

where  $\Psi(t) = \phi(t)^3/(1 + \phi(t)^2)$ ,  $t \geq 1$ , and similarly,

$$\begin{aligned} & \mathbf{P}(M_n \leq V_n/\phi(n) \text{ i.o.}) \\ & \geq \mathbf{P}(M_n/B_n \leq \phi(n)^{-1} - (\log_2 n)^{-3/2} \text{ i.o.}) - \mathbf{P}(M_n \leq B_n/\phi(n), \Delta_n \geq (\log_2 n)^{-3/2} \text{ i.o.}) \quad (2.3) \\ & \geq \mathbf{P}(M_n \leq B_n/\Psi'(n) \text{ i.o.}) - \mathbf{P}(M_n \leq B_n/\phi(n), \Delta_n \geq (\log_2 n)^{-3/2} \text{ i.o.}), \end{aligned}$$

where  $\Psi'(t) = \phi(t)^3/(\phi(t)^2 - 1)$ ,  $t \geq 1$ . It is easily checked that  $J(\phi) < \infty$  implies  $J(\Psi) < \infty$  and  $J(\phi) = \infty$  implies  $J(\Psi') = \infty$ , and by Theorem 1 of Einmahl (1993), we have

$$J(\Psi) < \infty \implies \mathbf{P}(M_n \leq B_n/\Psi(n) \text{ i.o.}) = 0$$

and

$$J(\Psi') = \infty \implies \mathbf{P}(M_n \leq B_n/\Psi'(n) \text{ i.o.}) = 1.$$

Now by Lemma 2.2 below,

$$\mathbf{P}(M_n \leq V_n/\phi(n), \Delta_n \geq (\log_2 n)^{-3/2} \text{ i.o.}) = 0$$

and

$$\mathbf{P}(M_n \leq B_n/\phi(n), \Delta_n \geq (\log_2 n)^{-3/2} \text{ i.o.}) = 0.$$

From these equations and (2.2), (2.3), hence we see that Theorem 1.1 holds true.  $\square$

We now present two lemmas used in the main proof of Theorem 1.1.

LEMMA 2.1. *For any  $x > 0$  there exist positive constants  $\eta = \eta(x)$  and  $A = A(x)$  such that*

$$\mathbf{P}\left(M_n \leq x\sqrt{n/\log_2 n}\right) \leq A(\log n)^{-\eta}.$$

PROOF. See the Lemma 2(b) of Einmahl (1993).  $\square$

LEMMA 2.2. *We have*

$$\mathbf{P}(M_n \leq V_n/\phi(n), \Delta_n \geq (\log_2 n)^{-3/2} \text{ i.o.}) = 0 \quad (2.4)$$

and

$$\mathbf{P}(M_n \leq B_n/\phi(n), \Delta_n \geq (\log_2 n)^{-3/2} \text{ i.o.}) = 0. \quad (2.5)$$

PROOF. Let

$$X_{2j} = X_j I\{\sqrt{j}/(\log_2 j)^2 < |X_j| \leq \sqrt{j}\}, X_{3j} = X_j - X_{1j} - X_{2j}, \quad j \geq 1$$

and

$$V_{1n} = \sum_{k=1}^n (X_{1k}^2 - \mathbf{E}X_{1k}^2), \quad V_{2n} = \sum_{k=1}^n X_{3k}^2, \quad n \geq 1.$$

First using  $\mathbb{E}X^2 = 1$ , we have

$$\begin{aligned} \sum_{n=1}^{\infty} \frac{(\log_2 n)^4}{n^2} \mathbb{E}(X_{1n}^2 - \mathbb{E}X_{1n}^2)^2 &\leq 2 \sum_{n=1}^{\infty} \frac{(\log_2 n)^4}{n^2} \mathbb{E}X^4 I\{|X| \leq \sqrt{n}/(\log_2 n)^2\} \\ &= 2 \sum_{k=1}^{\infty} \mathbb{E}X^4 I\{\sqrt{k-1}/(\log_2(k-1))^2 < |X| \leq \sqrt{k}/(\log_2 k)^2\} \sum_{n=k}^{\infty} \frac{(\log_2 n)^4}{n^2} \\ &\leq C \sum_{k=1}^{\infty} \mathbb{E}X^2 I\{\sqrt{k-1}/(\log_2(k-1))^2 < |X| \leq \sqrt{k}/(\log_2 k)^2\} \\ &\leq C \mathbb{E}X^2 < \infty \end{aligned}$$

and

$$\sum_{n=1}^{\infty} \mathbb{P}(X_{3n} \neq 0) = \sum_{n=1}^{\infty} \mathbb{P}(|X| > \sqrt{n}) \leq C \mathbb{E}X^2 < \infty.$$

Thus, it follows by applying Corollary 3.1 of Lin et al. (1999, P.95) and Borel-Cantelli lemma that

$$\frac{(\log_2 n)^2}{n} V_{1n} \rightarrow 0 \quad a.s. \quad \text{and} \quad V_{2n} = O(1) \quad a.s. \quad (2.6)$$

Using strong law of large numbers and Hartman-Wintner LIL, we have

$$\lim_{n \rightarrow \infty} \frac{V_n^2}{n} = 1 \quad a.s. \quad \text{and} \quad \limsup_{n \rightarrow \infty} \frac{M_n}{\sqrt{2n \log_2 n}} \leq 1 \quad a.s.$$

Thus, by  $\mathbb{E}X^2 = 1$ , we obtain that for large  $n$ ,

$$\begin{aligned} \Delta_n &= \left| \frac{M_n(V_n^2 - B_n^2)}{B_n V_n (B_n + V_n)} \right| \\ &\leq \frac{3\sqrt{\log_2 n}}{n} \left( |V_{1n}| + V_{2n} + \sum_{j=1}^n X_{2j}^2 \right) \\ &\leq \frac{3\sqrt{\log_2 n}}{n} \left( |V_{1n}| + V_{2n} + \sqrt{n} \sum_{j=1}^n |X_{2j}| \right) \quad a.s. \end{aligned} \quad (2.7)$$

Recalling that  $B_n^2 \leq n$ ,  $n \geq 1$  and  $\lim_{n \rightarrow \infty} V_n^2/n = 1$  a.s., in order to prove (2.4) and (2.5), by (2.1), (2.6) and (2.7), it suffices to show that

$$\mathbb{P}\left(M_n \leq 2\sqrt{n/\log_2 n}, \sum_{j=1}^n |X_{2j}| \geq \frac{1}{4}\sqrt{n}/(\log_2 n)^2 \text{ i.o.}\right) = 0. \quad (2.8)$$

Now, set  $m(n) := \lceil n/(\log_2 n)^9 \rceil$ ,  $n \geq 1$ . By  $\mathbb{E}X^2 = 1$ , we have

$$\sum_{j=1}^n \mathbb{E}|X_{2j}| \leq \sum_{j=1}^n \mathbb{E}|X_j| I\{|X_j| > \sqrt{j}/(\log_2 j)^2\} \leq \sum_{j=1}^n \frac{(\log_2 j)^2}{\sqrt{j}} \leq C\sqrt{n}(\log_2 n)^2.$$

Applying Kolmogorov's LIL and  $\mathbf{E}X^2 = 1$ , we have

$$\limsup_{n \rightarrow \infty} \frac{\sum_{j=1}^n (|X_{2j}| - \mathbf{E}|X_{2j}|)}{\sqrt{2n \log_2 n}} \leq 2 \quad a.s.$$

it easily follows from above inequalities that

$$\sum_{j=1}^{m(n)} |X_{2j}| = o(\sqrt{n}/(\log_2 n)^2) \quad a.s. \quad (2.9)$$

Hence observe that on account of (2.9) it is enough to show that

$$\mathbf{P}\left(M_n \leq 2\sqrt{n/\log_2 n}, \sum_{j=m(n)+1}^n |X_{2j}| \geq \frac{1}{5}\sqrt{n}/(\log_2 n)^2 \text{ i.o.}\right) = 0. \quad (2.10)$$

Let  $n_k = 2^k$  and  $m_k = \lceil 2^k/(\log k)^{10} \rceil$ ,  $k \geq 0$ , for large enough  $k$ ,

$$\begin{aligned} & \bigcup_{n=n_{k-1}+1}^{n_k} \left\{ M_n \leq 2\sqrt{n/\log_2 n}, \sum_{j=m(n)+1}^n |X_{2j}| \geq \frac{1}{5}\sqrt{n}/(\log_2 n)^2 \right\} \\ & \subseteq \left\{ M_{n_{k-1}} \leq 2\sqrt{2n_{k-1}/\log_2 n_{k-1}}, \sum_{j=m_k+1}^{n_k} |X_{2j}| \geq \frac{1}{10}\sqrt{n_k}/(\log_2 n_k)^2 \right\}. \end{aligned}$$

Thus, in order to prove (2.10), it suffices to show that

$$\mathbf{P}\left(M_{n_{k-1}} \leq 2\sqrt{2n_{k-1}/\log_2 n_{k-1}}, \sum_{j=m_k+1}^{n_k} |X_{2j}| \geq \frac{1}{10}\sqrt{n_k}/(\log_2 n_k)^2 \text{ i.o.}\right) = 0. \quad (2.11)$$

Let

$$M_{n_{k-1},j} = M_{j-1} \vee \max_{j < n \leq n_{k-1}} |S_n - X_j|, \quad 1 \leq j \leq n_{k-1} \quad \text{and} \quad n'_k = n_{k-1} - 1.$$

Notice that

$$M_{n_{k-1},j} \leq M_{n_{k-1}} + |X_j| \leq 3M_{n_{k-1}}, \quad 1 \leq j \leq n_{k-1}.$$

Using the independence and Lemma 2.1, it is clear that for some constant  $\eta > 0$  and large enough  $k$ ,

$$\begin{aligned} & \mathbf{P}\left(M_{n_{k-1}} \leq 2\sqrt{2n_{k-1}/\log_2 n_{k-1}}, \sum_{j=m_k+1}^{n_k} |X_{2j}| \geq \frac{1}{10}\sqrt{n_k}/(\log_2 n_k)^2\right) \\ & \leq \mathbf{P}\left(\bigcup_{j=m_k+1}^{n_k} \{|X_j| > \sqrt{j}/(\log_2 j)^2, M_{n_{k-1}} \leq 2\sqrt{2n_{k-1}/\log_2 n_{k-1}}\}\right) \\ & \leq \sum_{j=m_k+1}^{n_{k-1}} \mathbf{P}(|X_j| > \sqrt{j}/(\log_2 j)^2, M_{n_{k-1}} \leq 2\sqrt{2n_{k-1}/\log_2 n_{k-1}}) \\ & \quad + \sum_{j=n_{k-1}+1}^{n_k} \mathbf{P}(|X_j| > \sqrt{j}/(\log_2 j)^2, M_{n_{k-1}} \leq 2\sqrt{2n_{k-1}/\log_2 n_{k-1}}) \end{aligned}$$

$$\begin{aligned}
&\leq \sum_{j=m_k+1}^{n_{k-1}} \mathbf{P}(|X_j| > \sqrt{j}/(\log_2 j)^2, M_{n_{k-1},j} \leq 6\sqrt{2n_{k-1}/\log_2 n_{k-1}}) \\
&\quad + \sum_{j=n_{k-1}+1}^{n_k} \mathbf{P}(|X_j| > \sqrt{j}/(\log_2 j)^2) \mathbf{P}(M_{n_{k-1}} \leq 2\sqrt{2n_{k-1}/\log_2 n_{k-1}}) \\
&\leq \sum_{j=m_k+1}^{n_{k-1}} \mathbf{P}(|X_j| > \sqrt{j}/(\log_2 j)^2) \mathbf{P}(M_{n'_k} \leq 9\sqrt{n'_k/\log_2 n'_k}) \\
&\quad + \sum_{j=n_{k-1}+1}^{n_k} \mathbf{P}(|X_j| > \sqrt{j}/(\log_2 j)^2) \mathbf{P}(M_{n_{k-1}} \leq 3\sqrt{n_{k-1}/\log_2 n_{k-1}}) \\
&\leq Ck^{-\eta} \sum_{j=m_k+1}^{n_k} \mathbf{P}(|X| > \sqrt{j}/(\log_2 j)^2).
\end{aligned}$$

Finally, By Lemma 4 of Einmahl (1993), we have

$$\begin{aligned}
&\sum_{k=1}^{\infty} \mathbf{P}\left(M_{n_{k-1}} \leq 2\sqrt{2n_{k-1}/\log_2 n_{k-1}}, \sum_{j=m_k+1}^{n_k} |X_{2j}| \geq \frac{1}{10}\sqrt{n_k}/(\log_2 n_k)^2\right) \\
&\leq C \sum_{k=1}^{\infty} k^{-\eta} \sum_{j=m_k+1}^{n_k} \mathbf{P}(|X| > \sqrt{j}/(\log_2 j)^2) < \infty
\end{aligned}$$

and hence we obtain (2.11) from the Borel-Cantelli lemma.  $\square$

PROOF OF THEOREM 1.2. For each  $n \geq 1$  and  $1 \leq i \leq n$ , we have

$$\bar{X}_{ni} = X_i I\{|X_i| \leq n^{1/2}(\log n)^{-1/3}\}, \quad \bar{V}_n^2 = \sum_{j=1}^n \bar{X}_{nj}^2 \quad \text{and} \quad \bar{B}_n^2 = \sum_{j=1}^n \text{Var}(\bar{X}_{nj}).$$

By  $\mathbf{E}X^2 = 1$ , it is easy to show that  $\bar{B}_n^2 \leq n$ . Hence for some  $\frac{1}{7} < \delta < 1$  and any  $\varepsilon > 0$

$$\begin{aligned}
&\mathbf{P}(M_n \leq \varepsilon \sqrt{\pi^2 V_n^2 / (8 \log_2 n)}) \\
&\leq \mathbf{P}(M_n \leq \varepsilon \sqrt{\pi^2 (1 + \delta) \bar{B}_n^2 / (8 \log_2 n)}) + \mathbf{P}(V_n^2 \geq (1 + \delta) \bar{B}_n^2) \\
&\leq \mathbf{P}(M_n \leq \varepsilon \sqrt{\pi^2 (1 + \delta) / 8} \sqrt{n / \log_2 n}) + \mathbf{P}(\bar{V}_n^2 \geq (1 + \delta) \bar{B}_n^2) \\
&\quad + \mathbf{P}\left(\bigcup_{j=1}^n \{|X_j| > n^{1/2}(\log n)^{-1/3}\}\right) \\
&:= I_1 + I_2 + I_3.
\end{aligned}$$

In order to prove (1.5), it suffices to show that for any  $b > -1$

$$\sum_{n=1}^{\infty} \frac{(\log_2 n)^b}{n \log n} I_i < \infty, \quad \forall \varepsilon > 0, \quad i = 1, 2, 3. \quad (2.12)$$

By Lemma 2.1, there exists a positive constant  $\eta$  such that

$$\sum_{n=1}^{\infty} \frac{(\log_2 n)^b}{n \log n} I_1 \leq C \sum_{n=1}^{\infty} \frac{(\log_2 n)^b}{n \log n} (\log n)^{-\eta} < \infty, \quad \forall \varepsilon > 0.$$

Since  $\mathbf{E}X = 0$  and  $\mathbf{E}X^2 = 1$ , there exists a positive integer  $n_0$  such that for all  $n \geq n_0$

$$\mathbf{E}\bar{X}_{n1}^2 \geq \frac{3}{4} \quad \text{and} \quad \mathbf{E}\bar{X}_{n1} \leq \frac{1}{4}.$$

Hence using the Bernstein inequality, there exists a positive constant  $\beta < 1/3000$  such that

$$\begin{aligned} \sum_{n=1}^{\infty} \frac{(\log_2 n)^b}{n \log n} I_2 &\leq C + \sum_{n=n_0}^{\infty} \frac{(\log_2 n)^b}{n \log n} \mathbf{P}(\bar{V}_n^2 \geq (1 + \delta/2)n\mathbf{E}\bar{X}_{n1}^2) \\ &\leq C + \sum_{n=n_0}^{\infty} \frac{(\log_2 n)^b}{n \log n} (\log n)^{-\beta} \\ &< \infty. \end{aligned}$$

Finally, by  $\mathbf{E}X^2 = 1$ , we have

$$\sum_{n=1}^{\infty} \frac{(\log_2 n)^b}{n \log n} I_3 \leq \sum_{n=1}^{\infty} \frac{(\log_2 n)^b}{\log n} \mathbf{P}(|X| > n^{1/2}(\log n)^{-1/3}) \leq C\mathbf{E}X^2 < \infty.$$

Thus, (2.12) holds true. □

#### ACKNOWLEDGMENTS

This paper was supported by National Natural Science Foundation of China and Youth Talent Foundation of Zhejiang Gongshang University (Q07-07).

#### RESUMO

Nesta nota, obtemos um teste integral de Chung para somas auto-normalizadas de variáveis aleatórias i.i.d. (independentes e identicamente distribuídas). Além disso, obtemos uma taxa de convergência da lei de Chung do logaritmo iterado para somas auto-normalizadas.

**Palavras-chave:** teste integral de Chung, somas auto-normalizadas, taxa de convergência.

#### REFERENCES

- CHUNG KL. 1948. On the maximal partial sums of sequences of independent random variables. Trans Amer Math Soc 4: 205–233.
- EINMAHL U. 1989. The Darling Erdős theorem for sums of i.i.d. random variables. Probab Theory Relat Fields 82: 241–257.
- EINMAHL U. 1993. On the other law of the iterated logarithm. Probab Theory Relat Fields 96: 97–106.



- GRIFFIN PS AND KUELBS JD. 1989. Self-normalized laws of the iterated logarithm. *Ann Probab* 17: 1571–1601.
- GRIFFIN PS AND KUELBS JD. 1991. Some extensions of the laws of the iterated logarithm via self-normalized. *Ann Probab* 19: 380–395.
- JAIN NC AND PRUITT WE. 1975. The other law of the iterated logarithm. *Ann Probab* 3: 1046–1049.
- LIN ZY. 1996. A self-normalized Chung-type law of iterated logarithm. *Theory Probab Appl* 41: 791–798.
- LIN ZY, LU CR AND SU AG. 1999. *Limit Theoretical Basis of Probability*. Hight Education Press, Beijing.