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Equilibrium pricing and ordering policies in a two-echelon supply chain in the presence of strategic customers

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ABSTRACT

This paper studying the impact of strategic customer behavior on decentralized supply chain gains and decisions, which includes a supplier, and a monopoly firm as a retailer who sells a single product over a finite two periods of selling season. We consider three types of customers: myopic, strategic and low-value customers. The problem is formulated as a bi-level game where at the second level (e.g. horizontal game), the retailer determines his/her equilibrium pricing strategy in a non-cooperative simultaneous general game with strategic customers who choose equilibrium purchasing strategy to maximize their expected surplus. At the first level (e.g. vertical game), the supplier competes with the retailer as leader and follower in the Stackelberg game. They set the wholesale price and initial stocking capacity to maximize their profits. Finally, a numerical study is presented to demonstrate the impacts of strategic behavior on supply chain gain and decisions; subsequently the effects of market parameters on decision variables and total profitability of supply chain's members is studied through a sensitivity analysis.

Key words: pricing and revenue management, strategic customer, supply chain management, game theory.

INTRODUCTION

Retailer's decision variables are always affected by supply situations and sale policies, so they are the two most important challenges influencing the retailers. However, the complexity of these challenges has considerably increased by various factors such as customer awareness, economic variables, technological development, and competitive environment. For instance, internet development has inspired suppliers to sell their products or services directly to target consumers or those customers looking for some opportunities at lower prices because of growing global awareness. If retailers cannot overcome these challenges, their benefits will be significantly compromised. Therefore, supplying and selling policies are retailer's two major concerns.

In the real context, there are many applications, which help us better understand this issue; for example, travel agencies must book airline tickets and hotel rooms (e.g. supply side) and determine sales policies (e.g.

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demand side) based on their demand estimations; or style goods retailers should simultaneously consider supplying constraints and market behavior. Therefore, it seems imperative for firms to simultaneously focus on both sides of supply and demand.

On the demand side, nowadays, customers have largely learned to follow buying opportunities at fair prices. During the last decade, many research projects have emphasized on increasing the number of customers who buy their required goods during season sales. Change in the customer behavior as well as other factors such as competitive pressures or supplying constraints, have forced retailers to modify their pricing and stocking strategies (see e.g., Byrnes and Zellner (2004), Silverstein and Butman (2006)) of future prices and this strategic behavior has resulted in a plethora of studies in this area (for example, see, e.g., Su and Zhang (2008)).

According to strategic customer behavior in recent literature, customer purchasing decisions depend on their expectations of the market conditions. Specifically, some studies have focused on strategic customers with future price-depended purchasing time. Such papers have examined optimal markdowns pricing strategies in the presence of strategic customers. Shen and Su (2007) have presented a comprehensive review of these studies. Desai et al. (2004) compared centralized and decentralized distribution channels for durable goods and demonstrated that strategic decentralization could increase the manufacturer's profits under some specific conditions. Arya and Mittendorf (2006) proved that the benefits of decentralization are robust against fluctuations of manufacturer's commitments ability. Aviv and Pazgal (2008) proposed optimal pricing of a seasonal product at the market with forward-looking customers. Elmaghraby et al. (2008) investigated optimal markdown pricing in the presence of multi-unit rational demands. Gallien (2006) proved that when we sell a finite inventory to infinite strategic customer, the optimal prices should increase over time horizon. Su (2007) examined a market with heterogeneous myopic and strategic customers and demonstrated that the optimal pricing strategy is one of the two markup or markdown mechanisms, based on the customer composition.

Also, Yu et al. (2007) compared different demand scenarios by using advance selling with limited capacity. Some other studies have focused on the models in which strategic customers decide about their buy-or-wait decisions based on products availability as driven by firms' inventory decisions. Yin et al. (2009) determined the optimal pricing strategies, in the presence of strategic customers, by developing a game-theoretical model for a retailer who sells a limited inventory of a product by using either display all format (DA) or display one format (DO), over a finite horizon time. They also supposed that strategic customers arrive at the store according to a Poisson process with a constant rate. Liu and van Ryzin (2008) developed quantity decisions instead of price decisions for a firm with a single product in a capacity-rationing model in the presence of strategic customers with heterogeneous valuations, identical risk preferences, and knowledge of the price path and fill rate. Anily and Hassin (2013) studied a deterministic pricing and replenishment model in which the retailer advertised a fixed price and the selling schedule, and the strategic customers incurred holding or shortage costs. Jie et al. (2015) studied the impacts of the risk preference and the decreasing value of strategic customers on the ordering quantity, the sale price and the total profit in a single-period joint inventory pricing problem. They also compared the results with the classical newsvendor model. They found that strategic behavior leads to a lower ordering quantity, a lower price and a lower total profit. Besbes (2015) developed a dynamic programming approach to determine firm's optimal pricing policy under commitment and in presence of strategic heterogeneous customers. He studied the class of monotone pricing policies.

On the supply side, there are many related papers focusing on pricing strategies in supply chain framework with demand uncertainty. Cardenas-Barron et al. (2014) presented a comprehensive introduction to inventory management and supply chain scopes, and provided a basis for new directions in inventory management research. Chen et al. (2012) considered a supply chain with a single supplier and two retailers who competed with each other under a capacity allocation to determine the ordering strategy. Lai et al. (2012) studied the stocking decision of a downstream buyer with some private demand information under a general, single buyback contract as a supply chain framework. Using rational expectations framework, Tereyağoğlu and Veeraraghavan (2012) proposed a model that addressed pricing and production decisions for a firm under uncertain market demand including strategic consumers. Swinney (2011) examined the value of quick response production practices in the presence of forward-looking customers with uncertain, heterogeneous valuations who could have chosen between purchasing early and delaying the purchase decision until resolving valuation uncertainty. The market size is also uncertain to the firm and it may commit either to a single production run at a low unit cost or to a quick response strategy. Katsifou et al. (2014) presented an optimization model and an iterative heuristic to analyze the trade-offs between three decisions that are crucial for a retailer's commercial success: the product assortment, the inventory levels, and the pricing.

Cachon and Swinney (2009) considered a retailer who sells a product with uncertain demand including myopic, bargain-hunting, and strategic customers, over a finite selling season. They also determined retailer's initial stocking quantity and markdown pricing strategy. Cachon and Swinney (2011) also examined the performance of fast fashion system in comparison with three different systems: quick-response-only systems, enhanced-design-only systems, and traditional systems. They paid attention to the impact of each system on the strategic customer purchasing behavior. Levin et al. (2009) considered a monopolist seller who contingently prices a fixed stock of items over a finite time horizon. They proposed a dynamic pricing model for a finite population of strategic customers in a stochastic market by using a stochastic dynamic game. Hua et al. (2010) studied a service supply chain including a supplier and a retailer in which both of them face customers who have strategic behavior in choosing a purchasing channel. They assumed that a supplier has a limited capacity of a perishable product. They examined pricing and purchasing strategies under two alternative supply chain systems: centralized and decentralized. Su (2010) considered a monopolist seller with a finite capacity at the two-period season for determining pricing and the long-run capacity decisions in uncertain aggregate market included strategic customers, bargain hunters, and speculators with different valuations. Daojian et al. (2015) determined pricing and inventory decisions on decentralized supply chains with revenue-sharing contracts and centralized supply chains, and explored the impact of quick response on supply chain performance in presence of strategic customer. They found that if the extra cost of quick response is relatively low, the value of quick response would be greater in centralized systems than in decentralized ones. They also showed that revenue-sharing contracts can improve decentralized supply chain performance in comparison with centralized supply chain. Nita and Cardenas-Barron (2015) studied ordering and credit policies in a supply chain with a supplier who offers its retailer either a cash discount or a fixed credit period for enough great ordering quantity, and a retailer who offers credit period to its customer. They also assumed deteriorating constant rate for the retailer's inventory system.

As you can see, although many studies have distinctly investigated the strategic customer behavior and pricing strategies in a supply chain framework, few have probed into the performance of supply chain in the presence of strategic customers. The main purpose of this study, according to the literature, is to develop a modeling framework to study the strategic behavior of customers on supply chain gain and decisions and subsequently, supply chain members' decisions on strategic customer buying behavior. In this paper, the model is somewhat similar to the newsvendor problem, with the effects of supplier's decisions and the presence of strategic customers as the two major differences between them. Indeed, this model indicates the interaction among supplier, retailer, and strategic customers. Perhaps, Su and Zhang (2008) is the closest research to the model proposed in this paper. They developed a rational expectation to study the impact of capacity information on strategic customer purchase behavior by using a game-theoretical framework. They examined two seller strategies: commitment to a particular level of initial inventory, and providing customers with guarantees on product availability. The main differences between their model and this model are represented in supplier and his/her competition with retailer, and two-period pricing policies.

This study attempts to contribute to the earlier works in the following ways:

1. Many existing methods only consider a fixed and given initial capacity, while the model in this study analyzes pricing and ordering decisions in a decentralized supply chain.
2. As a new contribution to the field, the proposed model considers vertical and horizontal relationships among supplier, retailer, and customers.
3. This paper considers a two-segment market: certain and uncertain market with three myopic, strategic, and low-value customers.

This study also introduces the model with its assumptions, notations, and applications; develops a bi-level game approach to analyze the model and extracts results; presents a numerical example and reports the results. The model and its computational results are instantiated in a case with uniform distribution as uncertain demand function. Finally, concluding remarks summarize the contribution of the study. Also, Appendix A provides the evidence of all technical results.

MODEL DESCRIPTION

Consider a decentralized two-echelon supply chain with a supplier and a retailer in an uncertain market with myopic and strategic customers. The retailer makes an order for a single durable product (i.e. stylish goods) from a particular supplier and sells it to customers. The following describes these three groups of agents with their characteristics and other relevant assumptions,

1. The supplier supplies or produces a single durable product and wishes to maximize its total profit by determining the optimal wholesale pricing strategy with respect to its unit supply costs and retailer's ordering quantities. However, the wholesale price cannot be less than unit supply cost or more than customers' high valuation. We introduce C and Q_{Max} as unit supply costs (included purchasing, ordering and holding costs for each product) and supply constraint, respectively.
2. The monopolist retailer determines ordering quantity and pricing policies over two time periods, *t =1, 2*, to maximize his/her total expected profit. We assume there is only one purchasing opportunity because supply lead times are too long and also any inventory remaining at the end of the second period is equal to zero (Cachon and Swinney (2009)). Let P_t be the unit retail price in period *t* called as full-price and sale-price in the rest of the paper. Meanwhile, P_t must be between the wholesale price and customer high valuation in each period (it is a rational and obvious assumption).
3. Demand in the market is divided into two distinct segments, with three different types of customers (referred to as strategic, myopic, and low-value consumers in the rest of this paper). Each customer demands at most one unit of product but may have different valuations for it. Customer's valuations are either V_H (i.e. high-value) or V_L (i.e. low-value); in other words, it refers to their utility from using

the product so it can also be interpreted as the maximum willingness to pay for buying the product (Su (2010)).

- *Uncertain Demand:* First, the retailer faces uncertain demand segment where there is a non-negative random number X of customers who arrive at the first period into the market. Let f and F denote the probability density and cumulative distribution function of X, respectively. Also we assume that X satisfies the following property (Cachon and Swinney (2009)).

<u>*Definition 1.*</u> *A continuous, nonnegative random variable X with density f satisfies the monotone scaled likelihood ratio (MSLR) property if, for all* $\lambda \leq 1$ *and x in the support of X,* $f_X(\lambda x)/f_X(x)$ *is monotonic in x.*

This segment actually has two types of high-value customers: strategic and myopic. A fraction $\alpha \in [0,1]$ of these customers are myopic and the rest of them are strategic (Su (2010)). All these customers have high valuations V_H, but the strategic customer valuation in the second period is èV_H, where $\theta \in [0,1]$ is fixed non-increasing rate of strategic customers' valuation in the second period. They are also homogeneous in their valuations, their purchasing and their waiting risks. Myopic customers will just purchase at the first period if the retail price is less than their valuations (their surplus is positive) and their numbers are always less than Q. Strategic customers rationally decide about purchasing opportunities at both of the two periods and they will buy a product when price is sufficiently attractive. In other words, they consider their surplus from purchasing the product in each period and choose between them to maximize their expected surplus. But, there are two major points which influence on the strategic customers' decision: achieving the product and declining their valuations at the second period. Also, we assume that the customers, who wait, have the highest priority to receive the product at the second period. This is reasonable because customers who are interested in a particular product are more likely to get the product when the sale actually takes place.

- *Certain or Low-value demand:* The certain demand segment includes infinite low-value customers (even greater than initial capacity) who just enter into the market at the second period. They have the lowest priority compared with other customers to buy the product (equivalently, we can consider that low-value consumers show up only at the end of the second period). Hence, at a sufficiently low price (e.g., below V_L), the retailer can always sell all remaining capacity. It should also be noted that V_L can be lower than supplier wholesale price.
- The problem is modeled as a bi-level game. At the second level, retailer and strategic customers play together in a two-person non-cooperative simultaneous general game to determine their pricing and purchasing equilibrium strategies. The retailer does not exactly know about the customers' valuation, but s/he could estimate about customers low and high valuation based on his/her experiences so that we can assume s/he has enough knowledge about customers' valuation. Each strategic customer has also private knowledge of his/her own second-period valuation at the start of the game. While, at the first level, a supplier (as a leader) competes with a retailer (as a follower) to detect optimal pricing and ordering policies based on the Stackelberg game.

MODEL NOTATIONS

The table I summarizes the model notations and their definitions.

TABLE I
The model notations.

Parameters	
V_L	: Low-value customers' valuation
V_H	: High-value customers' valuation, where $V_H \geq V_L$
C	: Product's unit supply cost
α	: the proportion of myopic customers in uncertain demand, where $\alpha \in [,1]$
θ	: Strategic customers' valuation discount factor when they decide to wait until the second period to buy the product, where $\theta \in [0\ ,1]$ and è$V_H \geq V_L$
X	: Uncertain demand's non-negative continuous random variable
Q_{Max}	: Supplier's maximum capacity or retailer's maximum ordering quantity
Variables	
P_1	: Retailer's unit price at the first period (full-price)
P_2	: Retailer's unit price at the second period (sale-price)
P_w	: Supplier's unit wholesale price, where $P_w \geq C$
Q	: Initial stocking capacity (ordering quantity), where $Q \leq Q_{Max}$
β	: Strategic customers' decision variable, where $\beta \in \{0,1\}$ where $\beta = \begin{cases} 1, \ \textit{if strategic customers buy in the first period} \\ 0, \ \textit{otherwise} \end{cases}$
Functions	
$f_X(x)$	: Probability density function of X
$F_X(x)$	: Cumulative distribution function of X
$\Pi_r(Q,P_1,P_2)$	: Retailer's total expected profit function
$R(I,P_2)$	:Retailer's revenue function at the second period, where I is remaining capacity at the beginning of the second period
$\Pi_w(P_w)$	: Supplier's total profit function
$U_{Sc}(\ .)$	: Strategic customer's surplus at the i^{th} period

Chronology of Events

In this section, we briefly describe the sequence and the timeline of the proposed model. At the beginning of the season, a supplier sets the wholesale price P_w. Then, the retailer determines and orders the initial stocking quantity Q based on his/her prediction about the demand and the supplier's wholesale price. We suppose there is only one purchasing opportunity because production lead times are long enough. At the beginning of the first period, the retailer sets the full-price P_1 before that high-value random demand X is realized. After that, myopic customers buy the product and leave the market whereas strategic consumers must decide whether to buy or to wait until the sale period.

At *t=2*, low-value customers enter into the market and the retailer determines sale-price P_2 based on remaining capacity and market demand in the second period. Finally, trades take place at this price. Figure 1 presents the sequence of events in our model and also summarizes some key terms that will be used later.

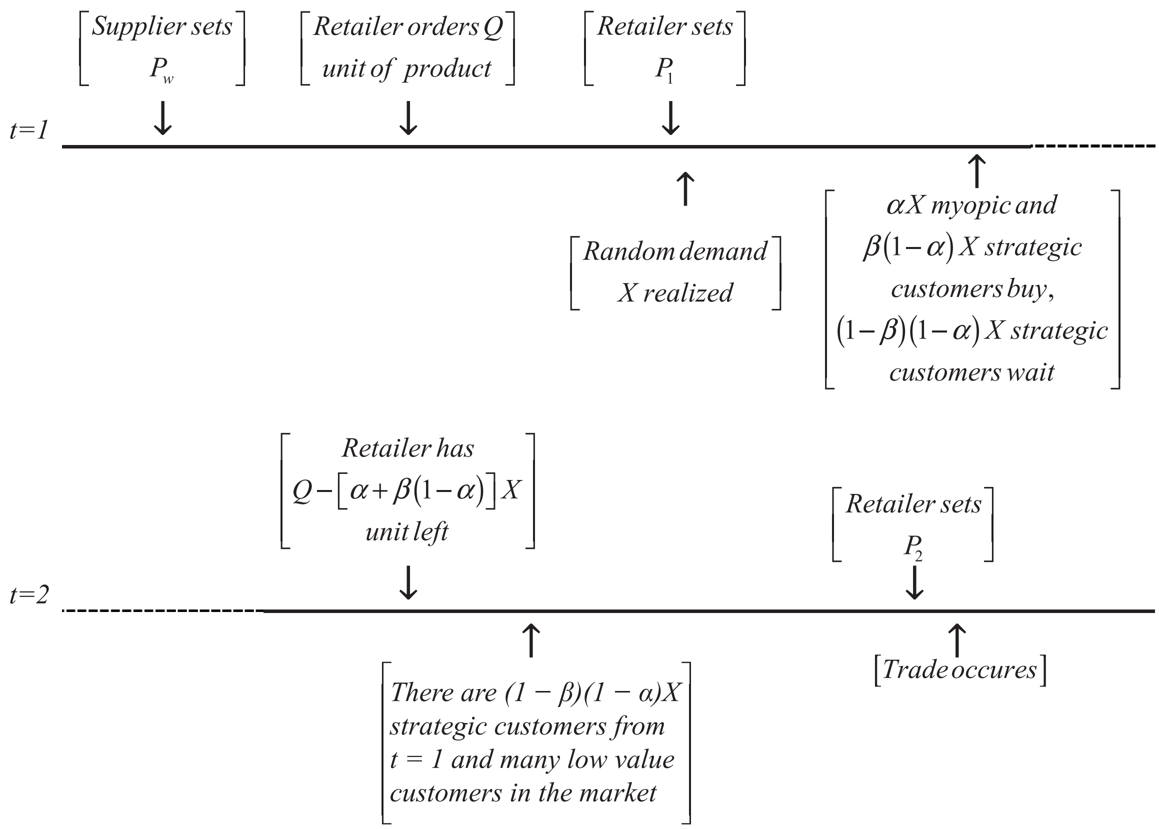


Figure 1 - The model's chronology of events.

IMPLICATIONS OF THE PROPOSED MODEL

There are several practical Implications of the proposed model such as tourism and fashion industries. In the tourism industry, Travel agencies always deal with customers in the demand side, and hotels and airlines in the supply side. Travel agencies can be never sure about the demand because customers' willingness about a special tour is always affected by many factors (such as tour's facilities and attractions of destination). Furthermore, they become aware of customers' price sensitivity. Usually, they expect that a relatively insensitive group of diehard tourists exist, and it is reasonably assumed to be myopic customers. There is also another price sensitive segment which has reliance on tour's price changes. This segment is known to be strategic customers in our model. Now, in regards to the uncertain and price sensitive demand, travel agencies must decide about tour's capacity which depends not only on demand but also on many other factors in suppliers' side such as airlines' ticket price or hotels' room price.

Our model can also be applied to study the fashion market or newly introduced products, such as electronic gadgets. In both of them normally there are some constraints such as production capacity or uncertain demand.

EQUILIBRIUM ANALYSIS

Now, by using a standard backward induction, a bi-level game among supplier, retailer, and strategic costumers is studied. At the second level, we model the game between the retailer and the strategic customers as non-zero sum simultaneous non-cooperative. In this situation retailer's gain/ loss of utility is not exactly balanced by the losses/gains of the utility of the strategic customers, and players make decisions independently without any enforcement to have cooperative behavior. Each player in this game has some beliefs about the actions of the other players. In other words, we look for a sub game perfect Nash equilibrium with a backward approach under rational expectations as the best solution which means that each player chooses optimal actions based on their beliefs about how others will play.

Finally, at the first level, the supplier and the retailer compete with each other in the Stackelberg game as a leader and a follower, respectively, to extract optimal pricing and ordering policies. In fact, in the Stackelberg game with a backward procedure, firstly, the retailer's (i.e. follower) problem must be solved to determine his/her decision variables as a function of the supplier's decision variables. In the next step, the supplier's decision problem is solved by attending the follower's possible reaction to maximize its utility. The retailer's optimal decisions can be determined by considering the supplier's decisions as input parameters in retailer's problem. Finally, the leader (i.e. supplier) finds its optimal decisions by assuming that the follower takes the optimal response.

SECOND LEVEL: A TWO-PERSON NON-COOPERATIVE SIMULTANEOUS GENERAL GAME

Second period

The retailer's optimal sale price must now be derived. In the first period, we suppose that the retailer has sold αX and $\beta(1-\alpha)X$ units to myopic and strategic customers, respectively (let $\beta \in [0,1]$, however, this assumption will be modified). Let $I = \left[Q-\left(\alpha+\beta(1-\alpha)\right)X\right]$ denote the retailer's on-hand inventory at the start of the second period. Therefore, the number of strategic consumers who have chosen to wait is $\left[(1-\alpha)(1-\beta)X\right]$. Further, we know that the infinite number of low-value customers are entered into the market at t=2. Thus, we have $\left[(1-\alpha)(1-\beta)X\right]$ high-value and infinite low-value customers at the sale market. Furthermore, it is obvious that the sale-price P_2 could never be less than V_L ($P_2 = V_L$ always yields greater profit) or greater than èV_H (if $P_2 > \theta V_H$, the strategic customer surplus will be negative so they will not buy any product). Therefore, if we consider that the retailer sets P_2 as sale-price, then his/her expected revenue function at the second period is equals to:

$$R(I,P_2) = P_2 * E\left[Min\left\{\begin{matrix}\left(Q-\left(\alpha+\beta(1-\alpha)\right)X\right), \\ \left((1-\alpha)(1-\beta)X\right)\end{matrix}\right\}\right], \tag{1}$$

where $P_2 \in [V_L, \theta V_H]$.

The following lemma demonstrates the set of the sale-prices given by revenue function.

<u>Lemma 1</u>. *In equilibrium,* $P_2 \in \{V_L, \theta V_H\}$

Proof. The proofs of all lemmas appear in the technical appendix. □

By attending to lemma 1, we can rewrite $R(I,P_2)$ in (1),

$$R(I,P_2)=\begin{cases} E\Big[Max\big\{\big(Q-(\alpha+\beta(1-\alpha))X\big),0\big\}\Big]V_L \ ,P_2=V_L \\ E\Big[Max\Big\{Min\big\{\big(Q-(\alpha+\beta(1-\alpha))X\big),\big((1-\alpha)(1-\beta)X\big)\big\},0\Big\}\Big]\theta V_H ,P_2=\theta V_H \end{cases}, \tag{2}$$

In other words, if the retailer chooses V_L as sale-price, s/he could absolutely sell all remaining capacity, otherwise, the strategic customer is only able to buy the product. Now, the retailer's equilibrium sale-pricing policy characterized by the following lemma can be determined:

Lemma 2. Let $D_m=V_LQ/\bar{\lambda}$ be critical minimum demand level where $\bar{\lambda}=\lambda\theta V_H+(1-\lambda)V_L$ and $\lambda=(1-\alpha)(1-\beta)$. Therefore, based on demand's random variable X, equilibrium sale-price (P_2^*) and expected revenue ($R^*(I,P_2^*)$) at the second period are equal to:

$$P_2^*=\begin{cases} V_L \ ,X\le D_m \\ \theta V_H ,X>D_m \end{cases},$$

$$R^*(I,P_2^*)=\begin{cases} E\big[Max\{Q-(1-\lambda)X,0\}\big]V_L, \\ P_2=V_L \\ E\left[Max\left\{Min\begin{Bmatrix}(Q-(1-\lambda)X), \\ (\lambda X)\end{Bmatrix},0\right\}\right]\theta V_H, \\ P_2=\theta V_H \end{cases}.$$

The form of the equilibrium policy shown at lemma 2 seems natural. Based on the results of lemma 2, there are two possible outcomes at the second period, corresponding to uncertain high-demand realizations. If uncertain demand is low or $X\le D_m$, retailer's expected revenue for $P_2=V_L$ is greater than when $P_2=\theta V_H$, because sometimes, clearing all remaining inventory by the lowest sale-price will be more beneficial than just selling some products (not all) by highest sale-price. But, when demand is high or $X>D_m$, the retailer sets the highest sale-price to sell the product only to the high-value strategic customers who had not bought at the first period. However there may be some unsold remaining inventory at the end of the season.

First period

At the beginning of the first period, the retailer proposes to maximize expected total profit by determining his/her pricing and ordering policies. In this section, primarily, the retailer's equilibrium pricing policy can be extracted, and then his/her optimal ordering policy which is set by competing with supplier is specified. The full-price is the most important point in the second level game because it not only influences strategic consumer decisions, but also, determines sale-price as described above. But before answering the question "what is the retailer's equilibrium full-price?" Therefore, the retailer total expected profit function and strategic customer surplus need to be introduced.

Retailer total expected profit function

Retailer total expected profit function consists of three parts: revenue in both of the first and second period, and the products' total purchase or supply cost. Suppose P_w and Q are supplier's unit wholesale price and retailer's ordering quantity, respectively. Therefore, P_wQ is the products' total supply cost that should be paid by the retailer. Then, with respect to lemma 2, $\Pi_r\left(P_1,P_2^*,Q\right)$ is equals to:

$$\Pi_r\left(P_1,P_2^*,Q\right)=P_1E\left[Min\left\{Q,(1-\lambda)X\right\}\right]+R\left(I,P_2^*\right)-QP_w,\Rightarrow$$

$$\begin{aligned}\Pi_r\left(P_1,P_2^*,Q\right)&=P_1E\left[Min\left\{Q,(1-\lambda)X\right\}\right]+E\left[Max\left\{Q-(1-\lambda)X,0\right\}\right]V_L\\&+E\left[Max\left\{Min\left\{\left(Q-(1-\lambda)X\right),(\lambda X)\right\},0\right\}\right]\theta V_H-QP_w,\end{aligned} \tag{3}$$

Or to put it another way,

$$\begin{aligned}\Pi_r\left(P_1,P_2^*,Q\right)&=P_1Q\bar{F}_X\left(D_u\right)+P_1\int_0^{D_u}\left[(1-\lambda)X\right]dF_X(x)+V_L\int_0^{D_m}\left[Q-(1-\lambda)X\right]dF_X(x)\\&+\theta V_H\int_{D_m}^{Q}\left[\lambda X\right]dF_X(x)+\theta V_H\int_{Q}^{D_u}\left[Q-(1-\lambda)X\right]dF_X(x)-QP_w,\end{aligned} \tag{4}$$

where $D_u=Q/(1-\lambda)$, $dF_X(x)=f_X(x)dx$ and $\bar{F}_X(.)=1-F_X(.)$.

Strategic customers' surplus

As mentioned before, the strategic customer's valuations at the first and second period are V_H and θV_H, respectively. Now, consider a particular customer who beliefs that s/he will get V_H-P_1 surplus, if s/he buys the product at the first period; but if s/he waits for the second period, s/he will obtain the product with [(availability probability of the product $)*\left(\theta V_H-P_2\right)$] surplus. Based on these expectations, the strategic customer's surplus is:

$$U_{Sc}=\begin{cases}\left(V_H-P_1\right),\beta=1\\ \left(\theta V_H-P_2\right)*\begin{pmatrix}availability\\ probability\,of\\ the\ product\end{pmatrix},\beta=0\end{cases}$$

Availability probability of the product can be calculated as follows. Consider an individual strategic customer who does not buy a product at the first period and waits instead. Because this customer is infinitesimal compared with the remaining customers, s/he will face a stockout if $X>Q$. In other words, when s/he waits, s/he will obtain the product if $X\le Q$ or with probability $F_X(Q)$. Of course, we implicitly know that the customers, who wait, have the highest priority to receive the product at the sale-price. Thus, we can rewrite the strategic customer's surplus as follows:

$$U_{Sc}=\begin{cases}V_H-P_1\ ,\beta=1\\ \left(\theta V_H-P_2\right)F_X(Q),\beta=0\end{cases}, \tag{5}$$

Now, we can solve the second level game and specify retailer's equilibrium full-price and strategic customer's equilibrium purchasing decision.

EQUILIBRIUM STRATEGIES OF RETAILER AND STRATEGIC CUSTOMER

According to customers' valuation at the first period (V_H) and supplier's unit wholesale price (P_w), we can derive $P_w \leq P_1 \leq V_H$ because if the full-price is greater than V_H or less than P_w, customers' surplus or retailer's profit, respectively, will be negative. Now, based on the above description and by attending to (4) and (5), the following theorem demonstrates the existence of an equilibrium solution. But before considering theorem 1, we should explain lemma 3 to introduce $\hat{P}$ as the strategic customer's indifferent full-price.

Lemma 3. There exists a full-price $\hat{P} = V_H - (\theta V_H - V_L) F_X(D_m) \in [P_w, V_H]$ *so that if retailer chooses one, all strategic consumers with first and second-period value* V_H *and* θV_H *are indifferent between purchasing in the first or second period.*

Actually, lemma 3 declares that strategic customers will purchase in the first period if $P_1 \leq \hat{P}$, otherwise they wait for the second period.

Theorem 1. In a two-person non-cooperative simultaneous general game between the retailer and strategic consumers, there is a subgame perfect Nash equilibrium so that,

1. *the equilibrium full-price,* P_1^**, and strategic customers' purchasing decision,* β^**, satisfy the following situations:*

$$P_1^* = V_H, P_2^* = \begin{cases} V_L, X \leq D_m \\ \theta V_H, X > D_m \end{cases}, \beta^* = 0,$$

2. *in equilibrium, retailer's total expected profit function and strategic customers' surplus equal to:*

$$\Pi_r\left(P_1^*, P_2^*, \beta^*, Q\right) = V_H Q \bar{F}_X\left(D_u\right) + V_H \int_0^{D_u} \left[(1-\lambda) X\right] dF_X(x) - QP_w$$

$$+ V_L \int_0^{D_m} \left[Q - (1-\lambda) X\right] dF_X(x) + \theta V_H \int_{D_m}^{Q} \left[\lambda X\right] dF_X(x) + \theta V_H \int_Q^{D_u} \left[Q - (1-\lambda) X\right] dF_X(x),$$

$$U_{Sc} = \left(\theta V_H - V_L\right) F_X\left(D_m\right),$$

Where $D_m = V_L Q / \bar{\lambda}$, $D_u = Q / (1-\lambda)$, $\bar{\lambda} = \lambda \theta V_H + (1-\lambda) V_L$ *and* $\lambda = (1-\alpha)$.

As we can observe from Theorem 1, at the first period, the retailer should charge a high full-price $P_1^* = V_H$ which is a reasonable result. One obvious reason is that the myopic customers stand in the market at this period and if retailer charges $P_1^* < V_H$, they will yield less benefit compared with $P_1^* = V_H$. But, at the second period, if uncertain demand is big enough or on the other hand, the number of strategic customers waited until second period are significant, the retailer should maintain the same strategy and charges high sale-price $P_2^* = \theta V_H$; otherwise, a markdown to V_L is preferable because of his clear policy at the end of season. This type of pricing format is commonly observed in practice.

Having extracted equilibrium retailer pricing and strategic customer purchasing strategies, the research analyzes first level game between retailer and supplier to specify the optimal initial stocking capacity and wholesale price in the next section.

First level: The Stackelberg game

This section models competition between the supplier and the retailer as a non-cooperative Stackelberg game in which the supplier acts as the leader and the retailer as the follower. Their net profits are considered as the players' payoff/utility functions for maximization. As mentioned before, the supplier produces or supplies a single product at the wholesale price according to the retailer's ordering quantity, while the retailer determines and orders his/her required initial stocking capacity based on market demand and supplier's wholesale price. However, according to the model assumptions, we know that $P_w \geq C$ and$Q \leq Q_{Max}$. The payoff function (i.e. net profit) for each player equals to its revenue minus total cost where these functions to the retailer (obtained in theorem 1) and the supplier are given, respectively, by

$$\Pi_r(Q) = V_H Q\overline{F}_X(D_u) + V_H \int_0^{D_u} \left[(1-\lambda)X\right] dF_X(x) - QP_w + V_L \int_0^{D_m} \left[Q-(1-\lambda)X\right] dF_X(x)$$
$$+\theta V_H \int_{D_m}^{Q} [\lambda X] dF_X(x) + \theta V_H \int_{Q}^{D_u} \left[Q-(1-\lambda)X\right] dF_X(x), \tag{6}$$

$$\Pi_w(P_w) = (P_w - C)Q \tag{7}$$

The Stackelberg equilibrium is obtained using a backward procedure. According to this procedure, at the first step, the retailer's (i.e. follower) problem must be solved to determine his/her initial stocking capacity as a function of the supplier's wholesale price (suppose that the supplier's wholesale price, in this step, is P_w). In the next step, the supplier's decision problem is solved by attending the follower's possible reaction to maximize its net profit. The retailer's optimal initial stocking capacity can be determined by considering the supplier's wholesale price as its input parameter. Finally, the leader (i.e. supplier) finds its optimal wholesale price by assuming that the follower takes the optimal response.

At the first step, based on rational expectations, both the supplier and the retailer know that there are some P_w^1 and P_w^2, so that $Max(C, P_w^1) \leq P_w \leq P_w^2$ and the retailer ordering quantity will always be Q_{Max} or 0, if the supplier chooses some P_w less than P_w^1 or more than P_w^2, respectively. Therefore, we can write the retailer's and supplier's maximization problems as follows,

$$Max_{0\leq Q\leq Q_{Max}}\ \Pi_r(Q)\ ; \tag{8}$$

$$Max_{Max(C, P_w^1)\leq P_w\leq Min(V_H, P_w^2)}\ \Pi_w(P_w); \tag{9}$$

According to above descriptions, theorem 2 demonstrates the retailer's response based on his/her estimation from the supplier's wholesale price, P_w.

Theorem 2. Consider $\overline{Q}$ as the retailer best response ordering quantity. Let $\hat{P}_w^1$ and $\hat{P}_w^2$ be retailer's lowerbond and upperbond estimations about P_w^1 and P_w^2. Define $Z(Q) = V_L F_X(D_m) + V_H \overline{F}_X(\gamma D_m)$, where $\overline{F}_X(\gamma D_m) = \theta V_H \overline{F}_X(Q) + (1-\theta) V_H \overline{F}_X(D_u)$ and $\gamma \in \left[\overline{\lambda}/V_L, \overline{\lambda}/\left(V_L(1-\lambda)\right)\right]$. Therefore,

1. *There are some $Q_L, Q_H \in [0, \mathrm{Q_{Max}}]$ so that Q_L and Q_H are minimum and maximum points of $Z(Q)$, respectively. Then, Z^* and Z^{**} can be defined as follows,*

$Z^{*} = Z(Q_L)$

$Z^{**} = Z(Q_H)$

and $\hat{P}_w^1 = Z^*$ *and* $\hat{P}_w^2 = Z^{**}$,

2. *a. If* $P_w \in \left[\mathrm{C},\ \mathrm{Max}\left(\mathrm{C},\ \mathrm{Z}^*\right)\right]$, *then* $\Pi_r(Q)$ *is a non-decreasing function of Q, and* $\bar{Q} = Q_{Max}$,

   b. If $P_w \in \left(\mathrm{Max}\left(\mathrm{C},\ \mathrm{Z}^*\right),\ Min\left(V_H, Z^{**}\right)\right)$, *then* $\Pi_r(Q)$ *is quasiconcave in Q and* $\bar{Q}$ *is determined by the unique solution to the first-order derivatives' equation of* $\Pi_r(Q)$,

   $d\Pi_r(Q)/dQ = V_L F_X(D_m) + \theta V_H \bar{F}_X(Q) + (1-\theta) V_H \bar{F}_X(D_u) - P_w = 0,$

   c. If $P_w \in \left[Min\left(V_H, Z^{**}\right), \infty\right)$ *then* $\Pi_r(Q)$ *is a non-increasing function of Q, and* $\bar{Q} = 0$.

Now, as mentioned above, the retailer supposes $P_w \in \left[Max\left(C, Z^*\right), Z^{**}\right]$ because s/he beliefs that the ordering quantity for $P_w > Z^{**}$ is the same as $P_w = Z^{**}$. Similarly, there is also no difference between ordering quantity at $P_w < Max\left(C, Z^*\right)$ and $P_w = Max\left(C, Z^*\right)$. So, consider $\bar{Q} = g(P_w)$ as the solution of equation $d\Pi_r(Q)/dQ = 0$ from theorem 2 which $g(P_w)$ is decreasing and reversible function of P_w. Further, let $g^{-1}(\bar{Q})\big|_{Q=Q_{Max}} = P_w^L$ and $g^{-1}(\bar{Q})\big|_{Q=0} = P_w^H$ (where $g^{-1}(\bar{Q})$ is reverse function). By substituting $\bar{Q}$, P_w^L, and P_w^H by the supplier's maximization problem in (9), we will be able to find the supplier's wholesale price decision and subsequently, the retailer's optimal ordering quantity.

Theorem 3. Consider the supplier's maximization problem in (9),

$$Max_{Max\left(C, P_w^1\right) \le P_w \le Min\left(V_H, P_w^2\right)} \Pi_w(P_w) = (P_w - C)\, g(P_w),$$

Therefore,

1. $P_w^1 = P_w^L$ *and* $P_w^2 = P_w^H$ *where* P_w^L, *and* P_w^H *are calculated by the following equations,*

   $g^{-1}(\bar{Q})\big|_{Q=Q_{Max}} = P_w^L$,

   $g^{-1}(\bar{Q})\big|_{Q=0} = P_w^H$,

2. *If* $g(P_w)$ *is a concave function,* $\Pi_w(P_w)$ *will be quasiconcave in* P_w *and the optimal wholesale price* (P_w^*) *is determined by solving the equation* $d\Pi_w(P_w)/dP_w = 0$. *Otherwise, the optimal wholesale price* (P_w^*) *is obtained by comparing the value of* $\Pi_w(P_w)$ *in critical points and boundaries.*
3. *The optimal initial stocking quantity can be calculated based on different values of the optimal wholesale price,*

   a. If $P_w^* \in \left[C, Max\left(C, P_w^L\right)\right]$, *then* $Q^* = Q_{Max}$ *and* $P_w^* = Max\left(C, Z^*\right)$,

   b. If $P_w^* \in \left(Max\left(C, P_w^L\right), Min\left(V_H, P_w^H\right)\right)$, *then* $Q^* = g\left(P_w^*\right)$,

   c. If $P_w^* \in \left[Min\left(V_H, P_w^H\right), \infty\right)$, *then* $Q^* = 0$ *and* $P_w^* = P_w^H$.

Indeed, theorem 3 demonstrates that the wholesale price P_w^* acts as a lever for persuading the supply chain to achieve a particular equilibrium ordering quantity because we know that the ordering quantity Q is decreasing in P_w and depends on it. So the supplier based on his/her capacity and his/her strategy can control the retailer's demand. In other words, by varying P_w between $Max\left(C, Z^*\right)$ and $Min\left(V_H, P_w^H\right)$, the system adjusts itself to appropriate equilibrium quantity within the corresponding range. Interestingly, as observed from the theorem 3, it is possible $P_w^* > \theta V_H$ but the retailer chooses the equilibrium pricing strategy according to theorem 2 despite of gaining negative profit at the second period.

DISCUSSION

In this section, firstly, in order to demonstrate the proposed model and the emphasis of the analytical results presented in the previous sections a numerical example is presented. In fact, this example helps us practically analyze and explain the results of the proposed bi-level game and its solutions. Secondly, a sensitivity analysis is also performed for some market-related parameters to study the model results' sensitivity in comparison with parameters' volatility. Finally we extract some meaningful managerial insights.

NUMERICAL EXAMPLE

Suppose that the market's uncertain demand follows up continues uniform distribution with a and b as distribution's parameters and following density and cumulative distribution functions,

$$f_X(x) = I_{[a,b]}(x)/(b-a), F_X(t) = P(X \le t) = (t-a)/(b-a) \tag{10}$$

Moreover, suppose strategic customers account for 80% of uncertain demand with $V_H = 250\$$ and $\theta = 85\%$ as high-value and discount factor valuation, respectively. The low-value customers' valuation is also 150$. The supplier's maximum capacity is 50 with 80$ as a supply unit cost. The values of our example's input parameters are given in Table II.

By using the model results and with respect to input parameters, the optimal decision variables for the retailer, supplier, and strategic customer are presented in Table III.

TABLE II
Value of input parameters.

parameters	value
V_L	150$
V_H	250$
C	80$
α	20%
θ	85%
Q_{Max}	50
a	10
b	60

TABLE III
Optimal decision variables for the retailer, supplier and strategic customer.

Parameters	value
P_1	250
P_2	$\begin{cases} 150, X \le 12.39 \\ 212.5, X > 12.39 \end{cases}$
P_w	175
Q	16.52
β	0
$\Pi_r(Q, P_1, P_2)$	764.78
$\Pi_w(P_w)$	1569.57

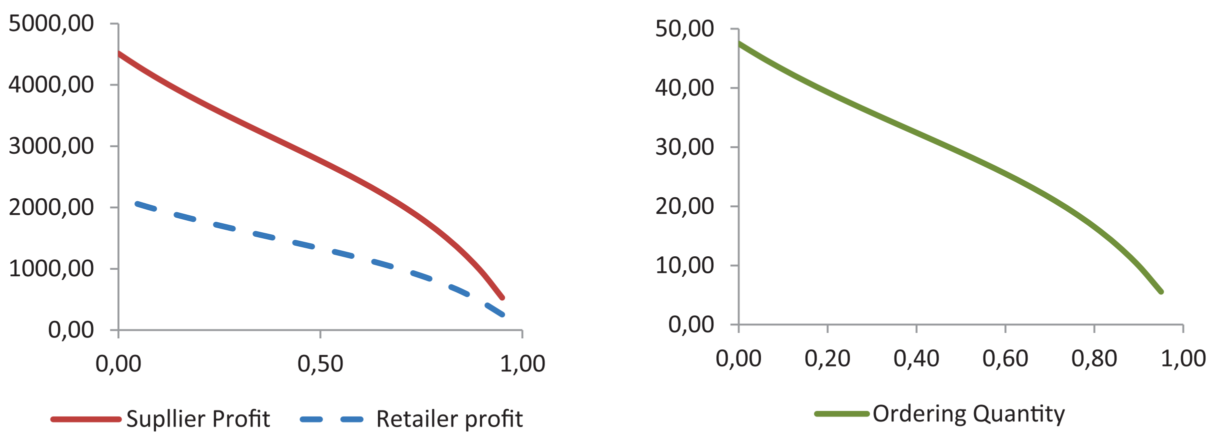
In other words, the equilibrium retailer's full-price and supplier's wholesale price are 250$ and 175$, respectively, and the equilibrium profits are obtained in *Q=16.52*. Furthermore, as much as $E(X) = 35 > D_m = 12.39$, we can consider $P_2 = 212.5\$$ as the equilibrium retail's sale-price, however, for uncertain demand less than 12.39, it is 150$.

Sensitivity Analysis

In this section, sensitivity analysis is also performed for market-related parameters (α,θ). In other words, we compare the effects of Parameters' volatility on the supplier's total profit, retailer's total expected profit and ordering quantity. Initially, the strategic customers' portion in the market $(1-\alpha)$ is considered. It is supposed that it varies in [0%, 95%]. Figure 2 shows the variations of supplier's profit, retailer's profit and ordering quantity with respect to $(1-\alpha)$.

As we can observe in Fig 2a, variations of supplier's total profit is more than the retailer's total expected profit and both of them are non-increasing with respect to $(1-\alpha)$'s fluctuations. But, the interesting point is decreasing the trend of ordering quantity's variations compared with $(1-\alpha)$'s fluctuations shown in Fig 2b.

As we can observe in Fig 2a, variations of supplier's total profit is more than the retailer's total expected profit and both of them are non-increasing with respect to $(1-\alpha)$'s fluctuations. But, the interesting point is decreasing the trend of ordering quantity's variations compared with $(1-\alpha)$'s fluctuations shown in Fig 2b.



a - The variations of supplier's and retailer's total profit

b - The variations of retailer's ordering quantity

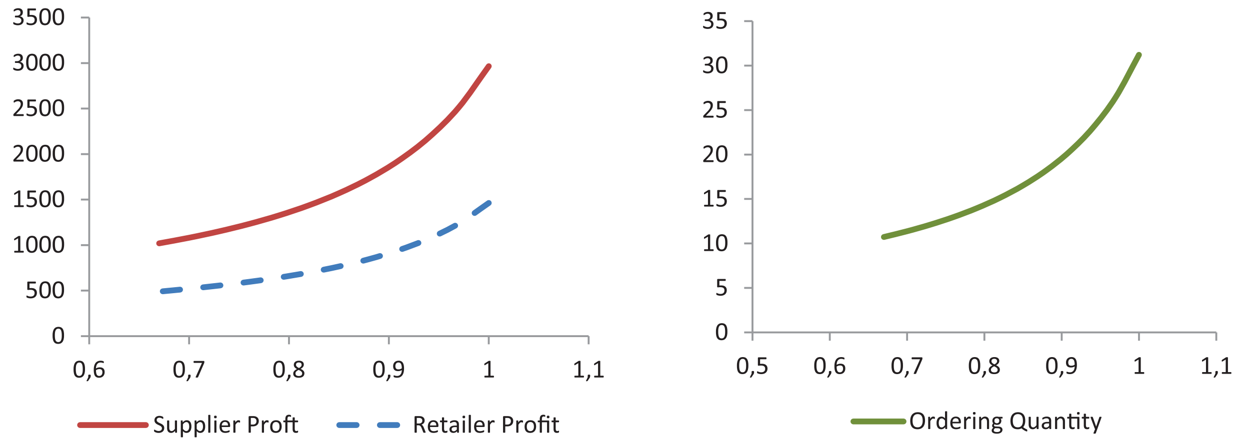
Figure 2 - The effect of $(1-\alpha)$'s variations.

Next, we evaluate the variations of supplier's profit, retailer's profit and ordering quantity with respect to variations of strategic customers' valuation discount factor, θ which is shown in Figure 3.

According to Fig 3a, by increasing θ, supplier's total profit and retailer's total expected profit raise, and vice versa, any decrease of θ will reduce the profit of supply chain members. The most important reason is the existence of direct relationship between ordering quantity and strategic customer valuation discount factor. In other words, any increase of θ influences positively on retailer's ordering quantity and subsequently, supplier's and retailer's profit will grow up.

As it can be observed in numerical example and sensitivity analysis sections, our achievements can be summarized in two important points:

1. More strategic behavior in the market, results in less retailer ordering quantity. However, practically, this happens when strategic customers buy a product earlier and as result, retailer's profit increases.
2. There is direct relationship between ordering quantity and strategic customer valuation discount factor. In other words, any increase of θ influences retailer's ordering quantity positively; subsequently, supplier's and retailer's profit will grow up.



a - The variations of supplier's and retailer's total profit. **b -** The variations of retailer's ordering quantity.

Figure 3 - The effect of θ's variations.

However, this result is reasonable because strategic customer valuation discount factor enhancement will stimulate the retailer to force strategic customers to buy later and without having to reduce his/her full-price. Generally, it is observed that customer strategic behavior has more affected on the supplier's decision in compare to the retailer.

CONCLUSIONS

In this paper, we presented a comprehensive study of the customers' behavior (i.e. myopic, strategic, and low value) in a two-echelon supply chain with a supplier and a monopolist retailer. The study attempted to bridge the existing gap, based on how customers' strategic behaviors influence supply chain's decision variable. The proposed model of this paper has been studied in two segments market including three types of customers. The first section presented a high-value uncertain demand with myopic and strategic customers who realized at the beginning of season, while there were infinite certain low-value customers at the second section who entered into the market at the end of the second period. We analyzed the model as a bi-level game where at the second level (horizontal game), the retailer competed with strategic customers in a non-cooperative simultaneous general game to determine the equilibrium pricing and purchasing strategy. While, at the first level (vertical game), the supplier and the retailer competed together as leader and follower in a Stackelberg game to set the optimal wholesale price and initial stocking capacity. In the other hand, by doing so, strategic customers had to decide about their purchasing time based on their expectations of the retailer's pricing strategies as well as reducing their valuation discount factor because of accessing late to the product in the season. Similarly, on the other hand, competitive environment, uncertain demand, and market's strategic behavior motivated retailers to compete with suppliers on ordering and pricing policies. Finally, the proposed model was also analyzed numerically using a uniform distribution demand and some managerial implications were derived.

As we can observe from the model results, the retailer's equilibrium pricing strategy is occurred in maximum valuation of strategic customer at the both period (based on uncertain demand's population). In other words, the retailer charges a high full-price $\left(P_1^* = V_H\right)$ at the first period which is a reasonable because of the presence of myopic customers in the market at this period. At the second period, the retailer keeps its previous strategy (charges high sale-price $P_2^* = \theta V_H$); if remaining uncertain demand in the market is big enough; otherwise, a markdown price $\left(P_2^* = V_L\right)$ is preferable because of retailer clear policy at the end of season. This type of pricing format is commonly observed in practice. As a result of retailer's decision in equilibrium, the strategic customer prefers to wait and purchase at the second period, hoping to reduce the price.

On the other hand, the results of proposed model demonstrates that the reverse dependence between wholesale price and ordering quantity can act as a leverage to control the retailer's demand, and consequently helps the supply chain to achieve a particular equilibrium. Particularly, at supplier's wholesale price that is the low/high enough, the retailer prefers to order maximum capacity or zero, respectively $\left(Q_{Max} / 0\right)$.

We can also see that customer strategic behavior influences the supplier's decision more than the retailer's decision. In other words, a market with strategic behavior has negative effects on the retailer's ordering policies; therefore, this situation brings about decrease in supplier's profit. Moreover, there is a direct relationship between ordering quantity and strategic customer valuation discount factor so that any increase of θ results in increasing the retailer's ordering quantity and supplier's and retailer's profit.

This study could be developed for more complicated situations such as multi supplier, oligopoly market, heterogeneous strategic customers with uncertain valuation, as well as market with Bargain-hunting customers. The model can also be modified and tailored for real implications such as tourism management problems.

RESUMO

Este artigo estuda o impacto do comportamento estratégico de clientes nos ganhos e decisões descentralizadas de cadeias de suprimentos compostas por um fornecedor e uma empresa monopolista, que funciona como uma revendedora de um único produto em duas temporadas finitas de vendas. Nós consideramos três tipos de clientes: míope, estratégico e de baixo valor. O problema é formulado como um jogo de dois níveis. No segundo nível (e.g., jogo horizontal), a revendedora determina seu ponto de equilíbrio na estratégia de precificação em um jogo geral não cooperativo com clientes estratégicos, que por sua vez definem sua estratégia de compra visando maximizar seu excedente esperado. No primeiro nível (e.g., jogo vertical), o fornecedor compete com a revendedora como líder e seguidor em um jogo de Stackelberg. Eles definem o preço de atacado e a capacidade de estoque inicial visando maximizar seus lucros. Finalmente, um estudo numérico é apresentado para demonstrar os impactos do comportamento estratégico nos ganhos e decisões da cadeia de suprimentos; subsequentemente os efeitos dos parâmetros de mercado nas variáveis de decisão e na lucratividade total dos membros da cadeia de suprimentos é estudado por meio de uma análise de sensibilidade.

Palavras-chave: políticas de precificação e lucros, clientes estratégicos, gerenciamento de cadeias de suprimento, teoria de jogos.

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APPENDIX

Proof of Lemma 1

According to (1), we know that $P_2 \in [V_L, \theta V_H]$. Let $P_2 = \tilde{P} \in (V_L, \theta V_H]$. Whereas retailer's expected revenue function at the second period is non-decreasing function with respect to P_2, then there is $\varepsilon > 0$ so that $R(I, \tilde{P}) \leq R(I, \tilde{P} + \varepsilon)$. In the other words, $P_2 = \tilde{P} + \varepsilon$ always yields greater or equal revenue in compare with $P_2 = \tilde{P}$, because for $P_2 > V_L$ only strategic customers with valuation θV_H are present at the market and earn non-negative surplus at these prices. Thus, reasonably, retailer desires to charge highest possible sale-price so that strategic customers' surplus still remains positive $(P_2 = \theta V_H)$; however some products maybe remain unsold. While, at $P_2 = V_L$ both of high-valuation and low-valuation customers could buy the product and retailer's all remaining capacity are sold. Therefore, in equilibrium, P_2 is equal to V_L or θV_H.

□

Proof of Lemma 2

Let $\lambda = (1-\alpha)(1-\beta)$ and $\overline{\lambda} = \lambda\theta V_H + (1-\lambda)V_L$. According to (2), $R(I,P_2)$ is equal to,

$$R(I,P_2) = \begin{cases} E\left[Max\{Q-(1-\lambda)X,0\}\right]V_L \ ,P_2 = V_L \\ E\left[Max\left\{Min\left\{\begin{matrix}(Q-(1-\lambda)X),\\(\lambda X)\end{matrix}\right\},0\right\}\right]\theta V_H \ ,P_2 = \theta V_H \end{cases}$$

Now, if $Max\{Min\{(Q-(1-\lambda)X),(\lambda X)\},0\} = Q-(1-\lambda)X$, then firstly $Q-(1-\lambda)X \le \lambda X$ or $Q < X$, and secondly $Q-(1-\lambda)X \ge 0$ or $X \le D_u = Q/(1-\lambda)$. Therefore, $R(I,\theta V_H) \ge R(I,V_L)$.

But if $Max\{Min\{(Q-(1-\lambda)X),(\lambda X)\},0\} = \lambda X$, then $Q-(1-\lambda)X \ge \lambda X$ or $Q \ge X$ (obviously $\lambda X \ge 0$), then $R(I,\theta V_H) \ge R(I,V_L)$ will be only established when,

$$E[\lambda X]\theta V_H \ge E\left[Q-(1-\lambda)X\right]V_L \Rightarrow$$
$$\theta V_H \int_0^Q \lambda X.dF_X(x) \ge V_L \int_0^Q \left(Q-(1-\lambda)X\right).dF_X(x) \Rightarrow$$
$$\int_0^Q \left[QV_L - \left(\theta V_H - \left((\theta V_H - V_L)(1-\lambda)\right)\right)X\right].dF_X(x) \le 0 \Rightarrow$$
$$\left[QV_L - \left(\theta V_H - \left((\theta V_H - V_L)(1-\lambda)\right)\right)X\right] \le 0$$

Or simply $X \ge D_m = QV_L/\overline{\lambda}$. Therefore, these results lead to following sale-pricing strategy and its expected revenue,

$$P_2^* = \begin{cases} V_L, X \le D_m \\ \grave{e}V_H, X > D_m \end{cases},$$

$$R^*(I,P_2^*) = \begin{cases} E\left[Max\{Q-(1-\lambda)X,0\}\right]V_L, P_2 = V_L \\ E\left[Max\left\{Min\left\{\begin{matrix}(Q-(1-\lambda)X),\\(\lambda X)\end{matrix}\right\},0\right\}\right]\theta V_H, \\ P_2 = \theta V_H \end{cases}$$

□

Proof of Lemma 3:

By attending to lemma 2, retailer's sale-pricing strategy is obtained by:

$$P_2^* = \begin{cases} V_L, X \le D_m \\ \theta V_H, X > D_m \end{cases},$$

Also consider P_1 as retailer's full-price at the first period. By substituting P_1 and P_2^* into (5), we can rewrite strategic customer's surplus U_{Sc}, as follows,

$$U_{Sc} = \begin{cases} V_H - P_1, \beta = 1 \\ (\theta V_H - V_L) F_X(D_m), \beta = 0 \end{cases},$$

The strategic customer is indifferent between buying at the full-price and waiting until the second period by hoping to get the product at the lower price, if only if his surplus at the first period be as much as it at the second period or:

$$V_H - P_1 = (\theta V_H - V_L) F_X(D_m) \Rightarrow P_1 = \hat{P} = V_H - \left((\theta V_H - V_L) F_X(D_m)\right).$$

Where $\hat{P} \in [P_w, \mathrm{V_H}]$. Therefore, $P_1 > \hat{P}$ means strategic customer's surplus at the second period is greater than ones at the first period so he wait until second period and otherwise, he buy the product at the first period.

□

Proof of Theorem 1:

Consider retailer's total expected profit function, $\Pi_r(P_1, P_2, Q)$ in (4). Define $d\Pi_r(P_1, P_2, Q)/dP_1 = Q\bar{F}_X(D_u) + \int_0^{D_u} (1-\lambda) X.dF_X(x)$ as first derivative of retailer's total expected profit function with respect to P_1. As you can see, $\Pi_r(P_1, P_2, Q)$ is non-decreasing function on P_1 so optimal full-price is on boundaries.

In addition, based on the lemma 3, there is a full-price $P_1 = \hat{P}$ that strategic customer is indifferent about his decision. So if retailer chooses $P_1 > \hat{P}$, strategic costumer waits until the second; otherwise he buys at the first period. Therefore, in this two-person non-cooperative simultaneous general game, both of the retailer and strategic consumer have two strategies where strategic customer's and retailer's strategies are $\beta \in \{0,1\}$ and $P_1 \in \{\hat{P}, V_H\}$, respectively. In the other words, clearly, strategic customer has always to decide choose about his buying time so $\beta \in \{0,1\}$.

While, retailer must choose either setting full-price at $P_1 > \hat{P}$ or $P_1 \leq \hat{P}$, but by notice to the non-decreasing property of $\Pi_r(P_1, P_2, Q)$, we found that his full-price's set of strategies is $P_1 \in \{\hat{P}, \mathrm{V_H}\}$. According to the above and by attending to (4), table of players' pay off could be obtained as table IV.

Based on the first part of theorem 1, the perfect Nash equilibrium (NE) strategy is $\{P_1^* = V_H, \beta^* = 0\}$, so it must satisfy the Nash equilibrium (NE) conditions. In the other words, according to the Nash equilibrium (NE) definition in two-person non-cooperative simultaneous general game, we know that the strategy profile $y^* \in S$ is a Nash equilibrium if no unilateral deviation in strategy by any single player is profitable for that player, that is

$$\forall i, y_i \in S_i : f_i(y_i^*, y_{-i}^*) \geq f_i(y_i, y_{-i}^*)$$

Where S_i and y_i are the strategy set and profile of player i, and $f_i(y_i)$ is the payoff function for $y_i \in S_i$. Also, y_{-i} is a strategy profile of all players except for player i.

Therefore, as you can see at the table IV, we have,

TABLE IV
Players' pay off under their different strategies.

		Strategic customer	
		$\beta=1$	$\beta=0$
Retailer	$P_1=\hat{P}$	$\boldsymbol{\Pi_r}(\boldsymbol{P_1},\boldsymbol{P_2},\boldsymbol{\beta},\boldsymbol{Q})=(\hat{P}-V_L)Q\bar{F}_X(Q)$ $+(V_L-P_w)Q+(\hat{P}-V_L)\int_0^Q XdF_X(x)$, $\boldsymbol{U_{Sc}}=(V_H-\hat{P})$,	$\boldsymbol{\Pi_r}(\boldsymbol{P_1},\boldsymbol{P_2},\boldsymbol{\beta},\boldsymbol{Q})=\hat{P}Q\bar{F}_X(D_u)+\hat{P}\int_0^{D_u}[\alpha X]dF_X(x)+$ $V_L\int_0^{D_m}[Q-\alpha X]dF_X(x)+\theta V_H\int_{D_m}^{Q}[(1-\alpha)X]dF_X(x)+$ $\theta V_H\int_Q^{D_u}[Q-\alpha X]dF_X(x)-P_wQ$, $\boldsymbol{U_{Sc}}=(\theta V_H-V_L)F_X(D_m)$,
Retailer	$P_1=V_H$	$\boldsymbol{\Pi_r}(\boldsymbol{P_1},\boldsymbol{P_2},\boldsymbol{\beta},\boldsymbol{Q})=(V_H-V_L)Q\bar{F}_X(Q)+$ $(V_L-P_w)Q+(V_H-V_L)\int_0^Q XdF_X(x)$, $\boldsymbol{U_{Sc}}=0$,	$\boldsymbol{\Pi_r}(\boldsymbol{P_1},\boldsymbol{P_2},\boldsymbol{\beta},\boldsymbol{Q})=V_HQ\bar{F}_X(D_u)+V_H\int_0^{D_u}[\alpha X]dF_X(x)+$ $V_L\int_0^{D_m}[Q-\alpha X]dF_X(x)+\theta V_H\int_{D_m}^{Q}[(1-\alpha)X]dF_X(x)+$ $\theta V_H\int_Q^{D_u}[Q-\alpha X]dF_X(x)-P_wQ$, $\boldsymbol{U_{Sc}}=(\theta V_H-V_L)F_X(D_m)$,

[1] $\lambda=(1-\alpha)$

$$1)\left\{\begin{array}{c}\Pi_r\left(\begin{array}{c}P_1^*=V_H,\\ P_2^*=\begin{cases}V_L, X\le D_m\\ \theta V_H, X>D_m\end{cases},\\ Q\end{array}\right),\\ U_{Sc}\left(\beta^*=0\right)\end{array}\right\}\ge\left\{\Pi_r\left(P_1,P_2,Q\right),U_{Sc}\left(\beta^*=0\right)\right\},$$

$$2)\left\{\begin{array}{c}\Pi_r\left(\begin{array}{c}P_1^*=V_H,\\ P_2^*=\begin{cases}V_L, X\le D_m\\ \theta V_H, X>D_m\end{cases},\\ Q\end{array}\right),\\ U_{Sc}\left(\beta^*=0\right)\end{array}\right\}\ge\left\{\Pi_r\left(\begin{array}{c}P_1^*=V_H,\\ P_2^*=\begin{cases}V_L, X\le D_m\\ \theta V_H, X>D_m\end{cases},\\ Q\end{array}\right),U_{Sc}\left(\beta\right)\right\},$$

Then, the strategy $\left\{P_1^*=V_H,\beta^*=0\right\}$ is a perfect Nash equilibrium (NE) strategy which is also unique because there is no strategy to satisfy Nash equilibrium (NE) conditions. Therefore, the retailer's equilibrium total expected profit function and strategic customers' equilibrium surplus are equal to,

$$\Pi_r(P_1,P_2,\beta,Q)=V_H\ Q\bar{F}_X(D_u)+V_H\int_0^{D_u}[\alpha X]dF_X(x)+V_L\int_0^{D_m}[Q-\alpha X]dF_X(x)+$$

$$\theta V_H\int_{D_m}^{Q}\left[(1-\alpha)X\right]dF_X(x)+\theta V_H\int_{Q}^{D_u}[Q-\alpha X]dF_X(x)-P_wQ,$$

$$U_{Sc}=(\theta V_H-V_L)F_X(D_m).$$

□

Proof of Theorem 2:

The retailer's total expected profit under the equilibrium pricing strategy is

$$\Pi_r(Q)=V_H\ Q\bar{F}_X(D_u)+V_H\int_0^{D_u}[\alpha X]dF_X(x)+V_L\int_0^{D_m}[Q-\alpha X]dF_X(x)+$$

$$\theta V_H\int_{D_m}^{Q}\left[(1-\alpha)X\right]dF_X(x)+\theta V_H\int_{Q}^{D_u}[Q-\alpha X]dF_X(x)-P_wQ,$$

First derivative of this expression with respect to Q yields

$$d\Pi_r(Q)/dQ=V_LF_X(D_m)+\theta V_H\ \bar{F}_X(Q)+(1-\theta)V_H\ \bar{F}_X(D_u)-P_w=0, \tag{A-1}$$

1. According to theorem 2 definitions, we know that $Z(Q)=V_LF_X(D_m)+\theta V_H\ \bar{F}_X(Q)+(1-\theta)V_H\ \bar{F}_X(D_u)$. By substituting these expressions in (A-1), we have

   $$d\Pi_r(Q)/dQ=Z(Q)-P_w=0, \tag{A-2}$$

   Furthermore, it is obvious that the functions $F_X(D_m)$, $\bar{F}_X(Q)$ and $\bar{F}_X(D_u)$ are continues and because of the continuity of continues functions' summation, $Z(Q)$ is continues. Therefore, according to extreme value theorem or Bolzano–Weierstrass theorem (Rusnock & Kerr-Lawson 2005), $Z(Q)$ has at least a maximum and minimum value in the closed and bounded interval $[0,Q_{Max}]$. In the other words, there exist some points $Q_L,Q_H\in[0,Q_{Max}]$ such that for all $Q\in[0,Q_{Max}]$,
   $Z^*\le Z(\mathrm{Q})\le Z^{**}$

   where $Z^*=Z(Q_L)$ and $Z^{**}=Z(Q_H)$.

2. a. suppose $P_w\in\left[C,\ Max(C,\ Z^*)\right]$. Because P_w is always lower than or equal to $Z(\mathrm{Q})$, so $d\Pi_r(Q)/dQ\ge 0$. This leads to $\Pi_r(Q)$ be a non-decreasing function of Q and gets the maximum value in its upperbound, $\bar{Q}=Q_{Max}$.

   b. Suppose $P_w\in\left(Max(C,\ Z^*),\ Z^{**}\right)$. By attending to $d\Pi_r(Q)/dQ\big|_{Q=Q_L}=Z^*-P_w<0$ and $d\Pi_r(Q)/dQ$ Z P $dQ\big|_{Q=Q_H}=Z^{**}-P_w>0$, so $d\Pi_r(Q)/dQ$ has certainly at least one root or encompasses at least one local maximum. To demonstrate quasi-concavity of $\Pi_r(Q)$ we must show that $d\Pi_r(Q)/dQ$ has a unique root or in the other words, $d\Pi_r(Q)/dQ$ has at most one local optimum (because of its asymptotic behavior). Infact, it means that $d\Pi_r(Q)/dQ$ is either quasi-concave or quasi-convex, and $d\Pi_r^2(Q)/dQ^2$ will include at most one interior zero. Therefore,

$$d\Pi_r^2(Q)/dQ^2 = V_L^2 f_X(D_m)/\bar{\lambda} - \theta V_H f_X(Q) - (1-\theta)V_H f_X(D_u)/\alpha,$$

By putting above expression equal to zero and after simplification, we could achieve the local optimum,

$$V_L^2 f_X(D_m)/f_X(D_u)\bar{\lambda} - \theta V_H f_X(Q)/f_X(D_u) - (1-\theta)V_H/\alpha = 0, \quad \text{(A-3)}$$

According to the MSLR assumption, Assume that $f_X(\lambda x)/f_X(x)$ is weakly increasing in x (The proof is identical if $f_X(\lambda x)/f_X(x)$ is weakly decreasing in x). So, in (A-3), the first term is positive and increasing in Q. Similarly, the second term is negative and increasing in Q and the third term is constant. Each term on the right hand side of (A-3) is increasing in Q, and if a solution to the equation exists, it is unique. This implies $d\Pi_r(Q)/dQ$ has at most one interior optimum, and consequently $\Pi_r(Q)$ is quasiconcave in Q. then, $\bar{Q}$ is determined by the unique solution to the first-order derivatives' equation of $\Pi_r(Q)$,

$$d\Pi_r(Q)/dQ = Z(Q) - P_w = 0.$$

c. Suppose $P_w \in [Z^{**}, \infty)$. Because P_w is always greater than or equal to $Z(\text{Q})$, so $d\Pi_r(Q)/dQ \le 0$. This implies $\Pi_r(Q)$ is a non-increasing function of Q and gets the maximum value in its lowerbound, $\bar{Q} = 0$.

□

Proof of Theorem 3:

The supplier's total profit is equal to,

$$\Pi_w(P_w) = (P_w - C)g(P_w), \quad \text{(A-4)}$$

Where $g(P_w)$ is decreasing ($dg(P_w)/dP_w < 0$) with reversible function of $g^{-1}(.)$. Because $0 \le \bar{\text{Q}} \le \text{Q}_{\text{Max}}$, so $g^{-1}(\text{Q}_{\text{Max}}) \le g^{-1}(\bar{\text{Q}}) \le g^{-1}(0)$ or simply, $Max(C, P_w^L) \le P_w \le P_w^H$.

In the other hand, the derivative of $\Pi_w(P_w)$ with respect to P_w can be written as

$$d\Pi_w(P_w)/dP_w = g(P_w) + (P_w - C)dg(P_w)/dP_w = 0, \quad \text{(A-5)}$$

By attending to the $d\Pi_w(P_w)/dP_w\big|_{P_w=C} = Q_{Max} > 0$ and

$$d\Pi_w(P_w)/dP_w\big|_{P_w=P_w^H} = (P_w^H - C)\left(dg(P_w)/dP_w\big|_{P_w=P_w^H}\right) < 0,$$

It is apparent that $d\Pi_w(P_w)/dP_w$ possesses at least one root. Furthermore, if $g(P_w)$ be a concave function or $d^2g(P_w)/dP_w^2 < 0$, then $dg(P_w)/dP_w$ will be decreasing function. Now by notice to (A-5), we have,

$$g(P_w) = -(P_w - C)dg(P_w)/dP_w,$$

As observe, the left-hand-side is decreasing (because $g(P_w)$ is decreasing function), and the right-hand-side is increasing in P_w (because $dg(P_w)/dP_w$ is decreasing function), so the first-order-condition has a unique solution. Therefore, $\Pi_w(P_w)$ is quasiconcave and has a uniquximizer and could be calculated by solving

$d\Pi_w(P_w)/dP_w = 0$.

Otherwise, if $g(P_w)$ does not be a concave function, the first-order-condition of $\Pi_w(P_w)$ might have a unique solution but certainly has at least one solution and if it happens, we must calculate the value of $\Pi_w(P_w)$. in all critical points and boundaries, then choose the point with maximum objective value as optimal wholesale price.

Now, if $P_w^* \in \left[C, Max\left(C, P_w^L\right)\right]$, then $Q^* = Q_{Max}$ and $P_w^* = Max\left(C, P_w^L\right)$. If $P_w^* \in \left(Max\left(C, P_w^L\right), Min\left(V_H, P_w^H\right)\right)$, then $Q^* = g\left(P_w^*\right)$, finally if $P_w^* \in \left[Min\left(V_H, P_w^H\right), \infty\right)$, then $Q^* = 0$ and $P_w^* = Min\left(V_H, P_w^H\right)$.

□