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**Erratum to “Bernstein-type Theorems in Hypersurfaces
with Constant Mean Curvature”
[An Acad Bras Cienc 72(2000): 301-310]**

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ABSTRACT

An erratum to Lemma 2.1 in Do Carmo and Zhou (2000) is presented.

Key words: Riemannian manifold, eigenvalue, hypersurface, mean curvature.

ERRATUM

Replace Section 2 in Do Carmo and Zhou (2000) by the following. The resulting change in the lemma will not affect the rest of the paper.

2. A RESULT ON NODAL DOMAINS

In this section we prove a result on the nodal domains of $|\phi|$ which will be needed in our proof of main theorems. We first need to recall the definition of nodal domains.

DEFINITION. An open domain D is called the nodal domain of a function f if $f(x) \neq 0$ for $x \in \text{int } D$ and vanishes on the boundary of ∂D . We denote by $N(f)$ the number of disjoint *bounded* nodal domains of f .

Now we have the following lemma which follows directly from Proposition 2.2 below. We want to thank the referee who provided the clearer proof of Proposition 2.2.

LEMMA 2.1. *Let M be a hypersurface in R^{n+1} with constant mean curvature H . Then*

$$\text{ind}(M) \geq N(|\phi|). \quad (2.1)$$

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PROOF. Let $N = N(|\phi|)$ and D_1, D_2, \dots, D_N be the N nodal domains of $|\phi|$ and let

$$\varphi(u) = u^2 + \frac{n(n-2)}{\sqrt{n(n-1)}}Hu - nH^2.$$

Then from (1.5) and Proposition 2.2 below we have functions f_1, f_2, \dots, f_N with supports in D_1, D_2, \dots, D_N respectively such that

$$I(f_i, f_i) = \int_{D_i} (|\nabla f_i|^2 - \varphi(u)f_i^2) < 0.$$

Denote W the linear subspace spanned by f_1, f_2, \dots, f_N . Since they have disjoint supports, they are orthogonal and thus the dimension of W is N . The index form $I(\cdot, \cdot)$ is negative definite on W so the Morse index is greater than or equal to N . \square

PROPOSITION 2.2. *Let (M, g) be Riemannian manifold and $u \geq 0$ be a continuous function satisfying the following inequality of Simons' type in the distribution sense*

$$u^2\varphi(u) \geq a|\nabla u|_g^2 - u\Delta_g u, \quad (2.2)$$

where $a > 0$ is a constant and φ is a continuous function on R . If u has a relatively compact nodal domain D , then there exists a function f_D with support in D such that

$$\int_D (|\nabla f|^2 - \varphi(u)f^2) < 0.$$

PROOF. Suppose that u admits a relatively compact nodal domain D . Write $q := \varphi(u)$ and $v := \log u$ on D . Thus (2.2) can be written as

$$q \geq a|\nabla v|_g^2 - \Delta_g v - |\nabla v|_g^2.$$

Then for any Lipschitz function f with support in D and vanishing at ∂D , we have

$$\int_D (|\nabla f|^2 - qf^2) \leq -a \int_D f^2 |\nabla v|^2 + \int_D |\nabla f - f \nabla v|^2.$$

Let $f = wu$, for some function w to be determined. We obtain

$$\int_D (|\nabla f|^2 - qf^2) \leq -a \int_D w^2 |\nabla u|^2 + \int_D u^2 |\nabla w|^2.$$

For all b such that $U/2 \leq b \leq U$, where $U := \sup_D u$, set

$$w_b(x) = \begin{cases} b & \text{as } u(x) \leq b, \\ u(x), & \text{as } u(x) > b. \end{cases}$$

Denote D_+ (resp. D_-) the set of points in D with $u(x) \geq b$ (resp. $u(x) \leq b$). A simple calculation leads to:

$$\int_D (|\nabla f|^2 - qf^2) \leq \int_{D_+} u^2 |\nabla u|^2 - \frac{aU^2}{4} \int_D |\nabla u|^2.$$

When b goes to U , the first term of right hand side tends to 0 (because $|\nabla u|^2$ is integrable), while the second term is fixed. It follows that $\int_D (|\nabla f|^2 - qf^2) < 0$ for all functions $f = w_b u$, when b is close to U . The conclusion is proved. \square

REFERENCE

DO CARMO MP AND ZHOU D. 2000. Bernstein-type Theorems in Hypersurfaces with Constant Mean Curvature. *An Acad Bras Cienc* 72: 301-310.