Non-Gaussianity and Loop Corrections in a Quadratic Two-Field Slow-Roll Model of Inflation. Part I

YEINZON RODRÍGUEZ*

Abstract. In this paper it is shown that it is possible to attain a high level of non-gaussianity in a particular quadratic two-field slow-roll model of inflation by taking care of loop corrections both in the spectrum and the bispectrum. A big $f_{NL}$ is obtained even if $\zeta$ is generated during inflation. Five issues are in consideration when constraining the available parameter space: 1. we must be sure that we are in a perturbative regime; 2. we must apply the correct condition about the (possible) loop dominance in $B_\zeta$ or $P_\zeta$; 3. we must satisfy the spectrum normalisation condition; 4. we must satisfy the spectral tilt constraint; 5. we must have enough inflation to solve the horizon problem.

Keywords: non-gaussianity, bispectrum, loop corrections.

Palabras y frases claves: no gaussianidad, biespectro, correcciones de lazo.

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* Centro de Investigaciones, Universidad Antonio Nariño, Cra 3 Este # 47A - 15, Bogotá, y Escuela de Física, Universidad Industrial de Santander, Bucaramanga, Colombia
e-mail: yeinzon.rodriguez@uan.edu.co
1. The Model

Our quadratic two-field slow-roll model is described by the potential

$$V = V_0 \left( 1 + \frac{1}{2} \eta_\phi \frac{\phi^2}{m_p^2} + \frac{1}{2} \eta_\sigma \frac{\sigma^2}{m_p^2} \right),$$

where the first term dominates and $\eta_\phi$ and $\eta_\sigma$ are the usual $\eta$ slow-roll parameters associated to the fields $\phi$ and $\sigma$. We are focused on the concave downward potential ($\eta_\phi$ and $\eta_\sigma < 0$) and for simplicity on the $\sigma = 0$ trajectory.

Since we are considering a slow-roll regime, the evolution of the fields in such a case is given by $\phi(N) = \phi_\star \exp(-N \eta_\phi)$ and $\sigma(N) = \sigma_\star \exp(-N \eta_\sigma)$, in terms of the field values $\phi_\star$ and $\sigma_\star$ at the time when the relevant cosmological scales exit the horizon. Such expressions, together with Eq. (1), seed the $\delta N$ formalism in order to calculate the spectrum and the bispectrum of the curvature perturbation including the tree-level and the one-loop contributions [1, 2, 3]:

$$P_{\zeta}^{\text{tree}} = \frac{1}{\eta_\phi^2 \phi_\star^2} \left( \frac{H_\star}{2 \pi} \right)^2, \quad P_{\zeta}^{1-\text{loop}} \simeq \frac{\eta_\sigma^2}{\eta_\phi^4 \phi_\star^4} \exp[4N(|\eta_\sigma| - |\eta_\phi|)] \left( \frac{H_\star}{2 \pi} \right)^4 \ln(kL),$$

$$B_{\zeta}^{\text{tree}} = -\frac{1}{\eta_\phi^2 \phi_\star^2} \left( \frac{H_\star}{2 \pi} \right)^4 4\pi^4 \left( \sum I_k^3 \right),$$

$$B_{\zeta}^{1-\text{loop}} \simeq \frac{\eta_\sigma^3}{\eta_\phi^6 \phi_\star^6} \exp[6N(|\eta_\sigma| - |\eta_\phi|)] \left( \frac{H_\star}{2 \pi} \right)^6 \ln(kL)4\pi^4 \left( \sum I_k^3 \right),$$

where $L$ is the infrared cutoff chosen so that the quantities are calculated in a minimal box.

The important factor in the loop corrections is the exponential. This exponential function is directly related to the quadratic form of the potential with a leading constant term. It will give a large contribution if $|\eta_\sigma| > |\eta_\phi|$. I have chosen the concave downward potential in order to satisfy the spectral tilt constraint, which makes either $\eta_\phi < 0$, if $P_\zeta \simeq P_{\zeta}^{\text{tree}}$, or $\eta_\sigma < 0$, if $P_\zeta \simeq P_{\zeta}^{1-\text{loop}}$, while keeping $|\eta_\sigma| > |\eta_\phi|$.

2. Constraints to Have a Reliable Parameter Space

2.1. Working in a Perturbative Regime

It has been proved by means of an analysis of the Feynman-like diagrams [2, 3], and also by an alternative non-perturbative approach [4], that there exists what it is called a
“coupling constant” $g$ for the potential in Eq. (1). Such a coupling constant allows us to work in a perturbative regime if it is much less than one:

$$g = \eta_\phi^2 \frac{1}{\phi_*^2} \left( \frac{H_*}{2\pi} \right)^2 \exp[2N(|\eta_\phi| - |\eta_\phi|)] \ll 1$$

$$\Rightarrow \left( \frac{\phi_*}{m_P} \right)^2 \gg \frac{rP_\zeta}{8} \eta_\phi^2 \exp[2N(|\eta_\phi| - |\eta_\phi|)] , \quad (4)$$

being $r$ the tensor to scalar ratio.

2.2. Tree-Level or Loop Dominance

Because of the exponential factors in Eqs. (2) and (3) it might be possible that the loop corrections dominate over $P_\zeta$ and/or $B_\zeta$. There are three possibilities in complete connection with the position of the $\phi$ field when the relevant scales are exiting the horizon. Here I will consider only the intermediate $\phi_*$ region, corresponding to the case when $B_\zeta$ is dominated by one-loop corrections and $P_\zeta$ is dominated by the tree-level term, because this is the only possibility which gives interesting and observationally relevant results.

$B_\zeta$ Dominated by One-Loop Corrections and $P_\zeta$ Dominated by the Tree-Level Term: the Intermediate $\phi_*$ Region

Looking at Eqs. (2) and (3) it is required in this case that

$$\frac{\eta_\phi^2}{\eta_\phi^3} \exp[4N(|\eta_\phi| - |\eta_\phi|)] \ll \frac{1}{\phi_*^2 \left( \frac{H_*}{2\pi} \right)^2} , \quad \frac{\eta_\phi^3}{\eta_\phi^6} \exp[6N(|\eta_\phi| - |\eta_\phi|)] \gg \frac{1}{\phi_*^2 \left( \frac{H_*}{2\pi} \right)^2} , \quad (5)$$

which combines to give

$$\frac{rP_\zeta}{8} \eta_\phi^2 \exp[4N(|\eta_\phi| - |\eta_\phi|)] \ll \left( \frac{\phi_*}{m_P} \right)^2 \ll \frac{rP_\zeta}{8} \eta_\phi^3 \exp[6N(|\eta_\phi| - |\eta_\phi|)] . \quad (6)$$

2.3. Spectrum Normalisation Condition

Since we are considering $\zeta$ being generated during inflation, we must satisfy the appropriate spectrum normalisation condition. According to Eq. (2) if $P_\zeta$ is dominated by the tree-level term, we have

$$P_\zeta^{tree} = \frac{1}{\eta_\phi^2 \phi_*^2} \left( \frac{H_*}{2\pi} \right)^2 = \frac{1}{\eta_\phi^2 \phi_*^2} \left( \frac{m_P}{\phi_*} \right)^2 \frac{rP_\zeta}{8} = P , \quad (7)$$
which reduces to
\[ \left( \frac{\dot{\phi}_*}{m_P} \right)^2 = \frac{1}{\eta_\phi^2} \frac{r}{8}, \] (8)

Notice that in such a situation, the value of the $\phi$ field when the relevant scales are exiting the horizon depends exclusively on the tensor to scalar ratio, once $\eta_\phi$ has been fixed by the spectral tilt constraint.

### 2.4. Spectral Tilt Constraint

The current WMAP value for the spectral tilt is $n_\zeta - 1 = -0.042 \pm 0.016$, and again we will consider only the case when $P_\zeta$ is dominated by the tree-level term. That means that the usual spectral index formula applies,

\[ n_\zeta - 1 = -2\epsilon - 2m_P^2 \sum_{ij} V_i N_j N_{ij}, \] (9)
giving the following result once the derivatives of $N$ with respect to $\phi_*$ and $\sigma_*$ have been calculated:

\[ n_\zeta - 1 = -2\epsilon + 2\eta_\phi. \] (10)

The effect of the $\epsilon$ parameter may be discarded in the previous expression, since $\epsilon$ is much less than $|\eta_\phi|$:

\[ \epsilon = \frac{m_P^2}{2} \frac{V_\phi^2 + V_\sigma^2}{V^2} = |\eta_\phi| \left[ \frac{1}{2} |\eta_\phi| \left( \frac{\phi}{m_P} \right)^2 \right] \ll |\eta_\phi|, \] (11)

according to the prescription that the potential in Eq. (1) is dominated by the constant term. Thus, using the central value for $n_\zeta - 1$, we get

\[ \eta_\phi = -0.021. \] (12)

### 2.5. Amount of Inflation

It is well-known that the number of e-folds of expansion from the time the cosmological scales exited the horizon to the end of inflation must be around or less than 70. The slow-roll evolution of the $\phi$ field tells us that such an amount of inflation is given by

\[ N = \frac{1}{|\eta_\phi|} \ln \left( \frac{\phi_{\text{end}}}{\phi_*} \right) \lesssim 70. \] (13)

Up to the moment we do not have a definite mechanism to end inflation in this model. It could not be by means of the violation of the $\epsilon < 1$ condition, since this would imply extrapolating the results to a region where the potential in Eq. (1) is no longer dominated
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by the constant term, and that, of course, would spoil the large non-gaussianity generated. Therefore, it will be assumed that inflation comes to an end when \( |\eta| \phi^2 / 2m_P^2 \sim 10^{-2} \) to be in a safe side, hoping that a mechanism to end inflation at this point may be implemented keeping, or perhaps enhancing, the generated non-gaussianity. Coming back to Eq. (13), we get

\[
N = \frac{1}{|\eta|} \ln \left( \frac{0.141 m_P}{|\eta|^1/2} \phi_* \right) \lesssim 70 , \tag{14}
\]

which leads to

\[
\frac{\phi_*}{m_P} \gtrsim \frac{0.141}{|\eta|^{1/2}} \exp(-70|\eta|) . \tag{15}
\]

3. \( f_{NL} \)

In this section I will calculate the level of non-gaussianity represented in the parameter \( f_{NL} \) by taking into account the constraints presented in Sections 1 and 2 [3]. The level of non-gaussianity, according to the expressions in Eqs. (2) and (3), is in this case given by

\[
-\frac{6}{5} f_{NL} = \frac{B_\xi^{1-loop}}{4\pi^4} \sum \frac{k_i^2 \zeta^2}{k_i^3 (P_{tree} \zeta)^2} \simeq \frac{n_B^3}{n_B^4} \exp[6N(|\eta| - |\eta|)] \left( \frac{H_*}{2\pi} \right)^2 \ln(kL) \\
= \frac{n_B^3}{n_B^4} \exp[6N(|\eta| - |\eta|)] \left( \frac{m_P}{\phi_*} \right)^2 \frac{rP_\zeta}{8} \ln(kL) \\
= \frac{n_B^3}{n_B^4} \exp[6N(|\eta| - |\eta|)] \mathcal{P}_\zeta \ln(kL) , \tag{16}
\]

so that

\[
-\frac{6}{5} f_{NL} \approx -2.5 \times 10^{-9} |\eta| \exp[286 \ln(5.78 \times 10^{-2} r^{-1/2}) (|\eta| - 0.021)] , \tag{17}
\]

where in the last line we have used expressions in Eqs. (8), (12), and (14).

In figure 1 we show lines of constant \( f_{NL} \) in the plot \( r \) vs \( |\eta| \) for the intermediate \( \phi_* \) region in agreement with the constraint in Eq. (6). Notice that by implementing the spectral tilt constraint in Eq. (12) to the spectrum normalisation constraint in Eq. (8) and the amount of inflation constraint in Eq. (15), we may conclude that the tensor to scalar ratio is bounded from below: \( r \gtrsim 1.77 \times 10^{-4} \).

The WMAP satellite observations provide an upper bound on the \( f_{NL} \) value: \( |f_{NL}| \lesssim 120 \), being one of main goals of the forthcoming observations of the anisotropies.
of the cosmic microwave background radiation to reduce even more such a bound. It is estimated that in the future WMAP will do it up to $|f_{NL}| \lesssim 20$, while the PLANCK satellite will do it up to $|f_{NL}| \lesssim 5$.

As it is seen in the plot, the PLANCK observationally allowed range of values for positive $f_{NL}$: $5 \lesssim f_{NL} \lesssim 120$ is completely inside the intermediate $\phi_*$ region, as required. So we conclude that if $B_\zeta$ is dominated by one-loop terms, but $\mathcal{P}_\zeta$ is dominated by the tree-level term, sizeable non-gaussianity is generated even if $\zeta$ is generated during inflation. We also conclude that for non-gaussianity to be observable, primordial gravitational waves must be undetectable.

![Figure 1](image-url) Lines of constant $f_{NL}$ in the $r$ vs $|\eta_\sigma|$ plot. The intermediate $\phi_*$ region is delimited by the LIf and Lff lines. The PLANCK observationally allowed range of values for positive $f_{NL}$: $5 \lesssim f_{NL} \lesssim 120$ is completely inside the intermediate $\phi_*$ region. Notice that the LIf line matches almost exactly the $f_{NL} = 1.75$ line.

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References


Yeinzon Rodríguez
Centro de Investigaciones, Universidad Antonio Nariño,
Cra 3 Este # 47A - 15, Bogotá, y
Escuela de Física, UIS,
Bucaramanga, Colombia
E-mail: yeinzon.rodriguez@uan.edu.co