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ABSTRACT: The aim of this paper is to elaborate a notion of explanation which is applicable to stochastic processes such as quantum processes. The model-theoretic approach was adopted in order to delimit appropriate classes, by defining set-theoretical predicates, of different kinds of physical transformations that quantum systems suffer, either of transitions or of transmutations, by interaction or in a spontaneous manner. To *explain* a singular quantum process consists in showing that it is feasible to *model* it as an indeterministic process of certain specified kind.

Keywords: explanation, model, stochastic process, random event.

1. *Introduction*

The dominant conception, from a logical point of view, of the scientific explanation has been *syntactic*. From Hempel, Popper, Braithwhite, Nagel to Railton it has be maintained that the explanations of physical events consist in *arguments*, either deductive or inductive, with scientific laws as their premises, universal or statistical, respectively. Salmon (1977) abandoned this conception, criticizing it as a dogma of logical empiricism, and worked out a theory of causal explanation, without offering, however, an alternative view of the formal structure of explanations. According to Salmon (1981), to explain physical processes is, ultimately, to fit them into structures formed by causal interactions that produce marks and causal processes that propagate marks. In order to explain a process it is neither necessary nor sufficient to derive a statement, describing it, from a set of premises, but one must conceptualize the process to be explained as a process that is caused by a physical interaction and capable of transmitting causal influences (for a sound critique to Salmon's causal theory see Dowe 2000).

In this paper a *semantic* or model theoretic approach has been adopted in order to provide a concept of explanation, which allows for the modelling of singular quantum processes as indeterministic processes, be they causal or spontaneous. Unlike Salmon, instead of drawing a *picture* whereby the process to be explained or *explanandum* must "fit" in a causal structure which plays the role of *explanans*, I maintain that it is feasible to conceptualise the *explanandum* process in the case of quantum processes as a *model* of a specific kind of indeterministic process, where the model takes the role of *explanans*.

In broad terms, the basic idea of explanation that I outline here consists in that explaining a singular quantum process is tantamount to modelling it as stochastic process, whose random transformations either are caused by certain physical conditions, responsible for production of the process, or stem from a certain physical situation when the process is not produced but rather occurs in a spontaneous manner; appealing, in both cases, to the nomological statements that assigns probabilities to each of the alternative effects or results of the transformation.

Although it is not syntactical, the present proposal shares with Hempel's nomological model the thesis that lawful statements are involved in all scientific explanations; in this sense, it is similar to that model in that it is a covering law view of explanation.

2. *Some Causal Theories*

Several theories of causality have been proposed which were intended to cover broad domains of application, perhaps unconstrained, in the physical world. Nevertheless, some of these theories that stand out cannot succeed in including spontaneous indeterministic processes, such as radioactive decay, in the quantum domain, because they are not productive processes. In order to clear the grounds for the introduction of some notions adequate for processes whose initial physical conditions do not produce alternative random effects but only give rise to them, a very broad review of three of those theories is presented here: The probabilistic theory of causality by Patrick Suppes (1970), the causal-mechanics theory by Wesley Salmon (1981) and the conserved quantity theory by Phil Dowe (2000). These authors assume the quantum indeterminism and recognize the irreducibility of quantum probabilities; hence it is pertinent to consider their theories here.

Suppes's starting point is the basic idea that causes increase the probability of effects, and the stipulation that the conditional probability of effect E, given cause C, is greater than the absolute probability of the effect E. In order to ensure that C is a genuine cause, and not a spurious one, of E, Suppes introduces the additional condition that there is no event F prior to C such that F screens C off from E because the conditional probability of E given both C and F equals the conditional probability of E given only F. A concise formulation of this notion of genuine cause is as follow (see Suppes 1970, pp. 12, 21 and 22):
Let E_t, and $C_{t'}$ be events, with $t' < t$, such that $P(C_{t'})$ is positive. Then $C_{t'}$ is a *genuine cause* of E_t if and only if

(1) $P(E_t \mid C_{t'}) > P(E_t)$.
(2) There is no event $F_{t''}$ with $t'' < t'$, such that $P(C_{t'}, F_{t''}) > 0$ and

$$P(E_t \mid C_{t'}, F_{t''}) = P(E_t \mid F_{t''}).$$

The clearest quantum example that contradicts Suppes's theory is given by Salmon:

> Suppose we have an atom in an excited state to which we shall refer as the 4th energy level. [...] Let $P(m \to n)$ stand for the probability that an atom in the mth level will make a direct transition to the nth level. Assume that the probabilities have the following values:
>
> $$P(4 \to 3) = \frac{3}{4} \qquad P(3 \to 1) = \frac{3}{4}$$
> $$P(4 \to 2) = \frac{1}{4} \qquad P(2 \to 1) = \frac{1}{4}$$
> $$P(3 \to 2) = 0$$

> It follows that the probability that the atom will occupy the 1st energy level in the process of decaying to the ground state is $\frac{10}{16}$; if, however, it occupies the 2nd level on its way down, then the probability of its occupying the 1st is $\frac{1}{4}$. Therefore, occupation of the 2nd level is negatively relevant to occupation of the 1st level. Nevertheless, if the atom goes from the 4th through the 2nd to the 1st level, that sequence constitutes a causal chain, in spite the negative statistical relevance of the intermediate stage. (Salmon 1984, pp. 200-201)[1]

That is, if it is assumed that occupying the second level is the cause C of occupying the first level E, then the conditional probability of E given C, $P(E \mid C) = \frac{1}{4}$, is not greater than the absolute probability of E, $P(E)\big(= \frac{10}{16}\big)$, and thus the former condition (1) fails.

From the former, it is noticeable that Suppes's demand, that the cause raise the probability of the effect, is too strong for the quantum domain, because it excludes some quantum processes, such as the previous, from the class of causal processes. The notion proposed here only requires the conditional probability of the effect, given the cause, to be positive (less that the unity), and it becomes weaker than Suppes' notion, but safe against the previous objection, because it includes a Markovian condition, which entails the intransitivity among indeterministic processes.

Salmon assumes a process ontology; entities that exhibit persistence of structure in a space-time interval, like waves, elementary particles and classical bodies. The processes that are capable of transmitting marks or local signals constitute causal processes while those that are incapable of doing so, such as shadows, are pseudoprocesses (see Salmon 1981, p. 286).

The causal processes propagate their own structure from one space-time region to another and hence are capable of transmitting causal influence. If a pair of causal processes overlap in a space-time point, each one is marked and marks the other, that is to say, both suffer a modification of structure, which constitutes a causal interaction. In general, causal processes transmit physical magnitudes such as charge, momentum and energy, whose values define its 'structure'.

Causal processes do not produce causal influence but only transmit it. The production of causal influence is in charge of causal interactions, in which the exchange of an amount of the former magnitudes takes place, according to the conservation laws of physics.

Salmon defines the fundamental idea of causal interaction as follows:

> CI: Let P_1 and P_2 be two processes that intersect with one another at the space-time point S, which belongs to the histories of both. Let Q be a characteristic that process P_1 would exhibit throughout an interval (which includes subintervals on both sides of S in the history of P_1) if the intersection with P_2 did not occur. Let R be a characteristic that process P_2 would exhibit throughout an interval (which includes subintervals on both sides of S in the history of P_2) if the intersection with P_1 did not occur. Then, the intersection of P_1 and P_2 at S constitutes a causal interaction if: (1) P_1 exhibits the characteristic Q before S, but it exhibits a modified characteristic Q' throughout an interval immediately following S; and (2) P_2 exhibits the characteristic R before S, but it exhibits a modified characteristic R' throughout an interval immediately following S. (Salmon 1984, p. 171)

[1] Salmon points out that although this example is fictitious is similar to term scheme of actual atoms.

In Salmon's theory these two types of mechanisms, propagation and production, constitute the causal structure of the physical world, which forms the basis for the explanation in causal terms of the processes within the domains of physical theories.

A clear counterexample for that causal theory has been presented by Dowe:

> [W]e would like to be able to include in any definition of causal interaction types of interactions other than those that produce modifications to two processes. [...] A genuine example of a Y-type interaction is the decay of radium-226 to radon:
>
> $${}^{226}_{88}\mathrm{Ra} \rightarrow {}^{222}_{86}\mathrm{Rn} + {}^{4}_{2}\mathrm{He}$$
>
> Salmon himself expresses a desire to incorporate λ-type and Y-type interactions (1984:182). Unfortunately, Salmon's causal interactions are defined in terms of two and only two processes. (Dowe 2000, p. 83)[2]

In this manner, Dowe argues, the reason why this type of disintegration process does not qualify as a causal interaction in Salmon's sense is the restriction imposed on the formulation of that notion to two and only two processes, either incoming or outgoing.

Dowe does not stipulate any probabilistic relation between causes and effects but rather puts forward the general idea that a physical interaction is causal if an exchange of a conserved quantity between the objects or processes involved occurs. A conserved quantity, says Dowe, is any magnitude ruled by a conservation law in the available physical theories. Thus, mass/energy, momentum and charge are conserved quantities. The nucleus of Dowe's theory can be expressed by the following two clauses:

> CQ1. A *causal process* is a world line of an object that possesses a conserved quantity.
> CQ2. A *causal interaction* is an intersection of world lines that involves exchange of a conserved quantity. (Dowe 2000, p. 90)

The primitive notions of clause CQ2 are those of world line, intersection and exchange of a conserved quantity. Dowe explains this as follows: "A *world line* is the collection of points on a space-time (Minkowski) diagram that represents the history of the object."; "An *intersection* is simply the overlapping in space-time of two or more processes.", finally, "An *exchange* occurs when at least one incoming, and at least one outgoing process undergoes a change in the value of the conserved quantity." (Dowe 2000, pp. 90, 91 and 92, respectively).

The former notion of causal interaction is not free of quantum counterexamples. As has been seen, Dowe objects to Salmon that the processes of radioactive decay do not fulfill his notion of causal interaction. This is correct, but it is surprising that just this same example does not satisfy the clause CQ2 of Dowe either, and precisely for the same reason, namely, that he requires as a necessary condition for qualifications as a causal interaction that two, or more, processes intersect, i.e., that they overlap in a region conformed by all the points that are common to both (or all) processes.

It is astonishing that Dowe uses this same type of process as an example of his own notions. Below, and after the reproduction of that example of radioactive decay, he argues that: "This qualifies as a causal interaction by CQ2 because there is an exchange

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[2] Note that Salmon's causal interactions are all X-type.

of charge, where the charge of the incoming process is divided between the two outgoing processes." (Dowe 2000, p. 93). Certainly, Dowe is limiting his notion of causal interaction to the requirement that at least one of the incoming processes and at least one of the outgoing processes suffer a modification of a conserved quantity, which is fulfilled by the former example, but he also demands that an *inter*-action occurs for at least two incoming processes.

As we have seen, the processes of spontaneous disintegration present a problem for causal theories, and Dowe's theory is not the exception. The previous objection may be decisive or not; what I try to show, however, is that spontaneous processes entail a challenge to any causal theory in the physical world, if it is intended to reach the quantum domain. Still, in any case, the point to remark on here is that, because of their generality, the former three causal theories are not capable of accounting for the peculiarities of quantum processes, when they exhibit random behaviors. This is mainly due to the following two reasons. Firstly, they do not make any distinction between deterministic classic interactions, of the X-type, such as the collision of two billiards balls, and indeterministic quantum interactions, of the X-type, such as the fusion process. Secondly, they do not depict that in the same physical conditions, in several individual trials, quantum systems of the same kind transform randomly in different routes.

3. Quantum Processes to Be Explained

Instead of assuming an ontology of events, like Hempel, or one of processes, like Salmon, I will assume an ontology of systems —i. e., sets of non separable elementary particles— because quantum processes can be considered to be transformations of quantum systems and these will be the physical entities to be explained.

From the perspective of the type of *natural* transformations that quantum systems endure, two kinds of processes may be distinguished: transformations by transition, which consist in the change of state of the system via a modification of the value of at least one magnitude or state variable of the system, and transformations by transmutation which consist in the conversion of the system itself into another type of system. The Compton Effect and the quantum jumps are examples for the former, while nuclear fission and fusion, radioactive decay and the annihilation of elementary particles are cases of the latter. This classification of natural quantum processes concerns *how* the systems transform but does not address *why* they do so. From the last perspective, quantum systems transform by interactions among particles and spontaneously, without the intervention of any factor that effect it. According to this, we may classify natural quantum processes into two sorts: transformation by interaction and spontaneous transformation. The Compton Effect, nuclear fission and fusion and particles annihilation are transformations through the interaction between particles, while discontinuous change of energy (quantum jumps) and the decay of radioactive elements are spontaneous transformations. Since the purpose here is to lend support to explaining natural quantum processes as stochastic processes, hereon the latter classification has been chosen but combined with the first in such a manner that four kinds of quantum processes result: (1) transition by interaction, (2) transmutation by interaction, (3) spontaneous transition and (4) spontaneous transmutation.

In order to be in a position to model singular processes as indeterministic processes a double task must be undertaken: first, to define a general notion of stochastic or indeterministic process that expresses the non deterministic but rather random behavior of quantum systems and, second, to characterize appropriate notions that allows for individualized quantum processes for each of the former kinds of transformations. The four specific notions of an indeterministic process are subsumed in one general notion of stochastic process and, hence, the models of this latter general notion are candidates for models of some of those specific notions.

The formulation outlined below, concerning that notions, follows the strategic lines sketched by Suppes (1957) to define a set-theoretical predicate in order to delimit a class of models as well as to use a scientific theory with the purpose to analyze causal connections and make causal claims (Suppes 1970, p. 13 and 1984, p. 52).

I wish to subscribe to Suppes's thesis, whereby the meaning of model, as defined by Tarski, is the same for diverse fields but what differs are rather the uses of that concept between one field to another (see Suppes 1960), and sustain, in this vein, that to claim that a singular physical processes is an indeterministic processes, causal or spontaneous, of some specific kind it is required: (i) to define a set-theoretical predicate for an appropriate notion; (ii) to construct a set-theoretical structure from the descriptions that physics provides for it; (iii) to demonstrate that this structure is a *model of* the predicate defined, in the logical sense, and (iv) to exhibit that the structure is a *model for* the physical process, that it fits the structure.

Physical systems may be represented as structures of the form $S = \langle P, M \rangle$ where P is a set of non-separable elementary particles and M is a set of magnitudes or state variables (such as energy, momentum, charge, and position) which intervene in the probabilistic specifications of the states of the system S, where each M_i in M is a state variable associated with a spectrum of possible values in a Borel set of the real numbers. It is convenient to specify the central notion of state description of a physical system: "By the static description of a physical system we mean the rules that assign specified mathematical objects to the states and to the physical quantities of the system, and the prescriptions for calculating the probability distribution of the possible values of every physical quantity when the state of the system is given." (Beltrametti and Cassinelli 1981, p. 3).

This notion applies to system states in a given moment t, specifying probability values to the physical quantities relevant for the description; thus, those quantities are probabilistic state variables. With regard to the evolving over time of quantum systems, which Schrödinger wave functions describes deterministically, one must assume the probabilistic interpretation of such wave functions, proposed by Born, in order to depict the rather random behavior of quantum systems. That probabilistic version of wave functions renders the random manner in which quantum systems transforms by assigning a probability to every component of the superposition of states of the system, which Schrödinger equation implies. Consistently with this, one may say that given a system in a certain state, its transformation is not deterministic to a unique future state, which is physically necessary, but rather it is physically possible, that the system transforms alternatively to various different future states in a random manner. Quantum theory provides the distributions of probability for such transformations that assigns probability values to the alternative states within a range of physically possible final states.

4. Formal Definitions and Models

All the aforementioned quantum processes exhibit a peculiar random behavior. In order to express it, I introduce a definition of a notion of stochastic or indeterministic physical processes. The quantum processes that fulfill that definition can be modeled as stochastic processes.

First, let it say that, the probabilistic state description of a quantum system S stands for physically possible events in S, which here are denoted by $d(S)$. A physical process in a system S, during a time lapse from t to t', consists of a transformation of the system such that the description of S in t is different from the description of S in t', i. e., $d(S)_t \neq d(S)_{t'}$.

Definition 1. *Let* $\boldsymbol{P}$ *be a physical process in a system* S. *Then* $\boldsymbol{P}$ *is a* stochastic (or indeterministic) process *only if*

(1) $\boldsymbol{P} = \langle d_0(S)_t, \{d_1(S)_{t'}, \ldots, d_n(S)_{t'}\}\rangle$ *such that*

(2) $d_0(S)_t$ *depicted the system* S *at a given time* t *and* $\{d_1(S)_{t'}, \ldots, d_n(S)_{t'}\}$ *is the range* R *of the descriptions of* S*, in a posterior time* t'*, that corresponds to possible future events, which are alternative and mutually exclusive, in* S.

(3) For any event $d_i(S)_{t'}$ *in* R, $P\big(d_i(S)_{t'} \mid d_0(S)_t\big)$ *is a positive real number less than 1, and in case that the set* R *is exhaustive:* $\sum P\big(d_i(S)_{t'}\big) = 1$, *with* $i = 1, 2, \ldots, n$.

(4) For any event $d(S)_{t''}$ *temporally prior to* $d_0(S)t$, *the following equality is satisfied:*

$$P\big(d_i(S)_{t'} \mid d_0(S)_t\big) = P\big(d_i(S)_{t'} \mid d_0(S)_t, d(S)_{t''}\big) = \\ = P\big(d_i(S)_{t'} \mid d_0(S)_t, -d(S)_{t''}\big).$$

This notion involves, that it is probable that a random transformation of the system S occurs, from a given event $d_0(S)_t$ to one of the various later possible event $d_i(S)_{t'}$ in R. The first clause specifies the set-theoretical structure of the physical process that would be stochastic, if the other three clauses are satisfied. The second one means, that the correspondence between the initial state and the events in R is *one to many*, while the third one expresses, that the occurrence of the initial event is *not sufficient* in order for the system to transform in an *univocal* manner to one of the posterior states, because none of those states has a probability equal to the unity. Finally, the fourth clause stipulates the Markovian property of a stochastic process, that is, if $d_0(S)_t$ is the present state of a system S, then the future transformation of S is defined only for $d_0(S)_t$ in such a manner that it is *independent* of the preceding states of $d_0(S)_t$ or, better, if the description of the present state is *complete*, then the evolution of S or its conversion to another type of system is independent of how S has become the state $d_0(S)_t$, which means, that the probabilities that the theory assigns to each $d_i(S)_{t'}$ in R, do not depend on any event $d(S)_{t''}$ prior to $d_0(S)_t$, from which the system has transited or transmuted to the present state $d_0(S)_t$.

It is necessary to briefly discuss a question that Hempel set forth as a condition for statistical explanations, namely, the high probability requirement, which might represent a potential objection to the proposal stated herein. According to this requirement, in order to explain an event, it is necessary that the probability value of the statistical law, which is part of the *explanans* as a major premise, is high, close to the unity. Otherwise, if that probability value is low, the event does not admit of an explanation; further, if there are two alternative and exclusive events, each equally probable, then neither can be explained. The consequence of such a requirement in the quantum domain is immediate: a variety of kinds of quantum events and processes whose probability values are not high but middle or low, lack an explanation.

Since that condition is too strong and restrictive, it may be expected to be rejected, as Salmon in effect does: "If one and the same probability distribution over a given partition of a reference class provide the explanation for two separate events —one with a high probability and the other with a low probability— the two explanations are equally valuable." (Salmon 1984, p. 89). The fact that innumerable cases of singular quantum processes have a low probability value is no obstacle for their eventual explanation. Thus, for example, in the case of a radioactive decay of polonium-216 to lead-212 or to astatine-216 —whose respective probabilities are 0.99987 and 0.00013—, if a model is adequate to account for the first process as an indeterministic process, then this same model is likewise adequate to account for the second as an indeterministic process, no matter how unlikely it is.

There is a broad spectrum of quantum processes that could be modeled by this notion, because they all have a random character, which makes them candidates for such stochastic processes.

In the transformations of transition by interaction, where the system develops from the present state to one of the various later states, the particles of the system do not undergo any conversion, while some of its magnitudes will change, varying its values. The Compton scattering is an example of this: the photon and the electron that suffer a collision persist as such after the impact, while the energy and momentum of both change and adopt different values within ranges of possible values.

In a system which is composed of an electron and a photon and whose state $d_0(S)_t$ at the moment of the collision, is defined by the values of energy and momentum, each $d_i(S)_{t'}$ represents a physically possible event in R after the impact, which corresponds to a change of the values of those relevant magnitudes within its respective ranges. Thus, it is possible to construct a set-theoretical structure of the form of the clause (1), which at the same time fulfills the clause (2). As all the events $d_i(S)_{t'}$ have a positive probability value, less than 1, also clause (3) is satisfied. The satisfaction of clause (4) resides in the fact, that the future transformations of the system, given the state $d_0(S)_t$, are independent of any event prior to it, in particular, of how either electron and photon have adopted their respective values of energy and momentum before the collision. In this manner, a model of the former notion of stochastic process may be obtained, which furthermore models this kind of physical processes.

The transformation of transmutation by interaction differs from the former, in that the elements of the system themselves convert into other elements. Fusion processes exemplify this. In the case of a fusion of two deuterons, ${}^{2}_{1}d + {}^{2}_{1}d$, it is physically possible for a transformation to occur by either of two routes: the formation of a nucleus ${}^{3}_{2}\mathrm{He}$, a light isotope of helium, plus a neutron, or the formation of a triton plus a proton. Both alternative routes are about equally probable, with exclusive and exhaustive results.

This kind of physical processes can be modeled as indeterministic processes as follows: The two deuterons form the set of particles P that, together with the relevant magnitudes (charge, energy and momentum), form a physical system S. The process $\boldsymbol{P}$ is formed by the state description of P at the moment of the interaction, which corresponds to the initial event $d_0(S)_t$, and by the results ${}^{3}_{2}\mathrm{He} + n$ and ${}^{3}_{1}t + p$ which constitutes the range R. The first clause is fulfilled, because $\langle\, {}^{2}_{1}d + {}^{2}_{1}d, \{\, {}^{3}_{2}\mathrm{He} + n,\ {}^{3}_{1}t + p\}\rangle$ is a structure such as specified by it. With regards to the second one, we shall consider, that the first element of that pair stands for the state of the system at the time t, at which the conversion of the same takes place and, that the two descriptions in the second element refer to the transformations of the system, at a later time t', which are physically possible, given $d_0(S)_t$. The probabilities of these last two results are positive and its addition is equal to the unity, which fulfills the third clause. Lastly, the fourth clause is satisfied because, again, the transformation of the system, given the event described by $d_0(S)_t$, is independent of the previous events the pair of deuterons comes from.

In these two kinds of transformations it is feasible to recognize certain events —physical interactions—, which may be considered to be the causes, which produce the modification of the systems. A notion of causal interaction, different from those in Salmon's notion, which do not involve the persistence of structure but only space-time continuity, would be adequate for characterizing the connection between the original system and the modified system in causal terms, either in a transformation of transition or of transmutation. A pair of necessary conditions would consist in that, first, in the cases of transformations by interaction, a space-time intersection of the particles of the original systems occurs and, second, in general, that the description of the original system is different from those of the system outcome of the transformation.

Such stochastic processes of either transition or transmutation would be considered of as *indeterministic causal* processes, wherever there is an event that provokes the system transformation in a random manner, adopting one of the possible alternative results.

The indeterministic character of certain quantum processes lies in that the event that gives rise to a process does not determine the transformation to a unique physically necessary result. One would attribute a causal nature to such processes, granted that the initial event produces the transformation of the system. This may be said of certain processes such as the Compton Effect and nuclear fusion, because one would think, that the physical interactions that occur among the particles that form the respective systems —the events in which the processes arise—, produce the state that the system later adopts or the conversion that the system undergoes.

However, cases of quantum processes such as radioactive decay contradict the idea of production as a means of explaining them as indeterministic causal processes, and hence, it is only feasible to claim that under certain physical conditions of some quantum systems, it is probable that these processes arise, without demanding any factor that produces it. The unstable nature of radioactive elements —because of their atomic constitution— gives rise to, but does not produce their decay. For this kind of processes, a notion of spontaneous transformation may be obtained if one recognizes that the physical condition of the system in a temporal lapse gives rise to its conversion.

In view of the above, the notion of physical conditions that give rise to the different kinds of transformation must include events of physical interaction as well as spontaneous events. Consequently, I will define below adequate notions, under which a variety of quantum processes may be subsumed, both induced by interactions and spontaneous.

For causal transformations, both transitions and transmutations:

Definition 2. *Let* $\boldsymbol{P}$ *be an indeterministic process. Then, for every* $d_i(S)_{t'}$ *in* R*, the pair* $\langle d_0(S)_t, d_i(S)_{t'}\rangle$ *is a* singular causal indeterministic process *only if:*

(1) The transformation $d_0(S)_t \rightarrow d_i(S)_{t'}$ *is prescribed probabilistically by a specific lawful statement.*

(2) The transformation $d_0(S)_t \rightarrow d_i(S)_{t'}$ *involves an interaction between, at least, two particles in the system* S.

If that transformation is merely a change in the values of some magnitudes, we say that it is a *transition by interaction* process. If, in addition, that transformation involves a conversion of some particle in S into another type of particle, it is said to be a *transmutation by interaction* process.

As for spontaneous transformation, both transitions and transmutations:

Definition 3. *Let* $\boldsymbol{P}$ *be an indeterministic process. Then, for every* $d_i(S)_{t'}$ *in* R*, the pair* $\langle d_0(S)_t, d_i(S)_{t'}\rangle$ *is a* singular spontaneous indeterministic process *only if:*

(1) The transformation $d_0(S)_t \rightarrow d_i(S)_{t'}$ *is prescribed probabilistically by a specific lawful statement.*

(2) The transformation $d_0(S)_t \rightarrow d_i(S)_{t'}$ *occurs in an spontaneous manner, without the intervention of any factor that effect it.*

We say that the transformation is a *spontaneous transition* process if the system S suffers just a change of some of the magnitudes. In case that the transformation involves, as well, a conversion of some particle in S to another type of particle, it said to be a *spontaneous transmutation* process.

The former specifications of the general notion of stochastic process, which appeals to specific lawful statements, are intended to apply to *singular* quantum processes. This is suitable to explain singular processes since as Hanson has noted: "[...] there should be one basic equation for every *type* of particle, not for every individual particle." (Hanson 1959, p. 357); and it seems that it is the most that can be done, because as van Fraassen argued: "[...] any explanation of an individual event must be an explanation of that event *qua* instance of a certain kind of event, nothing more can be asked." (van Fraassen 1993, p. 282).

The Compton scattering above was modeled as an indeterministic process. In order to see, that it is also a model of the former notion of transition by interaction process, let it suppose that a particular $d_k(S)_{t'}$ in R is the case, then the pair $\langle d_0(S)_t, d_k(S)_{t'}\rangle$ is modeling a singular process where $d_k(S)_{t'}$ differs from $d_0(S)_t$, in that the values of the relevant magnitudes —say, energy and momentum— of both particles have changed according to a probabilistic lawful statement, fulfilling the condition for being a transition process and the first clause. The satisfaction of the second clause follows from the fact that the collision of the electron and the photon is an interaction that provokes the transformation of the two-particle system. In this way, scattering processes may be modeled as causal indeterministic processes.

Also in the second kind of transformation, causal transmutation of a system (e. g., fission and fusion), a physical interaction is involved, but with the difference that the system itself is converted. In order to exhibit a model of the Definition 2 for this kind of transformation, the previous example of fusion reaction can be used again. It has been seen that the structure $\langle\, {}^2_1d +\, {}^2_1d, \{\, {}^3_2\mathrm{He} + n,\, {}^3_1t + p\}\rangle$ is a model of the notion of indeterministic processes, where the random transformations ${}^2_1d +\, {}^2_1d \rightarrow\, {}^3_2\mathrm{He} + n$ and ${}^2_1d +\, {}^2_1d \rightarrow\, {}^3_1t + p$ are about equally probable. In an individual system, each pair $\langle\, {}^2_1d +\, {}^2_1d,\, {}^3_2\mathrm{He} + n\rangle$ and $\langle\, {}^2_1d +\, {}^2_1d,\, {}^3_1t + p\rangle$ is modeling a singular indeterministic causal process, particularly of transmutation by interaction. With respect to clause (1), the former random transformations are prescribed by a quantum lawful statement as probable and with respect to the condition for being a transmutation process, notice that the second member of both pairs differs from the first in that, as a result of the fusion, the two deuterons convert into a different kind of element, along with an exchange of energy. Since the encounter of the two deuterons qualifies as a physical interaction, clause (2) is fulfilled.

In the third kind of transformation, which consists of the spontaneous transition of a system from an given state to one of the alternative final states $d_i(S)_{t'}$ (e. g., jumps of electrons from an energetic level to a lower one), $d_0(S)_t$ consists in a spontaneous event which modifies the state of the system.

To outline a model of the notion of spontaneous indeterministic process, particularly of the notion of spontaneous transition, let us suppose an excited atomic particle p in state Z_l with energy E_l that will descend spontaneously to the ground state, either to state Z_m, with energy E_m, or to state Z_n, with energy E_n, such that $E_l > E_m > E_n$. Thus, the transitions of the energetic state of the atom are $Z_l \rightarrow Z_m$ and $Z_l \rightarrow Z_n$, with an amount of energy emitted $E_l - E_m$ and $E_l - E_n$, respectively.

The corresponding physical system is represented by $\langle p, E\rangle$ while the process by $\langle Z_l, \{Z_m, Z_n\}\rangle$. The conditional probabilities $P(Z_m \mid Z_l)$ and $P(Z_n \mid Z_l)$ are positive and, if the state Z_l of p has not been induced by a radiation bundle, their values are independent of any event prior to the initial state Z_l of the system under consideration. The former satisfies Definition 1 above. With respect to Definition 3, those random transformations are prescribed by a probabilistic lawful statements and the value of the magnitude energy, in each singular case, is modified at the time of emission, satisfying the first clause and the condition of being a transition process. To fulfill the second clause, the state Z_l of p may be considered to be the event that gives rise to the spontaneous transitions of energy. In this way, the pairs $\langle Z_l, Z_m\rangle$ and $\langle Z_l, Z_n\rangle$ are modeling the two singular processes of change of energetic level, and each is a spontaneous transition process with respect to the third definition.

Finally, we have the fourth kind of transformation, spontaneous transmutation of a system. The disintegration of radioactive element bismuth-212 may be modeled, in accordance with this last notion, as a spontaneous transmutation process. In this kind of process, the bismuth, ${}^{212}_{83}\text{Bi}$, can decay by two alternative routes: a negative decay β, which converts to polonium, ${}^{212}_{84}\text{Po}$, or a decay α, which converts to thallium, ${}^{208}_{81}\text{TI}$. In both cases, the charge, number of protons, of the original element is modified. In the first instance, a neutron is converted into a proton and a beta particle, ${}^{1}_{0}n \rightarrow {}^{1}_{1}p + {}^{0}_{-1}e$, increasing the charge in one proton, while in the second instance, an alpha particle, ${}^{4}_{2}\text{He}$, is emitted, diminishing the charge in two protons. The transmutations arise from the change of the charge since the number of protons, which is equal to the atomic number, defines the elements.

The physical system is formed by the bismuth-212 and the values of the charge magnitude (subscript) and the atomic mass (superscript), along with the values of energy and momentum, that is, $S = \langle {}^{212}_{83}\text{Bi}, \{Z, A, E, m\}\rangle$. The physical process is conformed by the original element and the two elements, which result from the possible transmutations, that is, $\boldsymbol{P} = \langle {}^{212}_{83}\text{Bi}, \{ {}^{212}_{84}\text{Po}, {}^{208}_{81}\text{TI}\}\rangle$. The corresponding conditional probabilities are approximately $P({}^{212}_{84}\text{Po} \mid {}^{212}_{83}\text{Bi}) = 0.663$ and $P({}^{208}_{81}\text{TI} \mid {}^{212}_{83}\text{Bi}) = 0.337$.

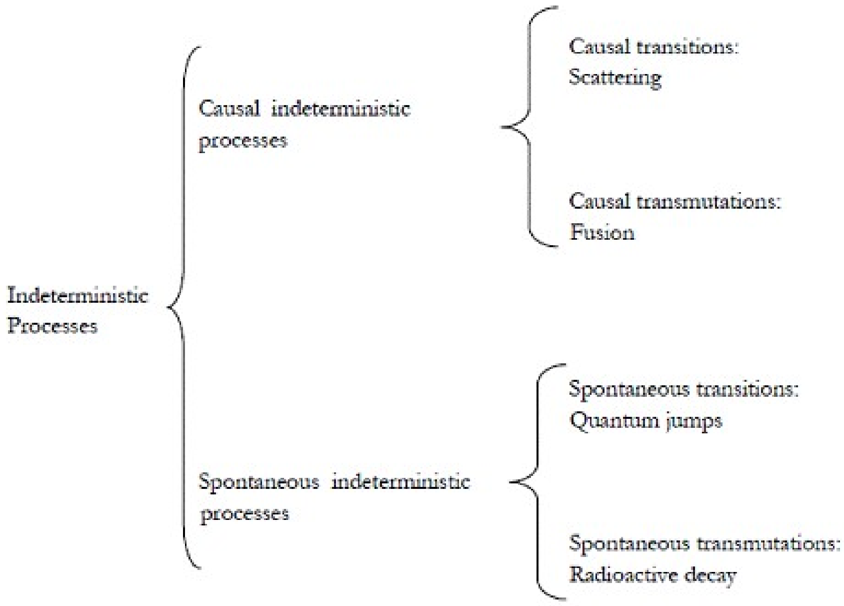
From the former, a model of the notion of stochastic or indeterministic process may be obtained. First, the process $\boldsymbol{P}$ has the set-theoretical structure specified by the clause (1). The set $\{ {}^{212}_{84}\text{Po}, {}^{208}_{81}\text{TI}\}$ is the range R of the alternative possible future events given the initial description of bismuth-212, corresponding to clause (2). The probabilities of both transmutations are positive, and when added together will equal 1, which fulfills clause (3). The satisfaction of the fourth and last clause resides, in that the former probabilities of the two possible transformations are the same, whether the bismuth-212 comes from an astatine element, ${}^{216}_{85}\text{At}$, by α-decay, or from a lead element, ${}^{212}_{82}\text{Pb}$, by β-decay, that is:

$$\begin{aligned} P\big({}^{212}_{84}\text{Po}_{t'} \mid {}^{212}_{83}\text{Bi}_{t}\big) &= P\big({}^{212}_{84}\text{Po}_{t'} \mid {}^{212}_{83}\text{Bi}_{t},\ {}^{216}_{85}\text{At}_{t''}\big) \\ &= P\big({}^{212}_{84}\text{Po}_{t'} \mid {}^{212}_{83}\text{Bi}_{t},\ {}^{212}_{82}\text{Pb}_{t''}\big) = 0.663; \\ P\big({}^{208}_{81}\text{TI}_{t'} \mid {}^{212}_{83}\text{Bi}_{t}\big) &= P\big({}^{208}_{81}\text{TI}_{t'} \mid {}^{212}_{83}\text{Bi}_{t},\ {}^{216}_{85}\text{At}_{t''}\big) \\ &= P\big({}^{208}_{81}\text{TI}_{t'} \mid {}^{212}_{83}\text{Bi}_{t},\ {}^{212}_{82}\text{Pb}_{t''}\big) = 0.337\,. \end{aligned}$$

This is an expression of the fact that the probabilities for a bismuth atom to transform, be it into one of polonium or be it into one of thallium, are independent of whether it is derived from a transformation of an atom of astatine or from a transformation of an atom of lead.

The former shows that this disintegration process may be modeled as an indeterministic process and, at the same time, that it is a candidate for modeling a process of spontaneous transmutation. The satisfaction of clause (2) of Definition 3 is settled for the former, if it is acknowledged that the unstable state of the original element bismuth-212 corresponds to the physical condition that allows for the spontaneous decay process to begin at a certain moment in time, either by negative beta decay or by an alpha decay. The lawful statements that prescribe the two possible transformations of the original system are: $\beta\colon\ {}^{212}_{83}\mathrm{Bi} \rightarrow {}^{212}_{84}\mathrm{Po} + {}^{0}_{-1}e$ and $\alpha\colon\ {}^{212}_{83}\mathrm{Bi} \rightarrow {}^{208}_{81}\mathrm{TI} + {}^{4}_{2}\mathrm{He}$, with the respective probabilities 0.663 and 0.337, which satisfy clause (1). With respect to fulfilling of the condition for being a transmutation process, the description of the transformed system differs from the description of the original system, in that the bismuth-212 particle is converted into a particle of another type of element, along with a modification of the value of the charge magnitude. This shows that the structure $\langle {}^{212}_{83}\mathrm{Bi}, \{ {}^{212}_{84}\mathrm{Po}, {}^{208}_{81}\mathrm{TI}\}\rangle$ is a model specifically of the notion of spontaneous transmutation process, where the pairs $\langle {}^{212}_{83}\mathrm{Bi}, {}^{212}_{84}\mathrm{Po}\rangle$ and $\langle {}^{212}_{83}\mathrm{Bi}, {}^{208}_{81}\mathrm{TI}\rangle$ model the two singular alternative decays.

The subsumption relationship among the models of the different kinds of processes defined here may be summarized in the following schema:



Assuming the irreducibility of quantum probabilities, the present proposal has been worked out in order to model certain quantum processes as stochastic processes, either causal or spontaneous. *Explaining* a singular quantum process consists in showing that it can be *modelled* as an indeterministic process of a specific kind which always involves a lawful component. The model theoretic explanations proposed acquire power from the lawful nature of quantum probabilities and its objective character resides in the conceptualist thesis of Roberto Torretti that: “Falling under general concepts is the very mark of objectivity.” (Torretti 1990, p. 175).

As is known, the statistical interpretation of probability, which is adopted by both Suppes and Salmon, is meaningless for individual cases, as a consequence of its own definition in terms of relative frequencies. Moreover, at the present time, experiments with an individual atom can be performed, hence it is not applicable empirically either. It come out to be unavoidable, to give probability statements interpretations that render particular statements both meaningfully and empirically appropriate for *singular* quantum processes or events.

The interpretation of probability as degree of physical possibility, which I proposed elsewhere (Rolleri 2002) and is applicable to singular cases, has been assumed herein. According to this, the intended meaning of the probabilistic statements consists in that a conditional probability like $P(d_k(S)_{t'} \mid d_0(S)_t) = r$ expresses, in quantitative terms, the degree of the physical possibility that the event described by $d_k(S)_{t'}$ occurs, under the condition that the event described by $d_0(S)_t$ occurs; or that, given $d_0(S)_t$, the occurrence of $d_k(S)_{t'}$ is physically possible to a degree r. Thus, the basic notion of stochastic process intends to assert that, given the initial state of a system S, it is physically possible that any event in the range R occurs; provided they all have a positive probability value —with different or equal degrees, low or high, does not matter. This aims to express the stochastic character of quantum processes; the occurrence of the event $d_0(S)_t$ displays a spectrum of alternative events, each of them physically possible, which occur in a random manner, but according to a probabilistic lawful statement.

5. *Conclusion*

What kind of explanation, if any, do the notions and its models proposed herein provide? In a similar way to the Laplacean explanations of Classical Mechanics events, there are modal, epistemic, nomological and ontic ingredients mixed in those notions. However, the modal element prevails, since the interpretation of probability statements intended above and the proposed explanations have a rather *modal* character. The epistemic element becomes manifest in the impossibility of avoiding the uncertainty in the state descriptions of quantum systems. The nomological ingredient comes from the paramount role of scientific laws in accounting for physical events and processes. The ontic element is rather a thesis about how physical systems are transformed. The ontological thesis consists in that there is a probabilistic regime in quantum domain, according to which the random transformations of quantum systems are prescripts by probabilistic lawful statements.

Salmon rejects all modal explanations of indeterministic processes. From the remark that under the modal conception, "[...] scientific explanations do their jobs by showing that what did happen had to happen" (Salmon 1985, p. 320), he concludes that: "This [modal] conception seems to be impaled on the horns of a trilemma: one must either (1) make an a priori commitment to determinism; (2) admit degrees of necessity; or, (3) grant that, to the extent that there are irreducibly statistical occurrences, they are inexplicable." (Salmon 1985, p. 322). As we can see, Salmon considers only the modality of necessity and overlooks possibility. The consequence of regarding that the irreducibly probabilistic quantum events and processes as inexplicable can be avoided by breaking the second horn of that false trilemma, accepting degrees of possibility for contingent events and processes and representing quantum probability statements as the expressions of the degree of physical possibility of random events and stochastic processes.

From the representation approach to probability (cf. Suppes 1974) assumed here, the usual philosophical view on necessity and possibility modalities that considers it as dual operators, mutually definable, has no relevance, since physical necessity corresponds rather to the upper bound of a closed interval. That is to say, within the extremes of a scale from physical necessity —corresponding to a probability equal to the unit— to physical impossibility —corresponding to the zero probability—, there is an interval of physical possibilities, which is the base for representing probability functions as the quantitative expressions of the degrees of those possibilities. For this reason, there is no demand for admit degrees of necessity.

Finally, we can understand, and eventually explain, that random events occur because, relatively to a scientific theory, it is physically possible that they occur. It can be said that scientific theories do their job of explaining by showing that what did happen *could* happen.

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