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Inconsistency as a Touchstone for Coherence Measures

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ABSTRACT: The debate on probabilistic measures of coherence has focused on evaluating sets of consistent propositions. In this paper we draw attention to the largely neglected question of whether such measures concur with intuitions on test cases with inconsistent propositions and whether they satisfy general adequacy constraints on coherence and inconsistency. While it turns out that, for the vast majority of proposals in their original shape, this question must be answered in the negative, we show that it is possible to adapt many of them in order to improve their performance.

Keywords: coherence; inconsistency; probability; Bayesianism

RESUMEN: El debate sobre las medidas probabilísticas de coherencia se ha centrado en evaluar conjuntos de proposiciones consistentes. En este artículo llamamos la atención sobre una cuestión largamente postergada, a saber, si tales medidas coinciden con las intuiciones sobre los ejemplos de prueba relativos a proposiciones inconsistentes, y si satisfacen condiciones generales de adecuación para la coherencia y la inconsistencia. Aunque esta cuestión se responde negativamente para la mayoría de propuestas, mostramos cómo es posible adaptar muchas de ellas para mejorar su rendimiento en este sentido.

Palabras clave: coherencia, inconsistencia, probabilidad; bayesianismo

1. Introduction

Looking at the flourishing history of formal explications of coherence, we see a pattern of growing complexity. The most prominent strand ranges from Ewing's (1934) early characterisation of coherence in terms of consistency and mutual derivability, over Lewis' (1946) qualitative explication of coherent propositions as "being so related that the antecedent probability of any one of them will be increased if the remainder of the set can be assumed as given premises" (1946, 338), to Douven and Meijs' (2007) sophisticated recipe for probabilistic measures of coherence in the sense of mutual confirmation.[1] Glass (2002) and Olsson (2002), on the other hand, have proposed a measure of coherence in the sense of relative set-theoretic overlap. Since this measure turned out to be too coarse-grained, Meijs (2006) put forward a refined version that is sensitive to subsets of the initial set whose coherence is to be assessed.

The latest contribution to this pattern is Schupbach's (2011) insightful paper on an alternative generalisation of Shogenji's (1999) measure of coherence in the sense of deviation from probabilistic independence. Surmising that "the problem with Shogenji's meas-

[1] For a recent survey of Bayesian confirmation theory see Crupi 2014.

ure has more to do with his means of generalising the measure than with the measure itself" (2011, 125), Schupbach shows that a more elaborate generalisation allows the measure to handle two troubling counterexamples. One of them, which is part of the so-called Depth Problem, will be the starting point of our current investigation. We will show that, while most coherence measures are able to cope with this problem as it stands, slightly modifying Schupbach's test case has a devastating consequence.

In more general terms, we will rivet on the hitherto largely neglected question of whether probabilistic measures of coherence adequately handle inconsistent sets of testimonies. To do so, we focus on two sets of inconsistent testimonies exhibiting different degrees of coherence (or incoherence, if you will). It will turn out that virtually *no* coherence measure adequately captures our intuition on this test case. Since in many cases the reason is that the given measures are not defined for sets of pairwise inconsistent propositions, we propose four ways of adapting them. For all but one measure our results are robust in the sense that all four adaption strategies provide adequate results. In the second last section, two further intuitions on our test case and two general adequacy constraints on coherence and inconsistency are taken into account. It will be shown how the measures fare with the additional intuitions and adequacy constraints under the four different adaption strategies; and it will be pointed out that one of these strategies does not go well with the idea that coherence is mutual confirmation. In the last section, a brief summary and an outlook on further questions concerning coherence and consistency is given.

2. *Probabilistic measures of coherence*

A *measure* of coherence is a (partial) function assigning sets of propositions real numbers representing the sets' degrees of coherence. When a coherence measure takes into account solely probabilistic information relating to the propositions in question, it is called a *probabilistic* measure of coherence. Without providing a motivation for the proposed functions, this section briefly lists some of them. Let $S = \{A_1, \dots, A_n\}$ be a set of propositions.[2]

The naïve deviation measure[3]

$$\mathcal{D}(S) = \frac{P(A_1 \,\&\ldots\&\, A_n)}{P(A_1)\cdot\ldots\cdot P(A_n)}$$

The naïve overlap measure[4]

$$\mathcal{O}(S) = \frac{P(A_1 \,\&\ldots\&\, A_n)}{P(A_1)\vee\ldots\vee P(A_n)}$$

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[2] Here and in what follows we assume that P is a *regular* probability measure so that all and only contradictions are assigned a minimum probability of 0. Accordingly, only tautologies have a probability of 1.

[3] This measure has been proposed by Shogenji (1999).

[4] This measure has been put forward independently by Glass (2002) and Olsson (2002).

Both the naïve deviation and the naïve overlap measure are based on a straightforward generalisation from the case of two to the case of n propositions: simply take the conjunction of *all* propositions in the set and divide it by the product of the marginal probabilities or the probability of the disjunction of all propositions, respectively. However, for both measures, more sophisticated generalisations have been proposed. The common idea behind these refinements is to take into account the coherence values of subsets of S and to consider the weighted average of all coherence values for all subsets with at least two propositions. To have a unified framework, let $[S]^k$ denote the set of all subsets of S with cardinality k. Then the cardinality of $[S]^k$ is $m_k = \binom{n}{k} = n!/k!\ (n-k)!$. Furthermore, let $m = \sum_{2 \leq k \leq n} m_k$.

The refined deviation measure[5]

$$\mathcal{D}^*(S) = \frac{1}{m} \sum_{k=2}^{n} \sum_{S_i \in [S]^k} \log_{10} \mathcal{D}(S_i)$$

The refined overlap measure[6]

$$\mathcal{O}^*(S) = \frac{1}{m} \sum_{k=2}^{n} \sum_{S_i \in [S]^k} \mathcal{O}(S_i)$$

Obviously, both refined ways of generalising the given measures are *subset-sensitive* in the sense that sets being assigned the same initial coherence values on $\mathcal{D}$ or $\mathcal{O}$ might differ regarding $\mathcal{D}^*$ or $\mathcal{O}^*$.

The mutual support account[7]

The family of approaches to coherence as mutual support is based on the following simple and appealing idea: a set's degree of coherence depends on the degree of confirmation (aka support) its elements provide for each other. To implement this idea, choose a probabilistic measure of support $\mathfrak{s}$ and calculate the extent to which each proposition and conjunction of propositions is supported by each remaining proposition and conjunction of them. Finally, the straight average of all results represents the set's degree of coherence. More formally, mutual support measures consider the degree of support between all pairs (S', S''), where S' and S'' are non-empty, disjoint subsets of S. For each set $S = \{A_1, \dots, A_n\}$, let $[S]$ de-

[5] Cf. Schupbach 2011. Note that Schupbach considers alternative weighting systems. To simplify comparison with the other approaches, we focus on the straight average.

[6] This measure is due to Meijs (2006).

[7] The first probabilistic measure based on the idea of coherence as mutual support is Fitelson's (2003, 2004). Later, Douven and Meijs (2007) systematically developed and generalised his account. For a recent constraint-based evaluation see Schippers 2014b.

note the set of all such pairs; then the cardinality l of $[S]$ is given by $l = \sum_{i=1}^{n-1} \binom{n}{i}(2^{n-1} - 1)$. Accordingly, for each support measure $\mathfrak{s}$, the corresponding coherence measure $C_{\mathfrak{s}}$ is defined as follows:

$$C_{\mathfrak{s}}(S) = \sum_{(S',S'')\in[S]} \frac{\mathfrak{s}(\wedge S', \wedge S'')}{l}$$

Obviously, the calculated degree of coherence crucially depends on the chosen measure $\mathfrak{s}$. The following list assembles prominent measures of the support A provides for B from the literature on Bayesian confirmation theory.[8]

Prior-posterior difference

$$d(B,A) = P(B|A) - P(B)$$

Prior-posterior ratio

$$r(B,A) = P(B|A) \,/\, P(B)$$

Counterfactual difference

$$s(B,A) = P(B|A) - P(B|\neg A)$$

Counterfactual ratio

$$j(B,A) = \frac{P(B|A)}{P(B|\neg A)}$$

[8] Note that this list does not exhaust all confirmation measures. Instead, we focus on what seem to be the most prominent ones. Since support-based coherence measures take into account *mutual* support, it is often the case that different confirmation measures yield the same coherence measure. This is due to the fact that some confirmation measures are so related that one can be generated from the other by systematically switching the argument positions. Hence, the above measures represent a wide range of confirmation-based coherence measures. Among the advocates of d are Gillies (1986) and Jeffrey (1992). Proponents of s include Christensen (1999) and Joyce (1999). Measure j is ordinally equivalent to Joyce's (2008) odds-ratio measure. The measure z has independently been proposed by Crupi et al. (2007) and Siebel (2006). Measures ordinally equivalent to r have been proposed by Horwich (1982), Keynes (1921), Kuipers (2000) and Milne (1996). Measure k or ordinally equivalent measures have been proposed by Kemeny and Oppenheim (1952), Good (1984) and Heckerman (1988). Measures similar to f are to be found in Schippers and Siebel (2012) and Roche (2013).

Relative distance

$$z(B,A)=\begin{cases}\dfrac{P(B|A)-P(B)}{1-P(B)} & \text{if } P(B|A)\geq P(B)\\[2ex] \dfrac{P(B|A)-P(B)}{P(B)} & \text{otherwise.}\end{cases}$$

Factual support

$$k(B,A)=\frac{P(A|B)-P(A|\neg B)}{P(A|B)+P(A|\neg B)}$$

The previous measures calculate the degree of *incremental* support, where a proposition A incrementally supports a proposition B iff $P(B|A) > P(B)$. In contrast, the following measure quantifies a separate kind of confirmation, sometimes called *absolute* confirmation (or firmness). According to this notion, a proposition A confirms a proposition B iff $P(B|A) > r$ for some threshold $r \geq 0.5$ (cf. Carnap 1962). To get a measure with the range $[-1, +1]$ and 0 as the neutral value for $P(B|A) = 0.5$, we propose to measure firmness not simply by the posterior but by the following function.

Firmness

$$f(B,A) = 2 \cdot P(B|A) - 1$$

Since this function is equivalent to $P(B|A) - P(\neg B|A)$, our firmness measure can also be seen as an analogue to the counterfactual difference measure of incremental support. Anyway, inserting one of the seven support measures into the above recipe yields a probabilistic measure of coherence. Thus, together with the first two measures and their refinements, we will consider eleven coherence measures.

Bovens and Hartmann's quasi-ordering

Although the main topic of this paper is probabilistic *measures* of coherence, we wish to include the *quasi-ordering* proposed by Bovens and Hartmann (2003a; 2003b, ch. 1f.). They argue that the following function enables us to determine the relative coherence of two sets of propositions:

$$F_r(S)=\frac{a_0+a_n(1-r)^n}{\sum_{i=0}^{n} a_i(1-r)^i},$$

where a_i is the probability that i of the n propositions in set S are false, and r is the reliability of the propositions' sources. (The sources are supposed to be independent in a specific way; and they are partially reliable such that, on the definition of r, $0 < r < 1$). To be sure, F_r does not represent the propositions' degree of coherence because it is functionally dependent on the credibility of their sources. Bovens and Hartmann's claim is rather that supplementing this formula with a simple assumption makes it possible to compare systems of statements with respect to coherence:

$$S \text{ is at least as coherent as } S' \text{ iff for all values of } r,\ F_r(S) \geq F_r(S').$$

Thus, if the F_r-values for S are, for all degrees of partial reliability, greater (smaller) than the corresponding values for S', then S is more (less) coherent than S'.

3. *The Depth Problem redux*

Schupbach's Depth Problem is based on an objection raised by Fitelson (2003, 196f.) against the naïve deviation measure. Remember that this measure calculates the degree of a set's coherence in terms of the deviation from probabilistic independence. Fitelson points to the fact that there are sets of propositions being n-wise independent (i.e. $P(A_1 \wedge \ldots \wedge A_n) = P(A_1) \cdot \ldots \cdot P(A_n)$) but k-wise dependent for $k < n$ (so that $P(A_{i_1} \wedge \ldots \wedge A_{i_k}) \neq P(A_{i_1}) \cdot \ldots \cdot P(A_{i_k})$ for a subset $\{i_1, \ldots, i_k\} \subsetneq \{1, \ldots, n\}$). Intuitively, these dependencies should also been taken into account when calculating a set's degree of coherence. Hence, "Shogenji's measure does not dig deeply enough into the probabilistic information of the scenario" (Schupbach 2011, 129). The following example stems from Schupbach (2011).

Imagine a court case in which three independent and equally reliable witnesses testify on the culprit of a robbery. All that is known for sure is that the culprit is one out of eight suspects who have been collected by the police and that each of the suspects is equally likely to have committed the robbery. In scenario 1, witness i provides the following information W_i:

W_1: The criminal was either suspect 1, 2 or 3.
W_2: The criminal was either suspect 1, 2 or 4.
W_3: The criminal was either suspect 1, 3 or 4.

In scenario 2, witness i gives the information W_i':

W_1': The criminal was either suspect 1, 2 or 3.
W_2': The criminal was either suspect 1, 4 or 5.
W_3': The criminal was either suspect 1, 6 or 7.

Let $E = \{W_1, W_2, W_3\}$ and $E' = \{W_1', W_2', W_3'\}$. Then, "intuitively, the testimonies of the witnesses in the first scenario 'hang together' much more so than the testimonies of the witnesses in the second scenario" (Schupbach 2011, 129), so that E is more coherent than E'. However, according to Shogenji's measure, both scenarios are on a par, i.e. $C_{Sh}(E) = 64/27 = C_{Sh}(E')$. This example constitutes one of the key motivations for Schupbach's refined generalisation.

As we know from our overview of coherence measures, the majority of them are as fine-grained as Schupbach's measure. Hence, it comes as no surprise that most of them cope with Schupbach's example. Table 1 summarises the measures' performances (for a proof see appendix A). Hence, besides the naïve deviation measure $\mathcal{D}$, only the ratio-based coherence measures C_r and C_j and Bovens and Hartmann's quasi-ordering do not pass the test. The reason for the latter's failure is not that it judges the testimonies in scenario 1 equally or less coherent than the testimonies in scenario 2, but that it abstains from judgement. As a first result, note that the given test case does not only promote Schupbach's refined variant of the naïve deviation measure but also many further proposals.

Table 1: Performance of coherence measures in Schupbach's robber case. A "+" indicates that the measure concurs with the intuition that the testimonies are more coherent in scenario 1, while "–" indicates that the measure violates this intuition.

Coherence measure	$\mathcal{D}$	$\mathcal{O}$	$\mathcal{D}^*$	$\mathcal{O}^*$	C_d	C_r	C_S	C_j	C_z	C_k	C_f	F_r
Schupbach's robber case	–	+	+	+	+	–	+	–	+	+	+	–

4. *A slight modification: inconsistent testimonies*

The testimonies in Schupbach's robber case are consistent because, in both scenarios, all of them were true if suspect 1 would have committed the crime. But consider a variant of Schupbach's example. Assume there to be six suspects where each is equally likely to be the culprit. In scenario 1, witness *i* provides the following information V_i:

V_1: The criminal was either suspect 1 or 2.
V_2: The criminal was either suspect 2 or 3.
V_3: The criminal was either suspect 1 or 3.

In scenario 2, witness *i* testifies to V_i':

V_1': The criminal was either suspect 1 or 2.
V_2': The criminal was either suspect 3 or 4.
V_3': The criminal was either suspect 5 or 6.

The crucial difference between $F = \{V_1, V_2, V_3\}$ and $F' = \{V_1', V_2', V_3'\}$ on the one hand and E and E' from Schupbach's example is that F and F' are both *inconsistent* sets. Nonetheless, it seems that the testimonies in the first scenario are still more coherent (or in other words, less incoherent) than the ones in the second scenario. This is due to the fact that, while in the second scenario the testimonies are also *pairwise* inconsistent, all pairs of testimonies in the first scenario are consistent.[9] Put another way, there is an overlap between

[9] Similarly, the testimonies in the second scenario seem to exhibit a higher degree of *inconsistency*. The relationship between degrees of incoherence and degrees of inconsistency is investigated in Schippers 2014a.

the accused suspects for all pairs of testimonies in the first scenario, whereas in the second one pairwise overlap is empty. We thus conclude that F should be assigned a higher degree of coherence (or alternatively, a lower degree of incoherence) than F'. However, as table 2 shows, this demand is violated by all measures except one (proof in appendix A). We thus have to realise that hardly any account handles inconsistent sets of testimonies in an adequate way.[10]

Table 2: Performance of coherence measures in the modified robber case.

Coherence measure	$\mathcal{D}$	$\mathcal{O}$	$\mathcal{D}^*$	$\mathcal{O}^*$	C_d	C_r	C_S	C_j	C_z	C_k	C_f	F_r
The modified robber case	–	–	–	+	–	–	–	–	–	–	–	–

5. *Adapting the measures*

The foregoing result looks quite devastating. Does it mean that $\mathcal{O}^*$ is the only coherence measure worth further investigation? No, there is still hope – at least for Schupbach's measure $\mathcal{D}^*$ and the mutual support measures.

An analysis of the calculations shows that the first two measures fail our test because they are insensitive to pairwise agreement within inconsistent sets of propositions. More precisely, all inconsistent sets are assigned the *minimal* coherence value. These measures are thus ruled out because they are clearly not sophisticated enough (cf. Siebel 2005). Bovens and Hartmann's quasi-ordering must also be dismissed because, as a matter of principle, it is not defined for inconsistent sets. In Bovens and Hartmann's (2005, 368) own words, "our criterion is meant to impose a quasi-ordering on *consistent* information sets. *Nostra culpa*, we should have made this explicit". This account is thus too limited from the start.

However, the situation is quite different for the remaining measures. For example, Schupbach's refined version of Shogenji's deviation measure fails because of the kind of normalisation built into it. $\mathcal{D}^*$ is an average of *logarithmised* $\mathcal{D}$-values. Since $\mathcal{D}$ is 0 for all sets of inconsistent testimonies, and since the logarithm for 0 is $-\infty$, Schupbach's measure states that both scenarios include maximally incoherent testimonies. One possibility to adapt this measure would be to dispense with the logarithm and simply average over the initial $\mathcal{D}$-values.[11] However, while this can indeed be considered a solution to the test case at hand, it provokes a problem initially motivating Schupbach's choice of the logarithm. Note that $\mathcal{D}$ ranges from 0 to infinity, where all values exceeding 1 indicate coherence. Hence, when we combine various degrees of coherence and incoherence for subsets of a given set of testimonies, it will happen that a high degree of coherence of *one* subset offsets a large

[10] Note that we are concerned with only two test cases in this paper. For an extensive analysis of a wide range of test cases see Koscholke (2014).

[11] Shogenji (2005) anticipated the need to adapt his coherence measure for inconsistent sets of testimonies. His proposal is based on the pair-wise coherence of the elements of a given set of testimonies.

number of high degrees of incoherence of other subsets. Therefore, we propose the following rescaled version $\mathcal{D}\#$ of the naïve deviation measure:

$$\mathcal{D}\# (A_1, \dots, A_n) = 1 - 2^{-\mathcal{D}(A_1, \dots, A_n)}$$

This measure's range is [0,1) with 0.5 indicating neutrality. Accordingly, our preferred refinement of the naïve deviation measure reads as follows:

$$\mathcal{D}^*_\#(S) = \frac{1}{m} \sum_{k=2}^{n} \sum_{S_i \in [S]^k} \mathcal{D}_\#(S_i)$$

This variant passes the test because it assigns a value between 0.5 and 0 to the first scenario and the minimum 0 to the scenario with inconsistent testimonies (see appendix A).

The mutual support measures fail for a similar reason. They remain silent on the scenario with pairwise inconsistent testimonies because the underlying confirmation measures do not cope with such inconsistencies. For example, $d = (V_1', V_2' \,\&\, V_3')$ does not have a value because $P(V_2' \,\&\, V_3')$ is 0 and thus $P(V_1' \mid V_2' \,\&\, V_3')$ not defined. Accordingly, one could try to put forward adapted versions of the confirmation measures in order to expand their domain to inconsistent evidence. From the perspective of classical logic, it might be tempting to adjudicate the *maximal* degree of confirmation when the evidence is inconsistent because, classically, inconsistent propositions deductively entail anything. From the perspective of a coherentist, however, this is an unfortunate attempt because it means conceding a coherence-boosting role to inconsistent propositions. Since inconsistencies are usually taken to have a negative impact on coherence, it seems more adequate to assign *minimal* degrees of confirmation in the case of inconsistent evidence.

However, this adaption does not solve the problem that some measures exhibit singularities for inconsistent *hypotheses*. Fortunately, there is a recipe accounting for both of these problems at once (cf. Fitelson 2004, Roche 2013). Note that, again classically, evidence A implies the negation of hypothesis B not only if A is inconsistent but also if B is inconsistent. This fact can be used for a case differentiation combining inconsistent evidence and inconsistent hypotheses. If A implies $\neg B$, we assign the greatest lower bound of the corresponding support measure, that is, -1 for d, s, z, k and f, and 0 for r and j. In all other cases, we just take the value provided by the given measure in its original shape.[12]

Table 3 summarises the performance of the adapted measures regarding our test case with inconsistent testimonies (proof in appendix A). Hence, the first adaption strategy, namely assigning minimal degrees of confirmation in the case of inconsistent evidence or hypotheses, leads to improved outcomes of all support-based coherence measures.

[12] The resulting measure C_k is Fitelson's (2003, 2004). Roche (2013) proposed a measure similar to the corresponding variant of C_f.

Table 3: Performance of support-based coherence measures on the first adaption strategy.

Coherence measure	C_d	C_r	C_S	C_j	C_z	C_k	C_f
The modified robber case	+	+	+	+	+	+	+

However, since the first strategy might smack of adhocness, we consider three alternative proposals.[13] The first two try to solve the problem from a confirmational standpoint. The issue then is how much inconsistent evidence confirms or disconfirms a hypothesis, and a natural answer is that it has *no* evidential impact at all. The same seems to hold for inconsistent hypotheses: whatever the evidence may be, it can neither support nor undermine such a hypothesis. Given these thoughts, we suggest to assign not the minimum but the *neutral* degree of confirmation once the evidence or the hypothesis is inconsistent, viz., 0 for d, s, z, k and f, and 1 for r and j. If both evidence and hypothesis are consistent, then again the original measure's value will be chosen. The results for this second proposal can be seen in table 4 (proof in appendix A). As this table indicates, the second adaptation strategy yields improved results for all coherence measures based on *incremental* support. However, this is not the case for the measure based on the *absolute* concept of confirmation because here we get identical values for the scenarios.

Table 4: Performance of support-based coherence measures on the second adaption strategy.

Coherence measure	C_d	C_r	C_S	C_j	C_z	C_k	C_f
The modified robber case	+	+	+	+	+	+	–

The third strategy is a mixture of the first and the second. One of the basic ideas here is that *consistent* evidence maximally disconfirms hypotheses whose negation follows from the evidence. Since, classically at least, the negation of an inconsistency follows from any evidence, we should therefore, parallel to the first strategy, assign the lowest confirmation value if the hypothesis is inconsistent and the evidence consistent. In the case of *inconsistent* evidence, however, the second strategy comes into play insofar as such evidence is taken to have neither a positive nor a negative evidential impact. That is, in such a case the neutral degree of confirmation is assigned. The results for this mixed strategy are given in table 5.

Table 5: Performance of support-based coherence measures on the third adaption strategy.

Coherence measure	C_d	C_r	C_S	C_j	C_z	C_k	C_f
The modified robber case	+	+	+	+	+	+	+

13 We thank two anonymous referees for suggesting the last two strategies to us.

There is yet a fourth possibility to improve the measures' performance: simply neglect the troublesome elements entering the calculation of each measure's average. That is, in order to determine the coherence of a set of propositions, we simply take the average of *defined* confirmation values. As is shown in table 6, this strategy yields the same positive results as the first and third one (proof in appendix A).

Table 6: Performance of support-based coherence measures on the fourth adaption strategy.

Coherence measure	C_d	C_r	C_S	C_j	C_z	C_k	C_f
The modified robber case	+	+	+	+	+	+	+

6. *Including two further intuitions and two adequacy constraints*

We have offered four proposals for adapting the measures of coherence built after Douven and Meijs' recipe. For the vast majority of proposals, the results are robust insofar as they do not depend on the chosen modification: no matter which suggestion we adopt, all of the given measures let the pairwise consistent testimonies be more coherent than the pairwise inconsistent ones, the only exception being the firmness-based measure. So far, so good. But let us broaden the perspective now by including two further intuitions on our test case and by proceeding to general adequacy constraints on coherence and consistency.

Up to now, we were only concerned with one intuition:

(Int1) The testimonies in scenario 2 have a lower degree of coherence than the testimonies in scenario 1.

It seems that we can go beyond this comparative claim by adding a qualitative and a quantitative claim. First, even if the testimonies in scenario 1 are not pairwise inconsistent, they are still inconsistent as a whole. This could be seen as a reason for judging them *incoherent*:

(Int2) The testimonies in scenario 1 are incoherent.

Second, since the testimonies in scenario 2 are pairwise inconsistent, they exhibit an extremely strong kind of inconsistency. It thus appears that they are not only incoherent but incoherent *to a large extent*:

(Int3) The testimonies in scenario 2 are highly incoherent.

Table 7 shows that, regardless of what type of adaption we choose, there are support-based measures not complying with both of these further intuitions (proof in appendix A). For example, C_j invariably delivers the neutral value for scenario 1 and is thus in conflict with the second intuition in any case. Even worse, when we adopt the second strategy by assigning the neutral degree of confirmation in the case of inconsistent evidence or hypotheses, we are left with no measure confirming to intuition 3. Here the values of all measures are either in the middle or the upper half of their incoherence ranges. They thus judge the testimonies in the second scenario not highly but only moderately incoherent.

Table 7: Performance of support-based coherence measures as to the two further intuitions.

Coherence measure	C_d	C_r	C_S	C_j	C_z	C_k	C_f
Minimum value strategy, Int2	+	+	+	–	+	+	+
Minimum value strategy, Int3	+	+	+	+	+	+	+
Neutral value strategy, Int2	+	+	+	–	+	+	+
Neutral value strategy, Int3	–	–	–	–	–	–	–
Mixed value strategy, Int2	+	+	+	–	+	+	+
Mixed value strategy, Int3	+	+	+	+	+	+	+
Defined value strategy, Int2	+	+	+	–	+	+	+
Defined value strategy, Int3	–	+	–	+	+	+	+

The second intuition might be considered an instance of a general adequacy constraint to be found, among other things, in BonJour's coherentist classic *The Structure of Empirical Knowledge*. "A system of beliefs is coherent", we read there, "only if it is logically consistent" (BonJour 1985, 95; cf. Bartelborth 1999, 136). Strictly speaking, this formulation leaves open the possibility that inconsistent sets are neither coherent nor incoherent; but it is presumably meant to be understood in the slightly stronger sense that such sets are incoherent. Moreover, a set of propositions can be inconsistent because it *contains* an inconsistent proposition, and it can be inconsistent because two or more of its elements *entail* an inconsistency. We will focus on the former type so that our adequacy constraint reads as follows:

(AC1) Sets with an inconsistent proposition are incoherent.

Note, however, that this is not plain sailing. BonJour himself suggests in a footnote that a sufficient number of coherence-boosting relations between the propositions in question could compensate for inconsistencies (cf. BonJour 1985, fn. 7, 240; Amaya 2007, 441). The first adequacy constraint is thus debatable, and the same holds for the second intuition. Note furthermore, that a positive result for a particular measure regarding constraint AC1 does not mean that the measure also satisfies the general constraint that *all* inconsistent sets are incoherent. On the other hand, a measure's violation of AC1 entails that the measure also violates the general constraint.

In contrast, the following adequacy constraint seems to be quite solid. Even if one allows inconsistent sets to be coherent under certain conditions, one will proscribe *raising* the coherence of an otherwise consistent set by adding an inconsistent proposition (cf. Bartelborth 1996, 193). That much seems evident. For example, if the testimonies of witnesses cohere and a further witness appears on the scene stating something inconsistent, then the resulting set of testimonies is less coherent than the original set. Likewise, adding an inconsistent testimony to an incoherent set brings about an even more incoherent set. There might be one case where the coherence does *not* go down: given that a consistent set can be maximally incoherent anyway, adding an inconsistent proposition cannot make such a set less coherent. However, in all other cases inconsistencies have a negative impact:

(AC2) Adding an inconsistent proposition to a consistent set lowers coherence (provided the set is not maximally incoherent).

As table 8 displays, the first adaption strategy is the only one leading to a general satisfaction of the second constraint. On the fourth strategy, this constraint is met only by the firmness-based measure; and on the second and third strategy, it is met by no measure at all. As to the second proposal applied to measures C_r and C_j, if the consistent set is incoherent, the set with the inconsistency is less incoherent. And C_d, C_s, C_z, C_k and C_f let the set with the inconsistency be just as coherent as the original set when the latter is neither coherent nor incoherent. The third account leads to results being a little weaker but nevertheless in conflict with the second constraint. Moreover, on the second, third and fourth strategy, all or almost all measures come into conflict with the first adequacy constraint because they rule that the set with the inconsistency is coherent if the original set is coherent (proof in appendix B).

Table 8: Performance of support-based coherence measures as to the two adequacy constraints.

Coherence measure	C_d	C_r	C_S	C_j	C_z	C_k	C_f
Minimum value strategy, AC1	+	–	+	–	+	+	+
Minimum value strategy, AC2	+	+	+	+	+	+	+
Neutral value strategy, AC1	–	–	–	–	–	–	–
Neutral value strategy, AC2	–	–	–	–	–	–	–
Mixed value strategy, AC1	–	–	–	–	–	–	–
Mixed value strategy, AC2	–	–	–	–	–	–	–
Defined value strategy, AC1	–	–	–	–	–	–	+
Defined value strategy, AC2	–	–	–	–	–	–	+

But which kind of adaption is the means of choice all things considered? If we take into account all intuitions and adequacy constraints and just count for every strategy how many measures satisfy these intuitions and constraints, the winner is clearly *strategy one*. It is the only strategy on which all but one measure satisfy the three normative intuitions; and there is no other strategy allowing more measures to satisfy both adequacy constraints. Hence, the first proposal, where inconsistent evidence and inconsistent hypotheses lead to the smallest possible confirmation value, seems to be the best option for adapting coherence measures in the light of inconsistent testimonies. What is more, this strategy was already established by scholars like Fitelson (2004) and Roche (2013) and thus owes some additional merits.

However, from a confirmational point of view this strategy is highly problematic. Remember that the measures in question are meant to quantify coherence in terms of mutual *support*. If we take that seriously, the underlying measures d, r, s, j, z, k and f must be fine-tuned in line with the principles regulating support. But then adopting the first strategy means burdening oneself with the claims that (1) inconsistent evidence *maximally disconfirms* any hypothesis and (2) an inconsistent hypothesis is *maximally disconfirme*d by any evidence. Since these presuppositions are questionable at least, it is far from clear that the coherence measures resulting from the first proposal can be considered measures of mutual support.

First of all, it seems to make much more sense to say that inconsistent *evidence* has *no* evidential impact on any hypothesis whatsoever. This intuition is the common core of strategies two and three. The second strategy assumes furthermore that an inconsistent *hypothesis* can neither be confirmed nor disconfirmed by any evidence. Given that this strategy produces devastating results as to intuition 3 and both adequacy constraints, however, opting for it seems to be a non-starter when it comes to the coherence of inconsistent testimonies. The mixed account three assesses inconsistent *hypotheses* in line with the first one, but inconsistent *evidence* in line with the second one. As to intuition 3, it improves on the second account, but it is equally bad regarding both adequacy constraints. Having to choose between these strategies, we appear to be on the horns of a dilemma. Either we stick to measuring coherence as mutual support and thus adopt the second or third strategy; but then we do not come to grips with inconsistencies because, among other things, we cannot satisfy the adequacy constraints. Or we choose the first strategy and thus come to grips with inconsistencies; but then we do not measure coherence in the sense of mutual support anymore.[14]

Does the fourth proposal provide a loophole? This proposal consists in remaining with the original support measures while neglecting the undefined cases when it comes to coherence. To accept this proposal means to take no stance on the degree of confirmation in the case of inconsistent evidence or hypotheses. Hence, it is attractive for people who want to adhere to the conception of coherence as mutual support but think that there is no true approach to inconsistent evidence or hypotheses. However, such people should drop all of the given measures except the firmness variant because the latter is then the only measure satisfying the adequacy constraints.

Remember that there are two further measures conforming to the thought that the pairwise inconsistent testimonies in scenario 2 are more incoherent, namely, Meijs' variant $\mathcal{O}^*$ of the naïve overlap measure and our Schupbach-like variant $\mathcal{D}^*_\#$ of the naïve deviation measure. How do they fare with the additional intuitions and adequacy constraints? $\mathcal{O}^*$ and $\mathcal{D}^*_\#$ fare well with both of these intuitions (proof in appendix A), and $\mathcal{O}^*$ also satisfies both constraints (proof in appendix B). But $\mathcal{D}^*_\#$ needs the same type of helping hand as the support-based measures because the underlying Shogenji measure $\mathcal{D}$ is not defined when one of the propositions is inconsistent. Unlike the support-based measures, $\mathcal{D}^*_\#$ is not an average of confirmation-values but of the coherence-values given by our rescaled version $\mathcal{D}_\#$ of the Shogenji measure $\mathcal{D}$. Thus, to adopt the first strategy means here to let $\mathcal{D}_\#$ assign minimum coherence to all sets containing an inconsistent proposition. The second strategy would be to let $\mathcal{D}_\#$ assign the neutral coherence value in these cases. The third strategy,

[14] However, see the remarks on pairs of inconsistent propositions in the conclusion.

where we distinguish between inconsistent evidence and inconsistent hypotheses, does not seem to make sense for deviation-based measures and will thus not be taken into account. The fourth strategy simply is to exclude these undefined cases from the calculation of the average. It can then be shown that the first strategy is the only one where $\mathcal{D}^*_\#$ satisfies both constraints (proof in appendix B).

Table 9 brings together all results. What is the meaning of it? First, if you want *all* intuitions and adequacy constraints to be met, C_r and C_j are ruled out. For both satisfy (AC1) under no strategy, and C_j also violates intuition 2 under all strategies. The measures remain-

Table 9: Performance of all coherence measures as to all intuitions and adequacy constraints.

Coherence measure	$\mathcal{O}^*$	$\mathcal{D}^*_\#$		C_d	C_r	C_S	C_j	C_z	C_k	C_f
Intuition 1	+	+	Strategy 1	+	+	+	+	+	+	+
			Strategy 2	+	+	+	+	+	+	–
			Strategy 3	+	+	+	+	+	+	+
			Strategy 4	+	+	+	+	+	+	+
Intuition 2	+	+	Strategy 1	+	+	+	–	+	+	+
			Strategy 2	+	+	+	–	+	+	+
			Strategy 3	+	+	+	–	+	+	+
			Strategy 4	+	+	+	–	+	+	+
Intuition 3	+	+	Strategy 1	+	+	+	+	+	+	+
			Strategy 2	–	–	–	–	–	–	–
			Strategy 3	+	+	+	+	+	+	+
			Strategy 4	–	+	–	+	+	+	+
Constraint 1	+	+	Strategy 1	+	–	+	–	+	+	+
		–	Strategy 2	–	–	–	–	–	–	–
			Strategy 3	–	–	–	–	–	–	–
		–	Strategy 4	–	–	–	–	–	–	+
Constraint 2	+	+	Strategy 1	+	+	+	+	+	+	+
		–	Strategy 2	–	–	–	–	–	–	–
			Strategy 3	–	–	–	–	–	–	–
		–	Strategy 4	–	–	–	–	–	–	+

ing are the refined overlap measure $\mathcal{O}^*$; the refined deviation measure $\mathcal{D}^*_{\#}$; C_d, C_s, C_z and C_k under strategy one, and C_f under strategy one and three. However, since strategy one does not go well with the notion that coherence is mutual support, advocates of this notion seem to be left with no other option than the mutual firmness variant C_f under strategy three.

Second, if you think that intuition 2 and constraint 1 are shaky because sets containing just one inconsistency can be coherent, then C_r and C_j remain in the running for the moment. Furthermore, like C_f, they allow for intuition 3 not only under strategy one but also under strategy three and four. However, since they conform to constraint 2 only under strategy one, which is in conflict with the notion that coherence is mutual support, again, they seem to provide no option for someone who wants to quantify this notion.

More generally, there are two stances one could take up vis-à-vis the results in table 9. On the one hand, one could try to devise arguments for the superiority of a particular adaption account and then dismiss measures violating certain intuitions or constraints on this account. On the other hand, one could argue for the superiority of a particular coherence measure and dismiss those adaption strategies giving rise to negative results for this very measure. For example, if one adheres to the firmness-based coherence measure C_f (like Roche 2013 seems to do), then the only possible kinds of adaption are the maximum disconfirmation strategy and the strategy based on defined values (where the former was indeed proposed in Roche 2013). It thus seems that our results should in general be considered a toolbox for coherentists who can draw on our work when arguing either for the superiority of an adaption strategy or the superiority of a coherence measure.

7. *Conclusion and outlook*

In their original shape, the vast majority of probabilistic measures did not properly quantify the degree of coherence of the inconsistent testimonies in our test case. The naïve deviation measure and the naïve overlap measure delivered the untoward judgement that both scenarios are equally incoherent. The measures of mutual support in line with Douven and Meijs' recipe, as well as the refined deviation measure and Bovens and Hartmann's quasi-ordering, were not even able to tackle the test case because they remain silent on pairwise inconsistent sets in general. The only measure coping with our example was Meijs' refined variant of the naïve overlap measure.

In the next step, we adapted the refined deviation measure by rescaling it, and we put forward four different proposals for extending the scope of the mutual support measures to pairwise inconsistent propositions. Although all but one of the measures provided the desired judgement on our test case under each proposal, they encountered serious difficulties when we included two further intuitions and two general adequacy constraints. For the first three adaption strategies ushered in the dilemma that the first one produces satisfying results but is in conflict with the conception of coherence as mutual support, whereas the second and third one are (more) in line with this conception but produce bad results. The fourth strategy offered a way out, the consequence being that coherence is to be quantified with the help of our firmness measure if all intuitions and constraints are to be satisfied. In other words, someone who takes this route is well-advised to view coherence not as mutual *incremental* confirmation but rather as mutual *absolute* confirmation.

However that may be, there is room for further thoughts. While our test case focused on sets of *individually consistent* propositions, it seems that there are also sets of *self-contradictory* propositions exhibiting different degrees of incoherence. These sets pose a threat for all probabilistic measures of coherence. To see why, consider the two sets $S_1 = \{A_1, A_2\}$ and $S_2 = \{A_1, A_3\}$, where

A_1: 2^7 is larger than 128.
A_2: 128 is lower than 2^7.
A_3: 2^7 is lower than 128.

Since $2^7 = 128$, all of these propositions are internally inconsistent. Nonetheless, while A_2 only rephrases what is asserted by A_1, the assertions made by A_1 and A_3 are diametrically opposed. There is thus a sense in which the former propositions, although inconsistent, are less incoherent than the latter. Nonetheless, all probabilities involving any of these contradictions are necessarily identical. Consequently, none of the above approaches can account for the intuitive difference in coherence.

Another example pointing in the same direction was given by Siebel (2005). Imagine a physicist who cannot recall the voltage of a power source recently utilised in an experiment. The only thing she knows for sure is that it was either 1 V or 2 V or … or 50 V. Accordingly, she assigns equal prior probabilities to these 50 alternatives. She asks three of her assistants and receives the following answers:

B_1: The voltage is 1 V.
B_2: The voltage is 2 V.
B_3: The voltage is 50 V.

Given the proximity of the voltages in B_1 and B_2, it seems appropriate to claim that the set $\{B_1, B_2\}$ is less incoherent than $\{B_1, B_3\}$. However, both sets involve pairwise inconsistent propositions. Hence, given equal priors and equal (zero) posteriors, all probabilistic coherence measures will assign identical degrees of coherence to both sets. Again, these measures cannot account for the intuitive difference in coherence. But these guesses need a thorough investigation we leave for further research. Up to this point, the only viable general conclusion is that bringing together coherence and inconsistency is a difficult task.[15]

APPENDIX

A Test cases

TC1 denotes Schupbach's original test case, TC2 is our variant with inconsistent testimonies. TC2 (min.) are the results for the minimum-based adaption, TC2 (neutr.) for the neutrality-based adaptation, TC2 (mixed) for the mixed strategy and TC2 (def.) for the

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approach where only defined constituents are taken into account. The calculations were made with a computer programme in GNU Octave written by Jakob Koscholke.

Measure	TC1		TC2		TC2 (min.)		TC2 (neutr.)		TC2 (mixed)		TC2 (def.)	
	SC 1	SC 2	SC 1	SC 2	SC 1	SC 2	SC 1	SC 2	SC1	SC2	SC 1	SC 2
C_d	0.198	0.188	−0.042	NaN	−0.042	−1.000	−0.042	−0.167	−0.042	−0.750	−0.042	−0.222
C_r	1.556	1.778	0.750	NaN	0.750	0.000	0.750	0.500	0.750	0.250	0.750	0.000
C_s	0.308	0.229	−0.038	NaN	−0.038	−1.000	−0.038	−0.250	−0.038	−0.750	−0.038	−0.333
C_j	2.458	NaN	1.000	NaN	1.000	0.000	1.000	0.500	1.000	0.250	1.000	0.000
C_z	0.311	0.254	−0.375	NaN	−0.375	−1.000	−0.375	−0.500	−0.375	−0.750	−0.375	−1.000
C_k	0.382	0.343	−0.333	NaN	−0.333	−1.000	−0.333	−0.500	−0.333	−0.750	−0.333	−1.000
C_f	0.084	−1.000	−0.500	NaN	−0.500	−1.000	−0.500	−0.500	−0.500	−0.750	−0.500	−1.000
$\mathcal{D}$	2.370	2.370	0.000	0.000								
$\mathcal{O}$	0.250	0.143	0.000	0.000								
$\mathcal{D}^*$	0.312	0.162	$-\infty$	$-\infty$								
$\mathcal{O}^*$	0.438	0.186	0.250	0.000								
$\mathcal{D}^*_\#$	0.354	0.230	0.323	0.000								

As to Bovens and Hartmann's quasi-ordering, remember that a_i is the probability that i of the given three testimonies are false. For scenario 1 in Schupbach's original case, we thus get:

$a_0 = P(\text{suspect } 1) = 1/8$
$a_1 = P(\text{suspect } 4) + P(\text{suspect } 3) + P(\text{suspect } 2) = 3/8$
$a_2 = 0$
$a_3 = P(\text{suspect } 5, 6, 7 \text{ or } 8) = 1/2$

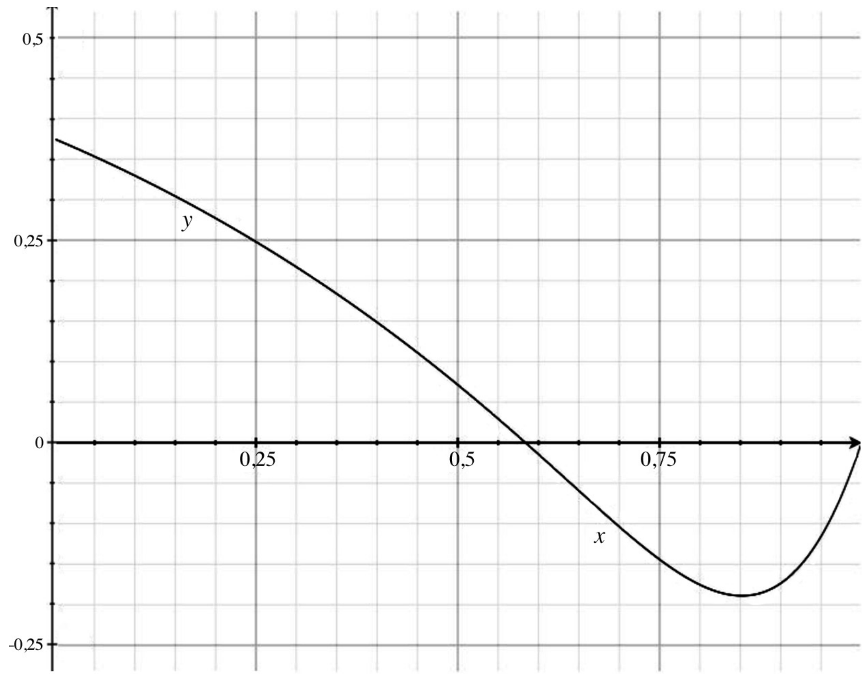
$$F_r(S) = \frac{1/8 + 1/2 \cdot (1-r)^3}{1/8 + 3/8 \cdot (1-r) + 1/2 \cdot (1-r)^3}$$

And for scenario 2:

$a'_0 = P(\text{suspect } 1) = 1/8$
$a'_1 = 0$
$a'_2 = P(\text{suspect } 6 \text{ or } 7) + P(\text{suspect } 2 \text{ or } 3) + P(\text{suspect } 4 \text{ or } 5) = 3/4$
$a'_3 = P(\text{suspect } 8) = 1/8$

$$F_r(S') = \frac{1/8 + 1/8 \cdot (1-r)^3}{1/8 + 3/4 \cdot (1-r)^2 + 1/8 \cdot (1-r)^3}$$

The reliability parameter r ranges from 0 to 1. By subtracting $F_r(S')$ from $F_r(S)$ for these arguments, we obtain the following function graph:



This means that $F_r(S)$ is for some values of r larger and for other values smaller than $F_r(S')$, with the result that Bovens and Hartmann's account does not reach a verdict on the given test case.

B Adequacy constraints

Mutual support measures

Let $S = \{A_1, \dots, A_n\}$ be a consistent set of propositions and $S^* = S \cup \{\bot\}$; and let $[S]$ and $[S^*]$ be the set of pairs of non-empty and non-overlapping subsets of S and S^*, respectively. Then we have the following cardinalities (see Roche 2013):

i) $|[S]| = 3^n - 2^{n+1} + 1$
ii) $|[S^*]| = 3^{n+1} - 2^{n+2} + 1$
iii) $|[S^*] \setminus [S]| = |[S^*]| - |[S]| = 2 \cdot 3^n - 2^{n+1}$

Given the partition of the pairs (T, T') within $[S^*]$ into the pairs (U, U') within $[S]$ and the pairs (V, V') within the remainder $[S^*] \setminus [S]$, we can rewrite the coherence value for an arbitrary coherence measure C based on confirmation measure $\mathfrak{s}$ as follows:

$$C_{\mathfrak{s}}(S^*) = \frac{\sum_{(T,T')\in[S^*]} \mathfrak{s}(T,T')}{|[S^*]|} = \frac{\sum_{(U,U')\in[S]} \mathfrak{s}(U,U')}{|[S^*]|} + \frac{\sum_{(V,V')\in[S^*]\setminus[S]} \mathfrak{s}(V,V')}{|[S^*]|} =$$

$$= \frac{|[S]|}{|[S^*]|} \cdot \frac{\sum_{(U,U')\in[S]} \mathfrak{s}(U,U')}{|[S]|} + \frac{\sum_{(V,V')\in[S^*]\setminus[S]} \mathfrak{s}(V,V')}{|[S^*]|} =$$

$$= \frac{|[S]|}{|[S^*]|} \cdot C_{\mathfrak{s}}(S) + \frac{\sum_{(V,V')\in[S^*]\setminus[S]} \mathfrak{s}(V,V')}{|[S^*]|}$$

We begin with the **first adaption strategy** where the degree of confirmation is the minimum in the case of inconsistent evidence or hypotheses. Since all pairs of the second addend in the equation above are such that $\perp \in V$ or $\perp \in V'$, we get:

$$C_{\mathfrak{s}}(S^*)=\frac{\|[S]\|}{\|[S^*]\|}\cdot C_{\mathfrak{s}}(S)+\frac{\|[S^*]\setminus[S]\|\cdot\min(\mathfrak{s})}{\|[S^*]\|}$$

a) Let $\mathfrak{s} \in \{d, s, z, k, f\}$, so that the minimum is -1 and the neutral value 0. Then:

$$C_{\mathfrak{s}}(S^*)\geq 0$$

$$\Leftrightarrow \frac{\|[S]\|}{\|[S^*]\|}\cdot C_{\mathfrak{s}}(S)+\frac{\|[S^*]\setminus[S]\|\cdot -1}{\|[S^*]\|}\geq 0$$

$$\Leftrightarrow \|[S]\|\cdot C_{\mathfrak{s}}(S)\geq\|[S^*]\setminus[S]\|$$

$$\Leftrightarrow C_{\mathfrak{s}}(S)\geq\frac{\|[S^*]\setminus[S]\|}{\|[S]\|}=\frac{2\cdot 3^n-2^{n+1}}{3^n-2^{n+1}+1}$$

But $3^n > 1$, and therefore $2 \cdot 3^n - 2^{n+1} = 3^n - 2^{n+1} + 3^n > 3^n - 2^{n+1} + 1$. The latter fraction is thus greater than 1, which is in conflict with the fact that the maximum of the given measures is 1. Hence, $C_{\mathfrak{s}}(S^*) < 0$, viz., inconsistent sets of the type S^* are incoherent; and thus the constraint (AC1) is satisfied. (AC2) is also satisfied. Given that $C_{\mathfrak{s}}(S) > -1$, adding an inconsistency lowers coherence because the opposite assumption entails a contradiction:

$$C_{\mathfrak{s}}(S^*)\geq C_{\mathfrak{s}}(S)$$

$$\Leftrightarrow \frac{\|[S]\|}{\|[S^*]\|}\cdot C_{\mathfrak{s}}(S)+\frac{\|[S^*]\setminus[S]\|\cdot -1}{\|[S^*]\|}\geq C_{\mathfrak{s}}(S)$$

$$\Leftrightarrow \frac{\|[S^*]\setminus[S]\|\cdot -1}{\|[S^*]\|}\geq C_{\mathfrak{s}}(S)\cdot\left(1-\frac{\|[S]\|}{\|[S^*]\|}\right)$$

$$\Leftrightarrow C_{\mathfrak{s}}(S)\leq\frac{\|[S^*]\setminus[S]\|\cdot -1}{\|[S^*]\|\cdot\left(1-\frac{\|[S]\|}{\|[S^*]\|}\right)}=\frac{-\|[S^*]\setminus[S]\|}{\|[S^*]\|\setminus\|[S]\|}=\frac{-\|[S^*]\setminus[S]\|}{\|[S^*]\setminus[S]\|}=-1$$

Since the minimum of the measures in question is -1, the latter inequality is satisfied only if $C_{\mathfrak{s}}(S) = -1$, that is, if S is maximally incoherent. Then the addition of an inconsistency leads to a set S^* with the same coherence value, namely the minimum. In all other cases, $C_{\mathfrak{s}}(S^*) < C_{\mathfrak{s}}(S)$.

b) Now let $\mathfrak{s} \in \{r, j\}$. Here the minimum is 0 and the neutral value 1, with the following result:

$$
\begin{aligned}
& C_{\mathfrak{s}}(S^*)>1 \\
\Leftrightarrow & \frac{\|[S]\|}{\|[S^*]\|}\cdot C_{\mathfrak{s}}(S)+\frac{\|[S^*]\backslash[S]\|\cdot 0}{\|[S^*]\|}>1 \\
\Leftrightarrow & C_{\mathfrak{s}}(S)>\frac{\|[S^*]\|}{\|[S]\|}
\end{aligned}
$$

Since S^* contains one element more than S, so that $|[S^*]| > |[S]|$, the fraction on the right side of the latter inequality is greater than 1. Hence, S^* can be coherent as long as S is coherent (to a certain degree depending on its size). But the coherence of S^* cannot be higher than the one of S because this would result in a contradiction:

$$
\begin{aligned}
& C_{\mathfrak{s}}(S^*)>C_{\mathfrak{s}}(S) \\
\Leftrightarrow & \frac{\|[S]\|}{\|[S^*]\|}\cdot C_{\mathfrak{s}}(S)+\frac{\|[S^*]\backslash[S]\|\cdot 0}{\|[S^*]\|}>C_{\mathfrak{s}}(S) \\
\Rightarrow & \|[S]\|>\|[S^*]\|
\end{aligned}
$$

The latter is impossible because $|[S^*]| > |[S]|$. On the other hand, S^* can possess the same coherence value as S; but this is the case only if $C_{\mathfrak{s}}(S) = 0$, that is, if S is maximally incoherent.

According to the **second adaption strategy**, the degree of confirmation is the neutral value when the evidence or the hypothesis is inconsistent. Again, since all pairs (V, V') in $[S^*] \setminus [S]$ are such that $\perp \in V$ or $\perp \in V'$, we get:

$$
C_{\mathfrak{s}}(S^*)=\frac{\|[S]\|}{\|[S^*]\|}\cdot C_{\mathfrak{s}}(S)+\frac{\|[S^*]\backslash[S]\|\cdot \mathrm{neutr}(\mathfrak{s})}{\|[S^*]\|}
$$

a) Let $\mathfrak{s} \in \{d, s, z, k, f\}$. These measures have the neutral value 0; therefore:

$$
\begin{aligned}
& C_{\mathfrak{s}}(S^*)>0 \\
\Leftrightarrow & \frac{\|[S]\|}{\|[S^*]\|}\cdot C_{\mathfrak{s}}(S)+\frac{\|[S^*]\backslash[S]\|\cdot 0}{\|[S^*]\|}>0 \\
\Leftrightarrow & C_{\mathfrak{s}}(S)>0
\end{aligned}
$$

Hence, if the consistent set S is coherent, the inconsistent expansion S^* is also coherent. On the other hand, S^* cannot be more coherent than S because, again, this assumption entails a contradiction:

$$C_{\mathfrak{s}}(S^*) > C_{\mathfrak{s}}(S)$$

$$\Leftrightarrow \frac{\|[S]\|}{\|[S^*]\|} \cdot C_{\mathfrak{s}}(S) + \frac{\|[S^*] \setminus [S]\| \cdot 0}{\|[S^*]\|} > C_{\mathfrak{s}}(S)$$

$$\Rightarrow \|[S]\| > \|[S^*]\|$$

However, if $C_{\mathfrak{s}}(S)$ is 0, then $C_{\mathfrak{s}}(S^*)$ is also 0. In other words, the addition of an inconsistency leads to a set with the same coherence value if the original set is neither coherent nor incoherent. This is also in conflict with the adequacy constraint that such an addition always lowers coherence.

b) Now let $\mathfrak{s} \in \{r, j\}$. Since the neutral value of these support measures is 1, we get:

$$C_{\mathfrak{s}}(S^*) > 1$$

$$\Leftrightarrow \frac{\|[S]\|}{\|[S^*]\|} \cdot C_{\mathfrak{s}}(S) + \frac{\|[S^*] \setminus [S]\| \cdot 1}{\|[S^*]\|} > 1$$

$$\Leftrightarrow C_{\mathfrak{s}}(S) > \frac{\|[S^*]\|}{\|[S]\|} - \frac{\|[S^*] \setminus [S]\|}{\|[S]\|} = \frac{\|[S^*]\| - \|[S^*] \setminus [S]\|}{\|[S]\|} = \frac{\|[S]\|}{\|[S]\|} = 1$$

This means that S^* is coherent if S is coherent. Still more, the coherence of S^* can be higher than the one of S because

$$C_{\mathfrak{s}}(S^*) > C_{\mathfrak{s}}(S)$$

$$\Leftrightarrow \frac{\|[S]\|}{\|[S^*]\|} \cdot C_{\mathfrak{s}}(S) + \frac{\|[S^*] \setminus [S]\| \cdot 1}{\|[S^*]\|} > C_{\mathfrak{s}}(S)$$

$$\Leftrightarrow \frac{\|[S^*]\| - \|[S]\|}{\|[S^*]\|} > C_{\mathfrak{s}}(S) \cdot \left(1 - \frac{\|[S]\|}{\|[S^*]\|}\right)$$

$$\Leftrightarrow \frac{\|[S^*]\| - \|[S]\|}{\|[S^*]\|} > C_{\mathfrak{s}}(S) \cdot \frac{\|[S^*]\| - \|[S]\|}{\|[S^*]\|}$$

$$\Leftrightarrow 1 > C_{\mathfrak{s}}(S)$$

Therefore, the coherence value of S^* is higher than the one of S if the latter set is incoherent.

The **third adaption strategy** is a mixture of the first two: according to it, $\mathfrak{s}(B, A)$ is neutral if A is inconsistent and minimal if B is inconsistent. Hence, we get

$$C_{\mathfrak{s}}(S^*) = \frac{\|[S]\|}{\|[S^*]\|} \cdot C_{\mathfrak{s}}(S) + \frac{1}{2} \cdot \frac{\sum_{(W,W') \in [S^*] \setminus [S],\, W \vDash \bot} \min(\mathfrak{s})}{\|[S^*]\|} + \frac{1}{2} \cdot \frac{\sum_{(W,W') \in [S^*] \setminus [S],\, W \vDash \bot} \mathrm{neutr}(\mathfrak{s})}{\|[S^*]\|}$$

a) Let $\mathfrak{s} \in \{d, s, z, k, f\}$, such that the minimum is -1 and the neutral value is 0. Then we get:

$$C_{\mathfrak{s}}(S^*) = \frac{\|[S]\|}{\|[S^*]\|} \cdot C_{\mathfrak{s}}(S) - \frac{1}{2} \cdot \frac{\|[S^*] \setminus [S]\|}{\|[S^*]\|}$$

Regarding adequacy constraint (AC1), this means:

$$C_{\mathfrak{s}}(S^*)_{\mathfrak{s}} > 0$$

$$\Leftrightarrow \frac{\|[S]\|}{\|[S^*]\|} \cdot C_{\mathfrak{s}}(S) > \frac{1}{2} \cdot \frac{\|[S^*] \setminus [S]\|}{\|[S^*]\|}$$

$$\Leftrightarrow \|[S]\| \cdot C_{\mathfrak{s}}(S) > \frac{1}{2} \cdot \|[S^*] \setminus [S]\|$$

$$\Leftrightarrow (3^n - 2^{n+1} + 1) \cdot C_{\mathfrak{s}}(S) > 3^n - 2^n$$

$$\Leftrightarrow C_{\mathfrak{s}}(S) > \frac{3^n - 2^n}{3^n - 2^n + 1}$$

The given inequation is satisfiable: if $n = 2$, then $C_{\mathfrak{s}}(S) > 5/6$, which lies in the range of the coherence measures in question. Hence, these measures violate (AC1) because they allow a set with an inconsistency to be coherent. Even more, as the following derivation shows, they also violate (AC2):

$$C_{\mathfrak{s}}(S^*) \geq C_{\mathfrak{s}}(S)$$

$$\Leftrightarrow \frac{\|[S]\|}{\|[S^*]\|} \cdot C_{\mathfrak{s}}(S) - \frac{1}{2} \cdot \frac{\|[S^*] \setminus [S]\|}{\|[S^*]\|} \geq C_{\mathfrak{s}}(S)$$

$$\Leftrightarrow -\frac{1}{2} \cdot \frac{\|[S^*] \setminus [S]\|}{\|[S^*]\|} \geq \frac{\|[S^*] \setminus [S]\|}{\|[S^*]\|} \cdot C_{\mathfrak{s}}(S)$$

$$\Leftrightarrow C_{\mathfrak{s}}(S) \leq -\frac{1}{2}$$

Accordingly, if S is fairly incoherent, that is, $C_{\mathfrak{s}}(S) < -1/2$, then adding a contradiction yields an increase in coherence; and if S's degree of coherence is equal to $-1/2$, then adding a contradiction has no impact on the degree of coherence.

b) Now let $\mathfrak{s} \in \{r, j\}$ so that the minimum is 0 while the neutral value is 1. Then

$$C_{\mathfrak{s}}(S^*) = \frac{\|[S]\|}{\|[S^*]\|} \cdot C_{\mathfrak{s}}(S) + \frac{1}{2} \cdot \frac{\|[S^*] \setminus [S]\|}{\|[S^*]\|}$$

We get the following result with respect to adequacy constraint (AC1):

$$C_{\mathfrak{s}}(S^*)_{\mathfrak{s}} > 1$$

$$\Leftrightarrow C_{\mathfrak{s}}(S) + \frac{1}{2} \cdot \frac{\|[S^*] \setminus [S]\|}{\|[S^*]\|} > \frac{\|[S^*]\|}{\|[S]\|}$$

$$\Leftrightarrow C_{\mathfrak{s}}(S) > \frac{\|[S^*]\| - \frac{1}{2} \cdot \|[S^*] \setminus [S]\|}{\|[S]\|}$$

$$\Leftrightarrow C_{\mathfrak{s}}(S) > \frac{1}{2} \cdot \frac{\|[S^*]\| + \|[S]\|}{\|[S]\|}$$

$$\Leftrightarrow C_{\mathfrak{s}}(S) > \frac{1}{2} \cdot \frac{(3^{n+1} - 2^{n+2} + 1) + (3^n - 2^{n+1} + 1)}{3^n - 2^{n+1} + 1}$$

To show that this latter inequation is satisfiable, let $n = 2$. Then $C_{\mathfrak{s}}(S) > 1$ if and only if $C_{\mathfrak{s}}(S) > 7/2$ Given that both measures in question range from 0 to ∞, a coherence value above 7/2 is not hard to achieve. We thus conclude that these measures violate (AC1). That the same conclusion holds with respect to (AC2) is shown by the following derivation:

$$C_{\mathfrak{s}}(S^*)_{\mathfrak{s}} \geq C_{\mathfrak{s}}(S)$$

$$\Leftrightarrow \frac{\|[S]\|}{\|[S^*]\|} \cdot C_{\mathfrak{s}}(S) + \frac{1}{2} \cdot \frac{\|[S^*] \setminus [S]\|}{\|[S^*]\|} \geq C_{\mathfrak{s}}(S)$$

$$\Leftrightarrow \frac{1}{2} \cdot \frac{\|[S^*] \setminus [S]\|}{\|[S^*]\|} \geq \frac{\|[S^*] \setminus [S]\|}{\|[S^*]\|} \cdot C_{\mathfrak{s}}(S)$$

$$\Leftrightarrow C_{\mathfrak{s}}(S) \leq \frac{1}{2}$$

Hence, for fairly incoherent sets S, i.e. $C_{\mathfrak{s}}(S) \leq 1/2$, adding an inconsistency does not yield a decrease in coherence and may even yield an increase (provided that the inequation is strict).

The **fourth adaption strategy** is to identify the coherence of a set with the average of *defined* support values. Due to the fact that we consider *mutual* support, in half of the pairs (V, V') within $[S^*] \setminus [S]$ the inconsistency is contained in the hypothesis V and in half of the pairs it is contained in the evidence V'.

a) Let $\mathfrak{s} \in \{d, s, z\}$. These measures are not defined for inconsistent evidence and provide the neutral value 0 for inconsistent hypotheses. Since there are $1/2 \cdot |[S^*] \setminus [S]|$ pairs with inconsistent hypotheses and thus $|[S]| + 1/2 \cdot |[S^*] \setminus [S]|$ pairs in total, we get:

$$C_{\mathfrak{s}}(S^*) = \frac{\sum_{(U,U')\in[S]}(U,U')}{\|[S^*]\| + \frac{1}{2}\cdot\|[S^*]\setminus[S]\|} + \frac{\frac{1}{2}\cdot\|[S^*]\setminus[S]\|\cdot 0}{\|[S]\| + \frac{1}{2}\cdot\|[S^*]\setminus[S]\|}$$

$$= \frac{\|[S]\|}{\|[S]\| + \frac{1}{2}\cdot\|[S^*]\setminus[S]\|}\cdot C_{\mathfrak{s}}(S)$$

Since the other terms in this fraction are positive, $C_{\mathfrak{s}}(S^*)$ will be positive if $C_{\mathfrak{s}}(S)$ is positive. That is, the inconsistent set S^* can be coherent. Furthermore, S^* can possess a greater coherence value than S. For the given fraction is smaller than 1. So, if $C_{\mathfrak{s}}(S)$ is a negative number, $C_{\mathfrak{s}}(S^*)$ will be greater than $C_{\mathfrak{s}}(S)$. In other words, if S is incoherent, S^* is less incoherent.

b) Now let $\mathfrak{s} \in \{r, j, k\}$. Then the values defined for S^* are just the values for S. For $[S^*] \setminus [S]$ consists of pairs (V, V') where at least one of these sets contains $\perp$. But if the evidence V' is inconsistent, the numerator $P(V|V')$ of r and j is not defined and the denominator $P(V'|V) + P(V'|\neg V)$ of k is 0. Similarly, if the hypothesis V is inconsistent, the numerator $P(V'|V) - P(V'|\neg V)$ of k is not defined and the denominators $P(V')$ of r and $P(V|\neg V')$ are 0. Since all of the additional support values for S^* are thus not defined, calculating the coherence of S^* boils down to calculating the coherence of S. Hence, first, inconsistent sets can be coherent; and second, adding an inconsistency does not lower coherence.

c) Finally, consider f. This measure is not defined if the evidence V' is inconsistent, and it provides the minimum -1 if the hypothesis V is inconsistent. We thus get:

$$C_f(S^*) > 0$$

$$\Leftrightarrow \frac{\|[S]\|\cdot C_f(S) + \frac{1}{2}\cdot\|[S^*]\setminus[S]\|\cdot -1}{\|[S]\| + \frac{1}{2}\cdot\|[S^*]\setminus[S]\|} > 0$$

$$\Leftrightarrow \|[S]\|\cdot C_f(S) - \frac{1}{2}\cdot\|[S^*]\setminus[S]\| > 0$$

$$\Leftrightarrow C_f(S) > \frac{\|[S^*]\setminus[S]\|}{2\cdot\|[S]\|}$$

$$\Leftrightarrow C_f(S) > \frac{2\cdot 3^n - 2^{n+1}}{2\cdot(3^n - 2^{n+1} + 1)} = \frac{3^n - 2^{n+1}}{3^n - 2^{n+1} + 1}$$

$3^n - 2^n > 3^n - 2^{n+1} + 1$ if and only if $2^n > 1$, which holds for all $n \geq 2$. But this would mean that the latter fraction is greater than 1 for all $n \geq 2$, which cannot be true because the maximum of f is 1. Hence, sets of the type S^* are not coherent. Even more, provided that

$C_f(S) > -1$, they must be less coherent than the original set S because, again, assuming otherwise leads to a contradiction:

$$C_f(S^*) \geq C_f(S)$$

$$\Leftrightarrow \frac{\|[S]\| \cdot C_f(S) + \frac{1}{2} \cdot \|[S^*] \setminus [S]\| \cdot -1}{\|[S]\| + \frac{1}{2} \cdot \|[S^*] \setminus [S]\|} \geq C_f(S)$$

$$\Leftrightarrow \|[S]\| \cdot C_f(S) - \frac{1}{2} \cdot \|[S^*] \setminus [S]\| \geq \left(\|[S]\| + \frac{1}{2} \cdot \|[S^*] \setminus [S]\|\right) \cdot C_f(S)$$

$$\Leftrightarrow \|[S]\| \cdot C_f(S) - \frac{1}{2} \cdot \|[S^*] \setminus [S]\| \geq \|[S]\| \cdot C_f(S) + \frac{1}{2} \cdot \|[S^*] \setminus [S]\| \cdot C_f(S)$$

$$\Leftrightarrow -1 \geq C_f(S)$$

Since the minimum of f is -1, the latter inequation is true only if S is maximally incoherent. In this case, adding an inconsistency does not lower coherence because it leads to a set which is likewise maximally incoherent. But if $C_f(S) > -1$, S^* is less coherent than S.

The refined overlap measure

Let $S = \{A_1, \ldots, A_n\}$ be a consistent set of propositions and $[S]^{\geq 2}$ the set of subsets of S with a cardinality of at least 2. Then:

$$\|[S]^{\geq 2}\| = \sum_{i=2}^{n} \binom{n}{i} = 2^n - (n+1)$$

If we add an inconsistency $\perp$, so that $S^* = \{A_1, \ldots, A_n, \perp\}$, we get:

$$|[S^*]^{\geq 2} \setminus [S]^{\geq 2}| = |[S^*]^{\geq 2}| - |[S]^{\geq 2}| = (2^{n+1} - (n+2)) - (2^n - (n+1)) = 2^n - 1$$

Since the sets in $[S^*]^{\geq 2} \setminus [S]^{\geq 2}$ contain an inconsistency and thus get the value 0 by the naïve overlap measure O, we have the following representation for the refined overlap measure:

$$\mathcal{O}^*(S^*) = \frac{1}{\|[S^*]^{\geq 2}\|} \cdot \sum_{i=2}^{n+1} \sum_{S_i \in [S^*]^i} \mathcal{O}(S_i) = \frac{1}{\|[S^*]^{\geq 2}\|} \cdot \left(\|[S]^{\geq 2}\| \cdot \mathcal{O}^*(S) + (2^n - 1) \cdot 0\right)$$

$$= \frac{\|[S]^{\geq 2}\|}{\|[S^*]^{\geq 2}\|} \cdot \mathcal{O}^*(S)$$

Assuming that 0.5 is the neutral value separating coherence and incoherence, we get:

$$\mathcal{O}^*(S^*) > \frac{1}{2}$$

$$\Rightarrow \frac{|[S]^{\geq 2}|}{|[S^*]^{\geq 2}|} \cdot \mathcal{O}^*(S) > \frac{1}{2}$$

$$\Rightarrow \mathcal{O}^*(S) > \frac{|[S^*]^{\geq 2}|}{2 \cdot |[S]^{\geq 2}|}$$

$$\Rightarrow \mathcal{O}^*(S) > \frac{2^{n+1} - (n+2)}{2^{n+1} - (2n+2)}$$

$$\Rightarrow \mathcal{O}^*(S) > 1$$

Since $\mathcal{O}^*(S) \leq 1$ for all S, the latter inequation cannot be true. Hence, S^* is always judged incoherent. Now we turn to the question whether S^* can be more coherent than S:

$$\mathcal{O}^*(S^*) > \mathcal{O}^*(S)$$

$$\Rightarrow \frac{|[S]^{\geq 2}|}{|[S^*]^{\geq 2}|} \cdot \mathcal{O}^*(S) > \mathcal{O}^*(S)$$

$$\Rightarrow |[S]^{\geq 2}| > |[S^*]^{\geq 2}|$$

The latter inequation is unsatisfiable because S^* contains one element more than S. Accordingly, S^* can never be judged more coherent than S if $\mathcal{O}^*(S) > 0$. Otherwise, $\mathcal{O}^*(S) = \mathcal{O}^*(S^*) = 0$.

The refined Schupbach-measure

We begin with the first adaption strategy. Given that $\mathcal{D}^*_\#$ is an average of coherence-values, we adapt this measure by assigning the maximum degree of incoherence 0 to all inconsistent subsets, i.e., all subsets of S^* containing $\perp$. Hence we get:

$$\mathcal{D}^*_\#(S^*) = \frac{1}{|[S^*]^{\geq 2}|} \cdot \sum_{i=2}^{n+1} \sum_{S_i \in [S^*]^i} \mathcal{D}^*_\#(S_i) = \frac{1}{|[S^*]^{\geq 2}|} \cdot \left(|[S]^{\geq 2}| \cdot \mathcal{D}^*_\#(S) + (2^n - 1) \cdot 0 \right)$$

$$= \frac{|[S]^{\geq 2}|}{|[S^*]^{\geq 2}|} \cdot \mathcal{D}^*_\#(S)$$

Accordingly, for the neutral value 0.5 we get the following result:

$$\mathcal{D}^*_{\#}(S^*) > \frac{1}{2}$$

$$\Rightarrow \frac{|[S]^{\geq 2}|}{|[S^*]^{\geq 2}|} \cdot \mathcal{D}^*_{\#}(S) > \frac{1}{2}$$

$$\Rightarrow \mathcal{D}^*_{\#}(S) > \frac{|[S^*]^{\geq 2}|}{2 \cdot |[S]^{\geq 2}|}$$

$$\Rightarrow \mathcal{D}^*_{\#}(S) > \frac{2^{n+1} - (n+2)}{2^{n+1} - (2n+2)}$$

$$\Rightarrow \mathcal{D}^*_{\#}(S) > 1$$

Given that $\mathcal{D}^*_{\#}(S) \leq 1$ for all S, the latter inequation leads to a contradiction. Accordingly, S^* is always judged incoherent. But can S^* nevertheless be more coherent than S?

$$\mathcal{D}^*_{\#}(S^*) > \mathcal{D}^*_{\#}(S)$$

$$\Rightarrow \frac{|[S]^{\geq 2}|}{|[S^*]^{\geq 2}|} \cdot \mathcal{D}^*_{\#}(S) > \mathcal{D}^*_{\#}(S)$$

$$\Rightarrow |[S]^{\geq 2}| > |[S^*]^{\geq 2}|$$

The latter inequation is, again, unsatisfiable. Therefore, S^* is judged less coherent than S if $\mathcal{D}^*_{\#}(S) > 0$. Otherwise, $\mathcal{D}^*_{\#}(S) = \mathcal{D}^*_{\#}(S^*) = 0$.

The second adaption is to assign the neutral value 0.5 to inconsistent subsets:

$$\mathcal{D}^*_{\#}(S^*) = \frac{1}{|[S^*]^{\geq 2}|} \cdot \sum_{i=2}^{n+1} \sum_{S_i \in [S^*]^i} \mathcal{D}^*_{\#}(S_i) = \frac{1}{|[S^*]^{\geq 2}|} \cdot \left(|[S]^{\geq 2}| \cdot \mathcal{D}^*_{\#}(S) + (2^n - 1) \cdot \frac{1}{2} \right)$$

Thus we get:

$$\mathcal{D}^*_{\#}(S^*) > \frac{1}{2}$$

$$\Leftrightarrow \frac{|[S]^{\geq 2}|}{|[S^*]^{\geq 2}|} \cdot \mathcal{D}^*_{\#}(S) + \frac{2^n - 1}{2 \cdot |[S^*]^{\geq 2}|} > \frac{1}{2}$$

$$\Leftrightarrow 2 \cdot |[S]^{\geq 2}| \cdot \mathcal{D}^*_{\#}(S) + (2^n - 1) > |[S^*]^{\geq 2}|$$

$$\Leftrightarrow \mathcal{D}^*_{\#}(S) > \frac{1 - 2^n}{|[S]^{\geq 2}|}$$

Given that $1 - 2^n < 0$ for all $n \geq 2$, $\mathcal{D}^*_{\#}(S^*)$ always exceeds the threshold 0.5 so that S^* is coherent. Furthermore:

$$\mathcal{D}^*_{\#}(S^*) > \mathcal{D}^*_{\#}(S)$$

$$\Leftrightarrow \frac{|[S]^{\geq 2}|}{|[S^*]^{\geq 2}|} \cdot \mathcal{D}^*_{\#}(S) + \frac{2^n - 1}{2 \cdot |[S^*]^{\geq 2}|} > \mathcal{D}^*_{\#}(S)$$

$$\Leftrightarrow \frac{2^n - 1}{2 \cdot |[S^*]^{\geq 2}|} > \mathcal{D}^*_{\#}(S) \cdot \frac{|[S^*]^{\geq 2}| - |[S]^{\geq 2}|}{|[S^*]^{\geq 2}|}$$

$$\Leftrightarrow \frac{2^n - 1}{2 \cdot |[S^*]^{\geq 2}|} > \mathcal{D}^*_{\#}(S) \cdot \frac{2^n - 1}{|[S^*]^{\geq 2}|}$$

$$\Leftrightarrow \mathcal{D}^*_{\#}(S) < \frac{1}{2}$$

Therefore, S^* is more coherent than S if and only if S is incoherent.

The third approach only considers defined constituents when calculating coherence. We thus get:

$$\mathcal{D}^*_{\#}(S^*) = \frac{1}{|[S]^{\geq 2}|} \cdot \left(|[S]^{\geq 2}| \cdot \mathcal{D}^*_{\#}(S)\right) = \mathcal{D}^*_{\#}(S)$$

Accordingly, S and S^* are always on a par with respect to their degrees of coherence.

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