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ARISTOTLE'S DOUBLE SOLUTION TO ZENO'S 'DICHOTOMY', SIGN OF A RADICAL REVISION?

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Abstract

In the *Physics* Aristotle offers two solutions to Zeno's 'Dichotomy'. Waterlow and Sorabji intend to show that the existence of two solutions indicates Aristotle's radical revision of the *Physics'* fundamental concepts. This article aims to criticize Waterlow's and Sorabji's arguments. An interpretation of the Aristotelian text is also offered that points to the two solution's compatibility and to the coherence of the *Physics'* fundamental concepts.

KEY WORDS: Aristotle; Zeno; Dichotomy; Infinity; Continuity.

Resumen

En la *Física*, Aristóteles ofrece dos soluciones a la 'dicotomía' de Zenón. Waterlow y Sorabji procuran mostrar que la existencia de esas dos soluciones trasunta una revisión radical por parte de Aristóteles de los conceptos fundamentales de la *Física*. El propósito de este artículo es criticar los argumentos de Waterlow y Sorabji. Se ofrece también una interpretación del texto aristotélico que señala la compatibilidad de las dos soluciones y la coherencia de los conceptos fundamentales de la *Física*.

PALABRAS CLAVE: Aristóteles; Zenón; Dicotomía; Infinito; Continuidad.

Introduction

Aristotle offers two solutions to Zeno's 'Dichotomy', the first in Ph.6.2, the second in Ph.8.8., after professing dissatisfaction with the first. This re-evaluation has been interpreted as a major revision of his theory of motion in Ph.6, in line with his theory of change in Ph.3,[1] but itself in conflict with Ph.3's theory of the infinite,[2] calling into question the theoretical consistency of the 'Physics' as a whole. I will set out Zeno's argument, 2), Aristotle's two solutions, 3), 4), and the two main arguments

[1] Waterlow (1982, pp. 131-158).
[2] Sorabji (1983, chs. 14, 21).
[3] Ferber (1995, p. 52); Ferber (2000, p. 139); Hussey (1995, p. 99); Shields (2007, p. 216).

in favour of the revision's indicating a radical theoretical revision, 5). I will then argue that the 'radical revision' interpretation represents a scholarly overreaction, 6), 7).

2) Zeno's argument

Zeno's arguments are generally understood as defending Parmenides' thesis that 'being'/reality is changeless.[3] The 'Dichotomy' is one of four arguments by which Zeno attempts to demonstrate that the reality of motion, so evident to our senses, is illusory. Since the reality of change is the 'Physics'' fundamental assumption,[4] Aristotle is keen on refuting Zeno's arguments.

Aristotle sets out the 'Dichotomy' in more detail at 263a5-7:[5] '… that before any distance can be traversed half the distance must be traversed, that these half-distances are infinite in number, and that it is impossible to traverse distances infinite in number,… .'[6]

The argument can be set out slightly more formally as follows:

(i) If motion is possible, then an object can traverse a finite distance in a finite time.
(ii) Traversal of any finite distance in a finite time requires traversal of infinitely many half-distances.
(iii) Traversal of infinitely many half-distances in a finite time is impossible.
(iv) So, traversal of any finite distance in a finite time is impossible.
(v) So, motion is impossible.

Zeno's crucial assumptions are a spatial distance's infinite divisibility, (ii), and the impossibility of completing infinitely many tasks in a finite time, (iii). Aristotle accepts (ii),[7] but targets (iii), 6.2.233a21-24: '… hence Zeno's argument makes a false assumption in asserting that it is impossible for a thing to pass over or severally come into contact with infinitely many things in a finite time.'

In his two solutions at 6.2.233a21-31 and 8.8.263a4-263b8 Aristotle qualifies the sense of 'infinite' in accord with his account in 3.5-7. There he distinguishes between the infinite by addition and the infinite by

[4] 2.2.194a27-32; 250b11-14.
[5] See also 239b11-13.
[6] Ross' edition, (Ross 1936), of the Greek text is used and the translation in Barnes (1995), except for 3), 4), where my own translation is used.
[7] Ferber (1995, pp. 6-8).

division, 206a14,15, and between actual and potential infinite,206a18-206b2. The infinite by addition is the indefinitely repeatable addition of a discrete unit to form an ever increasing sum or ensemble, 206a27,28,[8] the infinite by division is the indefinitely repeatable division of a continuous magnitude into ever smaller continuous magnitudes,207b1-5,16,17. There is no actual infinite in the sense that the processes of addition or division are ever completed, 206b6,7.[9] For Aristotle the infinite exists only potentially, 207b12,13.

Aristotle grants Zeno the truth of (iii), when understood as an injunction against an actual infinite, the impossibility to complete infinitely many discrete tasks in a finite time, 233a26,27, 263a7,10,11.[10] But he denies the truth of (iii), when a continuum's infinite by division and its potentiality are at issue.

3) Aristotle's first solution

Aristotle presents his first solution at 233a21-31, where he charges Zeno with an equivocation of the notion 'infinite', 'διχῶσ γὰρ λέγεται'('there are two senses'),a24. He contends that any continuous magnitude, a25, such as spatial distances or temporal periods, can be infinite by division, 'κατὰ διαίρεσιν', or by extent, 'τοῖσ ἐσχάτοισ', a25,26.[11] Zeno, so Aristotle, fails to specify correctly the sense of 'infinite' in (iii). Aristotle accepts the impossibility of traversing a distance that is infinitely long, infinite 'by extent', in a time finitely long, finite 'by extent', a26,27, but maintains that it is possible to traverse infinitely many sub-distances 'by division', a27,28. For just as a continuous distance, finite by extent, is infinite by division, so a time interval, finite by extent, is infinite by division, 'καὶ γὰρ αὐτὸσ ὁ χρόνοσ οὕτωσ ἄπειροσ'('for time itself is infinite in this way'), a28. So it is possible, Aristotle argues, to traverse infinitely many half-distances in a finite time, a28-30: the finitely long distance is divisible into infinitely many half-distances. To each half-distance there corresponds a temporal half-interval of the infinitely many half-intervals the finitely

[8] One might speak of an arithmetic sum or set-theoretic union, Ferber (1995, pp. 54, 102); Ferber (2000, p. 140).

[9] See also pp. 12, 15 below.

[10] See also Ferber (1995, pp. 32, 33, 107), and 7) below.

[11] Instead of 'extremities', (Barnes 1995, p. 394), I chose 'extent', following Waterfield, (Bostock 1999, p. 143), and Shields (2007, p. 219), to signal the close relation to 'infinite by addition' in 3.5-7, 204b4, 206a15, 207b29, Ross (1936, p. 642): an addition of spatially extended distances yields a spatial distance of a certain extent.

long time interval is divisible into, which is the finite time interval it takes to traverse the finite distance. Someone traversing a finite distance in a finite time thus never runs out of time to complete the traversal.

At 8.8.263a11-23 Aristotle considers this solution to be inadequate, failing to address the true nature of Zeno's problem, a17,18. He claims that what rendered the first solution successful, a16, had been the respective infinites' correct correlation in the correlated spatial and temporal magnitudes, a14,15.[12] Aristotle points out, a18-22, that the solution is not applicable when the correlates are regarded in isolation, for example time in isolation of spatial distance, 'ἀφέμενοσ τοῦ μήκουσ' ('when distance is taken away'), a18. Zeno's argument can be advanced again. For there to be a finite time interval, time must pass during the entire interval. But time is infinitely divisible, a21, so time must pass during infinitely many half-intervals. On pains of circularity, Aristotle can no longer correlate the time interval with another finite time interval, which, by way of being infinitely divisible, supplies sufficiently many temporal sub- intervals for time to pass during all the sub-intervals of the interval in question. Aristotle thus takes the inadequacy of the correlation-solution, a21, to indicate that Zeno's argument addresses the nature of continuous magnitudes as such, their possible constitution by the infinitely many parts issuable by way of their infinite divisibility.[13] Aristotle notably introduces his second solution speaking of the division of continuous magnitudes in general, 'ἐὰν γάρ τισ τὴν συνεχῆ διαιρῇ εἰσ δύο ἡμίση', ('for if one divides a continuum into two halves'), a23.

4) Aristotle's second solution

In presenting his second solution, 263a22-263b9, Aristotle refers back, a22,23, to a conception of continuous motion developed at 262a13-262b7. There he argues that eternal motion over a finite, straight distance cannot be continuous, since reversal of direction at the distance's termini necessitates a standstill for a period time, and thus rupture with the motion's continuity, a12-15. He draws a distinction between an intermediate point, as opposed to a continuous change's beginning- and end-point, a17,18, based on the distinction between actuality and potentiality, a19,20: during a continuous change the intermediate points are only potential mid-points, each single intermediate point being potentially two points, a20,21, the end of a sub-change, and the beginning

[12] See also 6.2.233a17-21,233a32-34.

[13] See also Ferber (1995, pp. 8, 32, 102); Ferber (2000, pp. 139, 140).

of a subsequent sub-change, a26,27. Becoming an actual intermediate point requires actual line division, a23,24, and entails its actually becoming these two termini. Their being distinct implies a period of rest between them: since having arrived at the intermediate point and having left it cannot be simultaneous, 262a27-b2, and since between any two moments in a continuous stretch of time there is an interval of time, b2,3, the object will rest there, b5-7, whence the change is no longer continuous. Aristotle thus has developed a way to retract the sub-changes' status of being genuine changes, for if they were, they would be bounded by periods of rest, and no longer be the sub-changes of the continuous change they were supposed to be sub-changes of. They thus do not count as genuine sub-tasks, completion of an infinite number of which is impossible according to (iii).

Against Zeno Aristotle argues as follows: a continuum's division into parts, such as the finite distance's bisection Zeno relies on, is an actual division, and involves 'treating as two', 263a23,24, a30-263b3, a potential mid-point. Its becoming an actual end and beginning distinct from each other, a24,25, entails rupture of the line's continuity, a27,28. Since continuous motion is motion along a continuous line, a27,a28, rupture of the line's continuity entails rupture of the motion's continuity, a26,a27. Aristotle concedes to Zeno that a continuum has infinitely many half-intervals or intermediate half-points, a28,29, but only potentially so, since rendering one of them actual entails rupture of continuity, in the case of continuous motion a period of rest, a29,a30.

Aristotle brings to bear this result on Zeno's argument, b3-9. He grants that it is impossible, b5,6, to traverse infinitely many half-distances or temporal intervals, b3,4, if they are actual, but maintains that it is possible to traverse them if they are potential, thus denying (iii). He further claims, b6-9, that the object moving continuously during a finite time span over a finite distance has traversed them 'incidentally', 'κατὰ συμβϵβηκὸσ', b6,7, but not 'simpliciter', 'ἁπλῶσ δ'οὔ',b7. The reason he offers is that '… the line has incidentally infinitely many halves, but its substance, ('οὐσία'), and essence, ('τὸ ϵἶναι'), are different', b7-9.

5) Waterlow's and Sorabji's argument

This second solution has been interpreted as signalling a radical revision of previously held theories in the 'Physics.' The 'radical revision' hypothesis is supported by two arguments I prefer to label 'identity'-argument and 'infinity'-argument.

The ‘identity’-argument, first raised by Waterlow,[14] proceeds as follows: in Ph.3.1-3.201a9-12,a27-30,b31-201a2, 202a7-9,a13-16 Aristotle defines change as an actuality of a potential to be in a new state qua such.[15] In 5.4.227b20-26 Aristotle lays down conditions for a change to be numerically one: a change is one, single change if and only if there is one object changing over a certain period of time in one respect from a certain beginning to a certain end. The specification of the change’s end is thus crucial in determining a change’s identity in both Ph.3.1-3 and 5.4. In Ph.6.6 Aristotle points to a continuous change’s infinite divisibility. At 237a18-20, 237a34-b8 he states that every object that has completed a change must have been changing before, and if it is changing, it must have completed a previous change. Aristotle infers that an object that has completed a change must have completed infinitely many changes, 237a15,16.

One might now ask, whether these sub-changes are themselves changes. They seem to satisfy the identity-conditions for change, being themselves changes of one object in one respect from an intermediate beginning to an intermediate end. If the identity-conditions identify the numerically single change of which they are sub-changes, then the identity-conditions do not determine the numerical identity of change, for they identify infinitely many others. As Waterlow puts it,[16] ‘…the singleness of the end-state specifying direction,…, dictated from the start is lost…’, for the change is directed no less to this end-state than to the infinitely many other intermediate end states. Ph.6’s ‘fundamental error’[17] is, so Waterlow, that in passing from beginning to end, ‘…no single change is occurring’,[18] and ‘…that there is no specific change or set of changes in which the change consists.’[19] Waterlow concludes: ‘This amounts to a proof that the concept of ‘change’ is incoherent: for change is nothing if not that by which something passes from one condition to the other.’[20] Waterlow suggests that Aristotle’s second solution in Ph.8 aims at correcting Ph.6’s ‘fundamental error’, and that Ph.6 might not originally have belonged to the ‘Physics.’[21]

[14] Waterlow (1982, pp. 131-158).

[15] The definition’s formulation provided is controversial, Heinaman (1994), but legitimate here, since it is the one endorsed by Waterlow (1982, pp. 109-131), who advanced the ‘identity’- argument.

[16] Waterlow (1982, p. 136).

[17] Waterlow (1982, p. 145).

[18] Waterlow (1982, p. 145).

[19] Waterlow (1982, p. 146).

[20] Waterlow (1982, p. 146).

[21] Waterlow (1982, pp. 131, 132, 145).

The 'infinity'-argument has been raised by Sorabji.[22] Sorabji takes Aristotle's concept of potential infinity espoused in Ph.3.6,7 to be that of an 'extendible finitude',[23] in the sense that there are never more than a finite number of actual existents. For example, there are never more than finitely many actual divisions on a line. Sorabji believes that, in order to retain consistency with this theory, Aristotle must hold that the number of makeable divisions must also be finite. For if there were more than finitely many, the line would have to be conceived as constituted of extension-less points, which Aristotle rejects.[24] But Aristotle's revised solution in Ph.8.8, Sorabji maintains, commits Aristotle to a 'startling…concession':[25] in Ph.8.8 Aristotle argues that an infinity of potentially existing points/divisions can be traversed, which seems to imply that there are more than finitely many potential points/divisions on a line, and thus more than finitely many potential existents. This implication is in direct conflict with Ph.3.6,7's theory of the potential infinite. As Sorabji puts it:[26] 'Aristotle needs to say,…, not merely that his divisions exist potentially,…, but also that their infinity is potential as well,…, and that in the sense which I defined earlier as not being more than finite.' The second solution's conflict with Aristotle's theory of the potential infinite betrays for Sorabji a deep inconsistency in the 'Physics' as a whole.[27]

6) Critique of Waterlow's argument

The identity-argument's plausibility would be weakened if Aristotle continued to apply his first solution in Ph.8, and developed his second solution in books preceding Ph.6 and in Ph.6 itself. Aristotle in Ph.8.10 extends Ph.6.2's correlation argument to forces. Magnitudes and changes, finite or infinite, by extent or divisibility, require the respective finite or infinite forces, 266a2-25,b6,7,b20-2. The second solution's key idea of distinguishing a change's genuine, actual beginning/end from a potential intermediate point, which, when being actualized becomes two points, beginning and end, with a period of rest in between, is found already at Ph.4.220a5-13: '… the point also both connects and terminates the

[22] Sorabji (1983,chs.14, 21).
[23] Sorabji (1983, p. 210).
[24] Sorabji (1983, p. 211).
[25] Sorabji (1983, p. 213).
[26] Sorabji (1983, p. 213).
[27] Sorabji (1983, p. 323).

length- it is the beginning of one and the end of another. But when you take it in this way, using the one point as two, a pause is necessary, if the same point is to be the beginning and the end.'

That periods of continuous change are bounded by periods of rest is stated explicitly in Ph.5.4.228a20-b10. Aristotle in 5.4 sets out the conditions for a change's *numerical* identity, 227b20-22, 228b1-7, its 'being one', as conditions for a change's *continuity*: a change is one if and only if it is continuous, 228a20-22.[28] The change's time period, in order not to violate a change's identity, must be one and not display gaps, 'μὴ διαλείπειν' ('not be intermittent'), 227b30-32, 228b8. Rupture of continuity engenders violation of identity, and such rupture Aristotle associates with a period of rest, 228b1-7.

In Ph.6 this key idea of Ph.8.8's solution is both present and developed further in terms of a change's being one/continuous and infinitely divisible, as announced at Ph.3.200b16-20, 207b15,16, 207b3435 and Ph.5.4.227a21. The key idea is present, for Aristotle repeatedly refers to a change's primary or immediate time, indicating the longest interval, during which the change is not interrupted by rest.[29] Aristotle associates the end of change with rest.[30] He repeatedly asserts that all change is in time, requiring a period of time,[31] and so is rest.[32] If Aristotle regarded Ph.6.6's infinite sub-changes as genuine changes, he would have to regard them as bounded by periods of rest. Ph.6's 'fundamental error' would amount to an inconsistency internal to Ph.6, and not merely an inconsistency with Ph.3.

But Aristotle did not regard these sub-changes as genuine changes. The identity-argument betrays ignorance of Ph.6's arguments' general intent. Aristotle argues not only that continuity entails infinite divisibility, 207b16,17,231b16,17, but also that infinite divisibility entails continuity.[33] In 6.2, 232b13-233a13, preceding the first solution, he proves that a change's time period is continuous. Aristotle assumes that all change requires time,232b20, proceeds at finite velocities, requiring definite ratios between spatial and temporal intervals,232a23-26, 232b14-16,[34] and that a faster object covers a distance in a shorter temporal period than a slower

[28]Ph.267a21-24; Met.1015b36-1016a1. Solmsen (1960, p. 184); White (1992, p. 104); Graham (1999, p. 135).

[29] 234b10-20, 235b31-34, 238b36-239a9, 240b24-30.

[30] 234a13-15, 239a23-29,a35-b4.

[31] 232b15,16,b20,21, 235a9-15, 236b19,20, 238b23-31, 239a23-25, 241a15-18.

[32] 234b9,10, 239a14,15.

[33] 200b16-20, 228a20,21, 232a22-24, 232b20-25, 233b15-17; Bostock (2006a, p. 165).

[34] Already assumed at 215b12-19, 216a8-12.

one. The fact that, during a change, shorter distances are always traversed in shorter periods of time he equates with infinite divisibility, inferring the change's time interval's continuity:[35] if the object rested for some time during the change, 232b21, covering no distance, it would not cover shorter distances in shorter times. Hence the change would not be continuous, b20-26, and thus not be one.

Similarly, in Ph.6.6 Aristotle assumes that change takes time, 237a16,17, its parameters obeying definite proportions,236b33-237a2. He proves that because of its primary time period's continuity and thus infinite divisibility, 236b33-237a2, 237a5-9, 237a25, change is continuous, 237b18,19. Aristotle associates the change's primary time, 236b19-31, with absence of rest, 236b28-30, 237a12-15, as in 6.2.232b20,21, and in accord with its oneness and continuity, Ph.5.4.228b1-7. Ph.6.6's infinite sub-changes thus do not qualify as genuine changes. The perfect tenses, 'κεκινῆσθαι', 237a2,a6,7, 'μεταβεβληκέναι', 237a15,17, should be non-misleadingly translated as 'having been changing', not as 'having completed a change', when all Aristotle intends to show is that infinite divisibility implies lack of a pause![36]

Finally, Aristotle grants Zeno the impossibility of contact with 'ἀπείρων' ('infinites'), 'κατὰ τὸ ποσὸν' ('by quantity/extent'), in a finite time, 233a26,27.[37] Since the contrast is with infinite divisibility,a27, and only a continuous magnitude is infinitely divisible,Ph.206a15-17,231b10-15, Met.1020a10-12, 'ποσὸν' refers to a denumerable plurality of discrete items. But at Ph.6.6 Aristotle seems to claim that an object having changed in a finite time has effected infinitely many discrete changes – a blatant contradiction?

At 233a23-34, following the first solution, Aristotle argues that a finite distance's traversal cannot take an infinite time, but must take a finite time, while traversal of an infinite distance requires an infinite time. He relies on Archimedes' axiom, assumed throughout the 'Physics', 206b9-11, 237b28-34, 238a28-31, 265a17-20, 266a15-22, that a finite magnitude is exhausted by a finite multiple of a finite part. He also assumes that what is infinite has no infinite parts, 188a2-5, 204a20-29, 233b1,2, 238b13-16, and, as before, that change takes time, obeying definite parametric proportions. Aristotle thus assumes that finite distances/time-periods are finite multiples of finite sub-distances/sub-intervals. Since any part of an infinite distance/period is finite, an

[35] Knorr (1982, pp. 117-119).
[36] See also White, (1992, pp. 103-106).
[37] Repeatedly so: 204b7-10, 207b27, 233a22, 238a33, 263a6,a20, DA.407a11-13.

infinite distance/period must be an infinite multiple, (extension, sum), of a finite sub-distance/sub-interval. Since a finite distance/interval cannot be both finite and infinite 'κατὰ τὸ ποσὸν', it cannot be the infinite multiple of a finite part. The implication is compatible with Ph.3.3's result that a finite magnitude's being infinite by addition/extent depends on the magnitude's being infinitely divisible, 206b3-6, the resultant ever-diminishing parts never exhausting the magnitude, 206b7-9. If Aristotle claims just such complete exhaustion for a finite change in Ph.6.6 he has contradicted both Ph.3 and Ph.6.2. If he is consistent in Ph.6, he cannot treat Ph.6.6's finite change as an infinite multiple of discrete, finite sub-changes. Since he argues for its infinite divisibility, wanting to retain its infinitude, 237b15, 'ἄπειρόν τι συνεχεῖ γε ὄντι' ('the continuous is infinite in a certain way'), he cannot retain an infinitude 'κατὰ τὸ ποσὸν', which implies regarding the sub-changes as genuine, discrete changes and component-parts of the change. But the 'identity'-argument requires just this of Aristotle to charge him with inconsistency regarding Ph.3's definition of change. So the identity-argument must attribute to Aristotle a blatant inconsistency between Ph.6.6 and Ph.6.2. Since the latter is incredible, so is the 'identity'-argument.

7) Critique of Sorabji's argument

Sorabji's infinity-argument is weakened by the fact that he fails to consider that Aristotle rejects a line's constitution of points regardless of their infinite number, because extension-less points cannot sum to form an extended magnitude, GC.316a31-34, and because contiguity of points entails their coincidence, Ph.231a29-b9.

Sorabji's key contention is that the actual/potential distinction is ineffective in allowing for a continuous line's traversal without violating Ph.3's strictures against actual infinity: having actually traversed infinitely many potential sub-distances amounts to having traversed an actually infinite number of them. Aristotle, to recall, denies this on account of the line's having only incidentally an infinity of potential points/sub-distances, and of the latter's traversal being incidental as well. Yet, Sorabji's contention retains its plausibility: if the line's having points/sub-distances only incidentally means that if it does not have them it is still a line, then actual traversal of infinitely many points/sub-distances, if it has them, still amounts to traversal of an actually infinite number.

How are the essential/incidental and actual/potential distinctions related? If x is essentially F it must be actually F, for if it was not, it would no longer be x. If x is incidentally F, F might still be an actual feature of

x. In the case of the continuous line's traversal, traversal of a sub-distance, an actual sub-traversal, requires actual division, and thus rupture with continuity and loss of the line's identity. Aristotle must believe that, in this context, if x is incidentally F, it is potentially F, and that, in the case of the line, having the points/sub-distances potentially does not imply rupture of continuity. At 263b7-9 incidental predication, being 'ἁπλῶσ δ'οὔ', b7, is contrasted with predication simpliciter, applicable to essence or 'οὐσία',b8. At Post.An.83a16-20[38] Aristotle develops such a contrast between 'natural', 'ἁπλῶσ', and 'unnatural', 'κατὰ συμβεβηκὸσ', predications.[39] The latter are 'things, being something else, are so and so.'[40]A predication 'x is F' is 'natural' in case x is the proper subject of F, 'ὑποκείμενον', 83a13,18,26,31, so that predications apply in accordance with categorical distinctions, 83a21-23 and that 'οὐσία', 83a25,26,30, or essence is characterised, 73a34-b4. A predication is 'unnatural' in case there is something else y that is F and y is ontologically parasitic on x, 73b9, 83a6, Met.1087a35.[41]

Aristotle at 263b7-9 thus adverts to a 'category'-mistake of Zeno's, indicating that 'the line has n sub-distances', n being a number, is an 'unnatural' predication. Only a plurality of discrete items, a 'πλῆθοσ', can be a number, whilst only a continuous magnitude, 'μέγεθοσ', can be a line, Cat.6.4b20-5a14, Met.1020a7-13. 'Πλῆθοσ' and 'μέγεθοσ' are sub-genera of quantity, Cat.6.4.b20-23, discrete, 'διώρισμένοσ', and 'continuous', 'συνεχήσ', being their defining characteristics. The line, a delimited, one-dimensional, continuous magnitude, Met.1020a12-14, can thus not also be a delimited plurality of discrete items, Met.1020a13.[42] The plurality of discrete sub-distances can be a number, and their existence is parasitic on the line, requiring actual division of the line. Once divided, the line no longer exists. Actual traversal of the line is thus not actual traversal of a 'πλῆθοσ' of n sub-distances. The n sub-distances are incidentally traversed in that the line's traversal is the traversal of a 'μέγεθοσ', which can become a 'πλῆθοσ' by undergoing substantial change in terms of its 'οὐσία' or essence, no longer being what it is.

Sorabji's mistake is to treat potential points/sub-distances as a 'πλῆθοσ.' But potential points/sub-distances are 'existents' in a very attenuated sense.[43] First, to be a discrete existent, a point on a continuous

[38] 83a1-33, 73b5-15.

[39] Barnes (1975, pp. 115-117, 168, 169).

[40] Barnes (1975, p. 116). An.Post.73b5-10, Met.987b23,24; 1001a6-10, 1087a33.

[41] Barnes (1975, p. 117).

[42] White (1992, pp. 31, 180, 181).

[43] Bostock (1987, pp. 263-265); Charlton (1991, pp. 133, 134).

line would have to be distinguishable and thus consecutive to its neighbour, which it cannot be.[44] Secondly, the existence of points depends on the existence of lines, as line-boundaries or locations of line-divisions,[45] whence constitution of a continuum by points amounts to a conceptual impossibility, 231b10-15.[46] A line is infinitely divisible, being always further divisible into continuous sub-distances, 231b16,17. But for sub-distances to form a continuum their boundaries have to be contiguous,[47] and have to become 'one.'[48] They no longer just coincide but form one unity, 'ἕν τι πέφυκε'('is naturally something that is one'), 227a15. Hence on a continuum there 'are' no longer sub-distances distinguishable from each other.[49] It is merely possible to mark them out, Ph.219a22-29, and to divide the continuum at any point, not everywhere, for 'everywhere' already implies distinguishing a point of division from its neighbour, GC.317a9-14.

Advertisement of a 'category'-mistake extends to the sense of 'infinite.' Infinity is a 'per se'-attribute, a necessary, but non-essential feature of quantity,[50] that is of number, a 'πλῆθοσ', and of 'μέγεθοσ.'[51] A continuous magnitude is infinitely divisible, and must be so, (per-se), because it is always further divisible into magnitudes, there being no smallest magnitude, Ph.206b15-20, 207b1-9,15-20; it is divisible into divisible magnitudes and not points, which cannot be consecutive, there being always a magnitude in between, Ph.231b8,9,16-19, Met.1020a10-12. A 'πλῆθοσ' is divisible into the discrete units constituting it, there being a smallest number, if counted, Ph.185b11,12, 207b1-14, 1020a7. It can be infinite by addition/extent, 204a7,11, being infinitely extendible, since for each unit another could be added, 206b3-11, 207b13-17. But it must be so extendible only in virtue of a finite, continuous 'μέγεθοσ" infinite divisibility.[52] The resultant sub-magnitudes never sum to the initial magnitude, 206b8,9. Aristotle assumes that what something can be divided into, it can be constituted by,[53] so he does not

[44] Ph.226b19-21,23, 231a18-21, 230a28-231b5, 231b7-9.

[45] Ph.226a6-17, 227a27-32, Cat.6.5a1,2, GC.316a29,30, DA.409a28-30, Met.1002a10ff, 1044b20,21. Aristotle even argues that the existence of sub-distances depends on the line's existence, Met.1019a4-11; Makin (2003, pp. 213-221).

[46] Miller (1982, pp. 99, 100).

[47] Ph.226b18-21,231a18-21,Cat.6.5.a2.

[48] Ph.227a12-16,21-24, 228a28-b1, 231b19.

[49] Furley (1982, pp. 28, 29); Bostock (2006b, p. 119).

[50] Top.102a18-30, An.Post.73a7, Met.1025b30-34.

[51] Ph.185a32-35,187b7, 203b30-204a4, 204a8-33. Bowin (2007, pp. 247-250).

[52] Ph.206b3-10, 206b20-23, 207b1-20.

[53] Ph.218a5-7, 231b10,11, Met.1023b19,20.

consider these sub-magnitudes to be genuine constituents or parts of the initial magnitude.

If a line having n sub-distances is an 'unnatural' predication, then so is its having infinitely many by addition/extent, for only the 'πλῆθοσ' of sub-distances can be infinitely many by addition/extent. But both the existence of the 'πλῆθοσ' and the infinity appropriate to it depend on the line's existence and its infinite divisibility. So the 'πλῆθοσ" number is not actually infinite. Its constituent, discrete elements are not parts of the line, constituting the latter's 'οὐσία.' Contrary to Sorabji's belief, a line's traversal is not traversal of a plurality of (potential, incidental) discrete sub-distances, finitely or infinitely many, but traversal of a continuous 'μέγεθοσ', whose infinite divisibility ensures the number of the 'πλῆθοσ' of actual discrete sub-distances to be infinitely extendible.

According to Aristotle, an entity can have infinitely many incidental features.[54] For example, a stone to be fashioned into a statuette has infinitely many potential shapes, Met.1026b7-9. These shapes do not exist as discrete, distinguishable statuettes in the stone, Met.1017b6-9, 1026b21-24. If I put the entire stone into my pocket I have not also put 'all' infinitely many statuettes into my pocket, 1026b6,7. I have put a single stone into my pocket, a continuous quantity, Cat.6.4b4, such that for any statuette a sculptor shapes it into he could shape it into a different one. The 'infinity'-argument fails.

In conclusion, neither the 'identity'-argument nor the 'infinity'-argument support the radical-revision hypothesis, which must thus be regarded as a short-sighted scholarly overreaction towards the complexity of a consistently evolving exploration of dynamic and kinematic concepts throughout the 'Physics'.

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[54] Ph.196b27-29,197a16,17, Met.1007a14,15.

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