THE YABLO PARADOX AND CIRCULARITY

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Abstract

In this paper, I start by describing and examining the main results about the option of formalizing the Yablo Paradox in arithmetic. As it is known, although it is natural to assume that there is a right representation of that paradox in first order arithmetic, there are some technical results that give rise to doubts about this possibility. Then, I present some arguments that have challenged that Yablo's construction is non-circular. Just like that, Priest (1997) has argued that such formalization shows that Yablo's Paradox involves implicit circularity. In the same direction, Beall (2001) has introduced epistemic factors in this discussion. Even more, Priest has also argued that the introduction of infinitary reasoning would be of little help. Finally, one could reject definitions of circularity in term of fixed-point adopting non-well-founded set theory. Then, one could hold that the Yablo paradox and the Liar paradox share the same non-well-founded structure. So, if the latter is circular, the first is too. In all such cases, I survey Cook's approach (2006, forthcoming) on those arguments for the charge of circularity. In the end, I present my position and summarize the discussion involved in this volume.

KEY WORDS: Yablo Paradox; Truth; Circularity; Finitism.

Resumen

En este artículo, describo y examino los principales resultados vinculados a la formalización de la paradoja de Yablo en la aritmética. Aunque es natural suponer que hay una representación correcta de la paradoja en la aritmética de primer orden, hay algunos resultados técnicos que hacen surgir dudas acerca de esta posibilidad. Más aún, presento algunos argumentos que han cuestionado que la construcción de Yablo no sea circular. Así, Priest (1997) ha argumentado que la formalización de la paradoja de Yablo en la aritmética de primer orden muestra que la misma involucra implícitamente circularidad. En la misma dirección, Beall (2001) ha introducido factores epistémicos en esta discusión. Más aún, Priest ha también argumentado que la introducción de razonamiento infinitario como complemento de la formalización en la aritmética sería de poca ayuda. Finalmente, se podría rechazar todo intento de dar definiciones de circularidad en términos de puntos fijos adoptando teoría de conjuntos infundados. Entonces, se podría sostener que la paradoja de Yablo y la del mentiroso comparten la misma estructura infundada. Por eso, si la última es circular, también lo es la primera. En todos los casos, presento el enfoque de Roy Cook (2006, en prensa) sobre estos argumentos que atribuyen circularidad a la construcción de Yablo. En el final, presento mi posición y un breve resumen de la discusión involucrada en este volumen.

PALABRAS CLAVE: Paradoja de Yablo; Verdad; Circularidad; Finitismo.

1. Yablo Paradox in Arithmetic

Imagine a denumerably infinite sequence of sentences S1, S2, S3,…, each of them claims that all sentences occurring later in the series are not truth:

(S1) For all k>1, Sk is untrue
(S2) For all k>2, Sk is untrue
(S3) For all k>3, Sk is untrue
…

According to Stephen Yablo (1985, 1993), this sequence generates a Liar-like paradox without any kind of circularity involved: no sentence in the Yablo list seems to refer to itself, and opposed to Liar cycles, no sentence seems to refer to sentences above it in the list. Nevertheless, this issue has been the focus of a fascinating discussion. Roy Sorensen (1998) and Otavio Bueno and Mark Colyvan (2003) have argued that the list produces a semantic paradox without circularity. Graham Priest (1997) and JC Beall (2001) have instead argued the paradox involves a fixed-point construction and as a result of this the list is basically circular. Roy Cook (2006, forthcoming) claims that the arithmetic variant of the Yablo's list is circular, but a slight modification of the original Yablo's construction allows us to generate a truly non-circular paradox.

Formalizing the sentences that appear in the Yablo Paradox with a truth predicate T, one gets that for all natural numbers n, S_n is the sentence ∀k>n, ¬ T(Sk). Since the sentences on the right hand side are the truth conditions for the sentences named in the Yablo sequence, one could formulate the list of Yablo's sentences by the set of biconditionals:

$$\{ S_n \leftrightarrow \forall k>n, \neg T(Sk): n \in \omega\}$$

Again, one could try to formalize the proof of the Yablo Paradox increasing first-order arithmetic. Being that some non-logical expressions that appear in the sequence are part of informal arithmetic (numerals, order relations), this option seems natural. Of course, since T is also a non-logical expression, one needs some kind of truth-theoretical principle. According to this, one can consider the local disquotational principle (DP):

$$T(S_n) \leftrightarrow S_n: n \in \omega.$$

Then, assume for *reductio*:

1.	$T(S_n)$	
2.	S_n	1, (D P) for S_n *.
3.	$\forall k>n, \neg T(S_k)$	2, eq.
4.	$\forall k>n+1, \neg T(S_k)$	3, Arith.
5.	S_{n+1}	4, eq
6.	$T(S_{n+1})$	5, (D P) for S_{n+1} *
7.	$\neg T(S_{n+1})$	3, Arith
8.	$\bot$	6 and 7.
8.	$\neg T(S_n)$.	7, I $\neg$.

But *n* is arbitrary,

9.	$\forall n, \neg T(S_n)$,	8, Universal Generalization (UG). **
10.	$\forall n>1, \neg T(S_n)$	9, Arith.
11.	S_1	10, Eq.
12.	$T(S_1)$	11, (D P)
13.	$\neg T(S_1)$.	9, Universal Elimination (UE)
14.	$\bot$	10 and 11.

Of course, there are some problems with this proof. Firstly, the demonstration uses UG in step 9 (line marked (**)) and (D P) in 2, 6 and 12 (lines marked (*)). Then, it's natural to suppose that n and k are variables. Otherwise, it is not possible to apply UG. Nevertheless, as Priest correctly (1997, p. 237) focus on, the application of the (D P) would not be possible in this case. This principle only applies to sentences, not to formulae with free variables in. In fact, Priest (1997) proposes constructing the Yablo list adding the predicate name *Y* to first order arithmetic. Simply put, according Priest, Yablo's Paradox consists of an ω-sequence of formulas: Y(1), Y(2), Y(3),… Y(n). In other words, this is just the infinite sequence:[1]

$$Y(1) \leftrightarrow (\forall x)(x > 1 \rightarrow \neg T(<Y(dot(x))>, n))$$
$$Y(2) \leftrightarrow (\forall x)(x > 2 \rightarrow \neg T(<Y(dot(x))>, n))$$

[1] This uses Feferman's dot notation, designed to allow quantification into formulae containing quotation terms. More precisely, the expression < Y(dot(x))> is a function term that containing the variable x free. Intuitively it means 'the result of substituting the numeral of the number x for all free variables in Y'. This implies that, for example, < Y(dot(n))> denotes the code of Y(n), for each number n. The function term <Y(dot(x))> is in fact definable in arithmetic, using function terms for substitution and the naming function (the function which takes a number to its canonical numeral).

$$Y(3) \leftrightarrow (\forall x)(x > 3 \rightarrow \neg T(<Y(dot(x))>, n))$$
::::
$$Y(n) \leftrightarrow (\forall x)(x > n \rightarrow \neg T(<Y(dot(x))>, n))$$
::::

Secondly, the application of UG in line 9 (step marked (**)) is wrong: only in case *n* is a constant, a numeral for an (unknown) particular natural number, disquotational principle can be applied. But, Priest concludes that in this case UG could not be applied correctly. UG could only be applied in the case the sentence Sn were arbitrary. But, it is not. We expected that adding the list of the Yablo biconditionals and the Yablo Disquotation Scheme to first order arithmetic yields an inconsistency. However, it can be shown that this theory is consistent, although ω-inconsistent. The last result should be evident: the set of numerical instances of $\{Y(n) \leftrightarrow \forall x{>}n, \neg T(< Y(dot(x))>): n \in \omega\}$ must be consistent. If it were not, by Compactness, this should mean that there is a proof of a contradiction from some finite subset of the Yablo sentences. Nevertheless, as Hardy claims,

> If we restrict ourselves to a finite collection of the Yablo sentences, then no paradox arises. The upshot of this is that there is no first-order derivation of a contradiction from Yablo's premises (The Yablo List) and the Tarski biconditionals. (1996, p. 197)

And Ketland adds:

> Each finite subset of Yablo biconditionals is satisfiable. By the Compactness Theorem, the whole set is satisfiable (2005, p. 165, note 1)

Moreover, Ketland shows that "with an appropriate definition of the extension of 'true", it is possible to satisfy this combination on any non-standard model of arithmetic" (2005, p. 165). Leitgeb (2001) and Barrio (2010) have argued against theories of truth that only have non-standard models. In particular, ω-inconsistency causes a dramatic deviation in the theory's intended ontology. In order to be able to express the concept of arithmetic truth, the theory has to abandon the possibility of speaking about standard natural numbers.[2]

[2] Bueno and Colyvan have only insisted that the list of Yablo biconditionals plus the local disquotation principle is sufficient for a formal derivation of a contradiction. However, as Ketland has shown, they are mistaken. See Bueno and Colyvan (manusc.) and Ketland (2004).

Of course, second-order arithmetic with standard semantics avoids the existence of non-standard models. So, as Barrio (2010) has shown, adding Yablo's sequence to this theory produces a theory of truth that doesn't have a model. However, Picollo (2012) has shown that even in higher-order cases, the theory is consistent: one is not able to derive a contradiction from the set of Yablo sentences.

In sum, contrary to what happens with the Liar paradox, the set of Yablo sentences formalized in first-order arithmetic is consistent and satisfiable, even though is ω-inconsistent and only has non-standard models. In second-order case with standard semantics, one has only standard models. So, adding the set of Yablo sentences to second-order arithmetic produces a theory that doesn't have a model. But there is not a finitary proof of a contradiction.

2. The Charge of Circularity

In this section, I briefly review the charge of circularity. I will present four arguments. In all of them, one attempts to show that there is no way of reformulating Yablo's construction that does not involve circularity implicitly. In each case, I will summarize the Cook's responses.

The Argument of Existence of the Sequence

An important point that has been discussed is how one knows that the Yablo list exists. Yablo seems to assume the existence of the list in order to show that the list generates a paradox. Nonetheless, as Priest claims:

> He [Yablo] asks us to imagine a certain sequence. How can one be sure that there is such a sequence? (We can imagine all sorts of things that do not exist.) As he presents things, the answer is not at all obvious. In fact, we can be sure that it exists because it can be defined in terms of Y(x): the n-th member of the sequence is exactly the predicate Y(x) with "x" replaced by <Y(x)>. (1997, p. 238, notation was changed to match that the one used by me)

Nevertheless, in this case, the fixed-point construction required to generate the sequence of Yablo involves an implicit circularity. So, from Priest's perspective, the list of Yablo's sentences itself is circular:

> … the paradox concerns a predicate Y(x) of the form (∀k > x)(¬ T(<Y(x)>, k)), and the fact that Y(x) = '(∀k > x)(¬ T(<Y(x)>, k)' shows

> that we have a fixed point, Y(x) here of exactly the same self-referential kind as in the Liar paradox. In a nutshell, Y(x) is the predicate ‘no number greater than x satisfies this predicate’. The circularity is now manifest. (1997, p. 238).

Specifically, moving onto Cook’s terminology, the predicate Y(x) is weak predicate fixed point of the predicate: ‘$\forall k>x\ (\neg T(<Y(z)>, k)$’. In other words, each member of the list Y(1), Y(2), Y(3),… Y(n) is implied by what Ketland (2005) calls the Uniform Fixed-Point Yablo Principle (UFPYP):

$$\forall x\ (Y(x) \leftrightarrow \forall k>x, \neg T(< Y(dot(x))>, k))$$

The point of Priest is that the UFPYP involves circularity, because it provides a definition for the predicate Y in terms of itself. And since this principle guarantees the existence of the sequence, the list itself involves circularity. As Ketland pointed out:

> To stress, it is a theorem of mathematical logic that the Yablo list exists. This is a direct and well-understood construction. Priest does not ‘presuppose the existence of the list, in order to establish that to derive a contradiction from the latter, a fixed-point construction is required’. (2004, p. 169)

Now, consider the stronger Uniform Yablo Disquotation principle:

$$\forall x\ (T(Y(dot(x)) \leftrightarrow Y(x))$$

It is important to note that adding UFPYP and the Uniform Yablo Disquotation principle to PA yields an inconsistency. Nevertheless, in that case, the infinity of the list of Yablo biconditionals would not play any important role in the paradox. So, this did not appear to be acceptable.

In any case, Cook concedes that the list of Yablo, as formulated within arithmetic, is circular. And he accepts that the circularity involved is not distinct from the sort found in the arithmetic Liar. In his words: “if the existence of fixed points is enough for a statement or predicate to be circular, then the Yablo paradox is circular” (forthcoming, p. 96). But, he rejects that this sort of circularity (fixed point) to be a plausible cause of the paradox. And what is more important, he argues that the circularity involved in both is too broad to be relevant. His argument takes into account that every unary predicate (in a strong enough language) is a weak fixed point of some binary predicate, and every statement is a weak sentential fixed point of some unary predicate. Hence, according to him, this mathematical fact seems to throw serious doubts on the prospects of explaining the roots of paradoxes in terms of the presence of (this sort of weak fixed point) circularity. Cook emphasizes that the sort of circularity found in both the Liar paradox and the Yablo paradox seems to be an innocuous type of circularity, inasmuch as this sort of circularity is endemic throughout arithmetic.

The Foundamentally Epistemically Circular Argument

Turning to the second argument, Beall (2001) has offered new reasons in support of the circularity of the sequence. He focuses on our knowledge of the meaning of the predicate Y. Then, he claims that we have no way coming to know what the predicate Y means without employing a circular fixed-point principle. From his position, the Yablo sequence is *epistemically* circular because "everyone, I think, will agree: we have not fixed the reference of 'Yablo's paradox' via demonstration. Nobody, I should think, has seen a denumerable paradoxical sequence of sentences, at least in the sense of 'see' involved in uncontroversial cases of demonstration" (Beall 2001, p. 179). But, for Beall, any such description is circular. So, any entity that can only be referred to by a circular description must itself be circular. Then, Beall concludes, Yablo's paradox is circular.

Cook's response has two dimensions. On the one hand, he claims that Beall has missed a crucial point regarding the fixed-point construction found in Priest's position: the existence of the sequence is guaranteed by the UFPYP. On the other hand, his point depends on the idea that the only way we can know that the Yablo sequence exists in PA is to apply the UFPYP. In this point, Cook shows that this is not the case: "we could have found a suitable predicate even if we had never been shown the diagonalization argument guaranteeing that the Yablo predicate exists" (forthcoming, p. 102). Because theorems of PA are enumerable using an enumeration of valid proofs, in order to construct the Yablo sequence, it would be enough to run through an enumeration of valid proofs until we get one whose final line is:

$$\forall x\ (\ \Phi(x) \leftrightarrow \forall k{>}x,\ \neg\ T(\ <\Phi(dot(x))>,\ k))$$

Then, one can apply countably many instances of UE to arrive at the ω-sequence of Yablo biconditionals. There is nothing circular in the process of carrying out proofs, enumerating them, or surveying the resulting enumeration. Thus, there is nothing circular in (this way of obtaining) the Yablo paradox.

Unless one also shows that any way of specifying the sequence has to use a fixed point construction, the list will not be circular. Of course, this moves on to sketching a way of specifying the sequence without a fixed point. At this point, the Cook's idea is to show that there are non-circular paradoxes, but Yablo's construction in PA is not one of them. Consequently, Cook introduces an infinitary language Lp. This language only allows conjunctions (possibly infinite) of predications of falsity to sentences names. In other words, every sentence is of the form $\wedge i_{\in I}$ $F(S_i)$, where $\{S_i : i \in I\}$ is a (possibly infinite) class of sentence names and where F is the falsity predicate. He also uses a denotation function δ in order to providing the denotation of sentence names in Lp. If C is the collection of every sentence name in Lp, for each of the sentence name in C, δ is denotation function such that δ: C → {formulas de Lp}. For example, the Liar sentence can be formulated in this system as δ(S1): F(S1).

The logic of Lp is the infinitary system D. Proofs within D admit possibly transfinite sequences of expressions, where each expression is either a finite or infinite conjunction of instances of the falsity predicate applied to sentence names or an instance of the truth predicate applied to a sentence name. System D has introduction and elimination rules. It's really important to note that Conjunction Introduction is in some applications an infinitary rule that plays the same function that omega-rule in formal arithmetic.

In Lp, the Yablo sequence is the set $\{\langle Si, \wedge k>i\ F(Sk)\rangle : i \geq 1\}$ (where i ranges over the integers). Cook shows that Yablo paradox is in the context of D provably inconsistent. The proof has $\omega^2 + 3$ steps. Semantically, the paradoxicality is apparent in the fact that no valuation can be found for these sentences if F is really interpreted as the falsity predicate. It is interesting to note that Cook's formalization of the Yablo Paradox in Lp is a genuinely non-circular paradox. Cook shows the absence of weak fixed points in Lp. This is evidence for the non-circularity of its construction. Interestingly, Cook defines an operation of 'unwinding' which transforms any set of formulas with an assignment of denotations to the sentence names into another such set which (i) does not involve any (direct or indirect) self-reference, but which (ii) shares important semantic properties with the 'original'. Cook's goal was to define the simplest framework in which Yablo's construction could be somewhat generalized.

Cook asks for the circularity of his construction. Then, he defines a notion of fixed point for D, and proves the following:

THEOREM 2.4.3: Given any denotation function δ such that $\delta(Yn) = \wedge \{F (Ym) : m \in \omega, m>n\}$, there is no $\kappa \in \omega$ such that $\delta(Y\kappa)$ is a weak fixed point in D of $< \{Yn\} n\in\omega, \delta >$.

According to him, the absence of a fixed point is evidence for the non-circularity in Lp. The proof that none of sentences of Lp involves in the construction are fixed points shows that there are non-circular constructions that are paradoxical.

The Argument against ω***-Rule***

Moving onto following argument on charge of circularity, Priest intends to support that resource to the ω-rule does not help prevent circularity. He claims:

> One might suggest the following. We leave the deduction as just laid out, but construe the n in the reductio part of the argument as schematic, standing for any natural number. This give us an infinity of proofs, one of $\neg T(S_n)$, for each n. We may then obtain the conclusion $\forall n \neg T(Sn)$ by an application of the ω-rule:
>
> $$\frac{\alpha(0), \alpha(1),\ldots}{\forall x \alpha(x)}$$
>
> The rest of the argument is as before. Construing the argument in this way, we do not have to talk of satisfaction. There is no predicate involved, a fortiori no fixed point predicate. We therefore have a paradox without circularity. (1997, pp. 238-239)

But, Priest adds:

> As a matter of fact, we did not apply the ω-rule [in his earlier sketch of the derivation of a contradiction], and could not have. The reason we know that $\neg T(S_n)$ is provable for all n is that we have a uniform proof, i.e. a proof for variable n. Moreover, no finite reasoner ever really applies the ω-rule. The only way that they can know there is such a proof of each $\alpha(i)$ is because they have a uniform method of constructing such proofs. And it is this finite information that grounds the conclusion the $\forall x \alpha(x)$. (1997, p. 239)

Priest's position against the use of ω-rule in Yablo's Paradox: being that no finite human being ever really applies the ω-rule (or any infinitary analogue such as our infinitary variant of conjunction introduction above), then the only way we can know that the Yablo Paradox truly is paradoxical is through a proof depending on fixed points of the sort described above. In contrast, Selmer Bringsjord and Bram van Heuveln defend:

> The point (…) is that in light of such arguments, Priest is in no position to simply assume… [that we are finite reasoners i.e. Turing machines] …and hence he hasn't derailed the infinitary version of Yablo's paradox. (2003, p. 65)

And they add:

> … also argued… specifically that logicians who work with infinitary systems routinely and genuinely use the ω-rule. Again, the claim isn't that such arguments are sound, and that therefore some human persons, contra Priest, genuinely use the ω-rule. The claim is a simple, undeniable one: if any of these arguments are sound, then we can really use the ω-rule, and the infinitary reasoning we gave above would appear to be above reproach. (2003, p. 67)

One can not just assume without argument that we are finite reasoner. Neither that a finite reasoner is a Turing machine. There are many arguments that show that we are not Turing machines. Hypercomputers, unlike Turing machines and their equivalents (and lesser systems), *can* make essential use of the ω-rule. And this is just a brute mathematical fact. Further, there are no compelling reasons for thinking that the performance of supertasks is a logical impossibility. Besides, according to Cook, Priest's objection "(…) relies on the idea that we might restrict the notion of *truth* (…) to natural languages or finitary languages (or both). The motivation for such a restriction, one assumes, would be the observation that all language users that we have come into contact with (and, importantly, all language users that matter) speak finitary languages that do not allow for the construction of the truly non-circular paradox sketched above." However, the main point of Cook is that restricting our account of truth (and our development of a view on the semantic paradoxes) to languages that we are able to speak looks worryingly provincial.

Cook changes the focus of the discussion. There are two different problems:

ONTOLOGICAL: Is there any infinite sequence that represents the truth predicate of some infinitary language and does not be circular?

EPISTEMOLOGICAL: Could a being with our epistemic capabilities knowing that infinite sequence by means no circular?

According to Cook, the main discussion about Yablo's paradox is on the ontological problem. Firstly, logic is modeling truth preservation and not all systems of logic are complete. Secondly, logic is used to describe mathematical structures. Infinitary languages, whose models are structures under study, might raise conceptual problems as Yablo's construction seems to show.

The Structural Collapse Argument

Hannes Leitgeb has recently suggested how to understand the notion of *circularity* rejecting definitions of the type discussed above (namely, in term of *fixed point*). Even though he does not endorse this approach, one could use non-well-founded sets to elaborate our intuitive concept of *circularity*. Using an analogy between non-well-founded sets and non-well-founded sentences, he claims:

> It may be shown that there are sets X and Y, such that X = {X} and Y = {Y1}, Y1 = {Y2}, Y2 = {Y3},...: intuitively, X is circular with respect to the membership relation whilst Y is not. However, according to Aczel's anti-foundation axiom, X is *identical* to Y, and thus either both are circular, or both are not, or the notion of *circularity* is to be abandoned. On the other hand, this is not necessarily the case if only some different set theory is chosen which allows for non-well-founded sets but which replaces the axiom foundation differently, such that X and Y do not turn out to be identical. (2002, p. 13)

Here, there is another way to argue for the circularity of Yablo's construction, showing that the Yablo paradox shares the same underlying structure as the Liar paradox. This is the *Collapse Argument*. Cook summarizes it clearly:

> There would be some operation that mapped each linguistic construction involving truth, satisfaction, or other semantic notions onto a particular pure (possibly non-well-founded) set. At a minimum, the result of applying such an operation to a set of statements should provide a (possibly non-well-founded) set whose membership relation is isomorphic (or, at least structurally analogous in some other well-defined and well-motivated manner) to the referential structure of the set of statements that served as input.

Yablo himself seems to have accepted the collapse argument:

> A point in favor of the structural collapse worry is that if we try to model the propositions involved in Aczel's non-well-founded set theory, they come out identical. This is because Aczel has one set per isomorphism type of directed graph, and the graphs here are isomorphic, each has the structure of a downward facing tree with omega branches descending from each node. (2006, p. 169)

Nevertheless, Cook replies that the structural collapse argument requires more than the mere existence of non-well-founded sets – that is, it requires more than the mere claim that some version of the anti-foundation axiom is true – at least, if we wish to use this strategy in order to demonstrate that all paradoxes, including all Yabloesque constructions, are circular. I am assuming some familiarity with the non-well-founded set. Roughly, there are different variants of non-well-founded set theory. And Cook adds that "the structural collapse account requires that AFA is the correct or 'best' anti-foundation axiom (and none of BAFA, FAFA, or SAFA is correct)". Further, Cook adds that the identity of the 'Liar' set and the 'Yablo' set does not entail the identity of the Liar statement and the Yablo paradox. And what is even worse, the structural collapse account seems self-defeating. If the argument depends on mobilizing a non-standard set theory that embraces that very same circularity – then, even if were the structural collapse account successful, it is not clear that it would provide what its defender presumably desires.

I sympathize with the response of Cook to Priest's point: the circularity involved in PA is too broad to be relevant. The sort of circularity found the Yablo paradox formulated in PA overgeneralizes: all arithmetic predicates turn out to be circular. But this does not be the case. So, one can use PA to formulate the Yablo paradox avoiding the risk of circularity. In my opinion, the problems associated with the ω-inconsistency in first order arithmetic are evidence to consider that one has a good non-circular representation of the list of Yablo's sentences. Moreover, second-order arithmetic with standard semantics avoids the existence of non-standard models. So, adding Yablo's sequence to this theory produces a theory of truth that doesn't have a model. I think that if a theory of truth that be ω-inconsistent is a bad thing, having a unsatisfiable theory is really bad. In this case, unlike the approach of Cook, one shows that adding Yablo's list to arithmetic produces serious problems. Of course, Cook is right: logic is modeling truth preservation. Infinitary languages might raise conceptual problems as Yablo's construction seems to show. But these are additionals problems to languages without quantifiers. I prefer not to limit the problems to infinitary languages.

The discussion that follows is the result of the visit of Roy Cook to SADAF in July 2011. It contains six short articles in which different positions concerning the Yablo Paradox are defended. Each one involving some divergence with respect to Cook's position. The first one, written by Picollo, argues that there is a non-circular formulation of the Yablo sequence in a first-order arithmetical language by providing a criterion for circularity that, she claims, avoids the flaws that other notions have, including the one embraced by Cook. Next paper, written by Teijeiro, establishes a difference between "natural paradoxes" and "formal paradoxes", and then argues that there is a natural, non-circular Yablo paradox, but there is not a non-circular formal one. The following articles discuss the adoption of infinitary logic to avoid circularity. Thus, Pailos explore two ways to argue that Cook's version of Yablo's paradox is not genuinely non-circular. The attempts to prove that it's not, lead to a very narrow conception of a theory of truth, or to deny that a paradigmatic case of paradox, such as the "Old-Fashioned Liar," is truly paradoxical. Tajer analyzes the links between anti-realism and finitism. Tajer's main claim is that, contrary to what is often said, anti-realism is not necessarily committed to finitism; the most important motivations for anti-realism are compatible with infinitary rules of inference. Rosenblatt's contribution discusses the possibility of a general purge of self-reference. His main points are that there is an unproblematic way to transform circular into non-circular constructions by using the method of unwindings and that this can be done even in the case of constructions that involve epistemic and/or modal expressions. Finally, Ojea claims that Cook's version of the Yablo's paradox in Lp is genuinely non-circular, but for different reasons than the ones alleged by Cook. He argues that the absence of fixed points in the construction is insufficient to prove the non-circularity of it, and suggests a way in which the structural collapse account may be useful to nevertheless show it is a genuine non-circular paradox. The discussion ends with Cook's responses to the objections involved in the afore-mentioned papers.

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