PERFORMANCE OF ARTIFICIAL NEURAL NETWORKS ON KRIGING METHOD IN MODELING LOCAL GEOID

Desempenho das redes neurais artificiais sobre o método da Krigagem no modelamento do geóide local.

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ABSTRACT

Transformation of ellipsoidal heights determined by satellite techniques into local leveling heights requires geoid heights at points of interest. However, the geoid heights at each point are not available. In order to determine them, the local geoid in the transformation area must be modeled or computed by an appropriate method, one way of doing it, is to use control points both of whose ellipsoidal and local leveling heights are available. In this study, performance of geoid by ANN compared to Kriging method in modeling local geoid was presented. Moreover, the transformation ability of the methods was investigated through a geodetic test network in Bursa Metropolitan Area of Turkey. The results suggest that the model by ANN exhibit better results than the one by Kriging Method.

Keywords: Geoid; Local Geoid Modeling; ANN; Kriging; Transformations.

RESUMO

A transformação das altitudes elipsoidais determinadas por técnicas satelitais em locais de altitudes niveladas requer o conhecimento da altitude geoidal no ponto de interesse. Entretanto, as altitudes geoidais de cada ponto nem sempre estão disponíveis. A fim de determiná-las, um geóide local de transformação deve ser determinado ou calculado através de métodos apropriados. Uma maneira de fazer isto, é usar pontos de controle de ambos cujas altitudes elipsoidais e niveladas estejam disponíveis. Neste estudo, apresenta-se os resultados da comparação entre o geóide obtido por ANN e o método da Krigagem na modelagem do geóide local.

Além disso, a habilidade da transformação dos métodos foi investigada através de uma rede geodésica teste na área metropolitana de Bursa na Turquia. Os resultados sugerem que o modelo por ANN exibe melhores resultados que o método da Krigagem.

Palavras-chave: Geóide; Modelamento do geóide local; ANN; Krigagem; Transformações.

1. INTRODUCTION

Global positioning system (GPS) makes three dimensional positioning possible at any time at any place on the earth and its atmosphere (SEEBER, 2003) . While horizontal positions are used directly on specific projections in local or global coordinate frame at a request, the ellipsoidal height component cannot directly be used without the prior knowledge of where the geoid is when the same points are considered.

In practice, for the use of ellipsoidal heights they must be transformed into a geoid-referenced height system as orthometric height. To obtain orthometric heights one can use geoid undulations that can be approximated by differences between ellipsoid and orthometric heights of points. That makes geoid undulations determination indispensable for GPS derived orthometric heights.

GPS derived geoid can be approximated provided that a reasonably large number of points whose ellipsoidal and orthometric heights known are available. This is described by a well-known geometrical relationship i.e. geoid undulation N is equal to an ellipsoid height h minus leveled height H (BOMFORD, 1979). The geoid is an irregular surface so it is not an easy task to approximate it with simple mathematical expressions. However, there exist a number of methods for approximating the local geoid surface with relatively simple expressions. These methods include the linear polynomial, the degree polynomial, the Kriging, etc., and alternatively the artificial neural network (ANN). One of the usages of the ANN is the function approximation (HASSOUN, 1995; DEMUTH and BEALE, 1998). It has been a method for local geoid determinations. A number of studies carried out to determine a local geoid (KAVZAOGLU and SAKA, 2005; STOPAR, et al., 2006; VERONEZ, et al., 2006; GULLU, et al., 2011). However, the geoid determined has not been compared with the one approximated by Kriging method.

This paper investigated the performance of estimated geoid by ANN compared to the one by Kriging method in Modeling Local Geoid. This was carried out on a test network established in Bursa Metropolitan area (Turkey).

2. MATERIAL AND METHODS

The ANN and Kriging methods were used in a local geoid determination using GPS/leveling data. Their descriptions are described briefly here.

2.1 Artificial Neural Network

An artificial neural network (ANN), generally known as a "neural network" (NN), is a mathematical model based on biological neural networks. The neural network consists of layers that contain processing units called "artificial neurons". An artificial neuron has input(s) multiplied by synaptic weight(s), which are to be determined by a training method, and an output through an activation function whose input is the sum of weighted inputs. The activation function determines the output of neurons, which is also taken as input to neurons in the subsequent layer.

The learning method in ANN is called as a training process. In training process experimental data is introduced to the neural network in order to establish a relationship between the input and output of the model. In this respect, neural network models the system under inspection and also can be used as a universal function approximator. Learning methods in neural networks is classified, based on the training strategy, into supervised and unsupervised training. The objective of the training algorithm is to reduce the total error between predicted output and given output (target) values. Mostly, gradient based algorithms are employed in a supervised learning process. In unsupervised learning, a neural network is assumed to classify input parameters based on features present in given input parameters.

Neural network is also classified based on their structures. The most encountered neural network structures include multi-layered perceptron (MLP), Radial Based Functions (RBF) Kohonen self-organized map (SOM) and Hopfield Neural Networks. Here only MLP is given the reader is referred to (Demuth and Beale, 1998) for other network types.

MLP neural network is organized in layers namely; input layer, one or more hidden layers and output layer. In each layer, a specific number of neurons with an activation function is placed. Together with the number of neurons, the number of the layers determines the topology of MLP neural network. A MLP neural network mostly contains one or two hidden layers. The number of neurons in the input and output layers is set to match the number of input and target parameters, which are used to model the process. As for the number of neurons in the hidden layer, there is not yet any given procedure to determine an optimum number of hidden layers and the number of neurons in each layer. The number of neurons in a layer and the number of layers are determined by experience and some rule of thumb.

To obtain sufficient results, synaptic weights have to be adjusted. This process is called learning methods. One of the common methods of learning methods in ANN is a backpropagation algorithm, details of which are presented next.

2.1.1 Backpropagation Algorithm

Back propagation (BP) algorithm is a supervised learning method for MLP neural networks. It is essentially a gradient decent based optimization technique. In this algorithm an error calculated from predicted and target values of output for the given input parameters, is propagated backward in order to adjust interconnection

(synaptic weights) between neurons so that the error is minimized. A fully-connected neuron has inputs from all the nodes (inputs) in the previous layer and has an output to neurons in the subsequent layer. In this structure, each neuron in a layer is mapping the sum-of-weighted input into an activation level that is input for an activation function. The activation function is a differentiable function. The most commonly used activation functions are the sigmoid, the tangent-hyperbolic and linear activation function, mostly at the output layer. One-layer ANN network is presented in Figure 1 as follows,

Figure 1. One-layer Artificial Neural Network.

$$v_k = \sum_{j=0}^{n} w_{kj} x_j$$
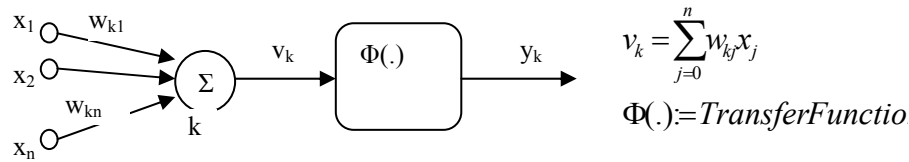
$$\Phi(.) := TransferFunctio$$

If a sigmoid function is used as transfer function, the output of the kth neuron in the layer is determined by the following equation,

$$y_k = \frac{1}{1 + e^{(-av_k)}} \tag{1}$$

$$v_k = \sum_j W_{kj} x_j$$

where yj is the output of a neuron k in the previous layer, "a" is a tuning parameter and W_{kj} is the weight between input xj and neuron k.

In this structure, the adjustable network parameters are optimized based on BP algorithm as follows,

$$W_{kj} = W_{kj} + \Delta W_{kj} \tag{2}$$

where Δ Wkj is weight update for the connection between jth and kth neurons in subsequent layers. The weight updates are calculated according to BP algorithm as follows,

$$\Delta W_{kj} = \eta \left(\frac{\partial E}{\partial W_{kj}} \right) \tag{3}$$

where E is the error (cost function) defined as,

$$E = \frac{1}{2} \sum_j \sum_k (T_{kj} - y_{kj})^2 \tag{4}$$

where j is the number of samples in training set and k is the number of output neurons. In order to elaborate implicit relations between the weight updates Δ W. (3) is given as follows,

$$\Delta W_{kj} = -\eta(T_k - y_k)x_j \qquad (5)$$

where η stands for learning rate, and the minus sign is added due to the opposite direction of the gradient is need for optimization.

As it can be seen from (5), the amount of the update signal produced by BP algorithm is a function of input and output and their derivatives.

2.2 Kriging Prediction Method

The Kriging is a statistical prediction method for variables to be predicted, i.e. local geoid undulation N in this study, which were computed as a linear combination of a set of control points. The method developed by a mine engineer D.G. Krige from South Africa has commonly used especially in modeling mine surface in mining and modeling over burden strata in geology. For all applications of the method, by using the criterion $\sum P_i = 1$ estimating error variance, the best linear equation system is provided to conclude interpolation with least error (Gedikoglu, 2000) where Pi is the weight determining the contribution of a sample value to the prediction.

Semi-variance is a measure of the degree of spatial dependence between samples. The basis of the method may consists of modeling graphics and obtaining semi-variances for each variable to be predicted. After obtaining the semi-variances the weights required can be determined so that the demanded variable at any point p with the coordinates x, y can be computed by the following general equation of Kriging,

$$N(x, y)_p = \sum_{i=1}^{n} P_i N_i = P_1 N_1 + P_2 N_2 + \dots + P_n N_n \qquad (6)$$

where Ni's are the sample values used in the prediction of $N(x, y)_k$, i.e. the geoid undulations at control points to be used in the prediction of the geoid undulation $N(x, y)_k$ at the point k.

In general, there are two different applications of Kriging method depending on the strategies of determining weights:

Weighting, known as simple, ordinary or punctual Kriging

2.2.1 Universal Kriging weighting

Before determining weights by one of these strategies, the distance depended variation of semi-variances must be modeled by using the points whose variable values are available. For this purpose, the double combinations of these points are taken into consideration, and the distance between the points of each combination is divided into appropriate intervals for modeling semi-variances. After that, the semi-variance values for each interval between n numbers of points are computed by,

$$\gamma_s = \sum_{i=1}^{n-s} (N_i - N_{i+s})^2 / 2n \qquad (7)$$

In this notation, Ni is a measurement of a regionalized variable (i.e. geoid undulation in this study) taken at location, i, and Ni+s is another measurement taken h intervals away (DAVIS, 1986) and γi is the semi-variance value obtained from linear or nonlinear graphics called semi-variogram. Then, using the semi-variances, a scattered diagram "semi-variogram" regarding distance is drawn. Finally, a curve as in Figure 2 is fitted to the scattered semi-variance values in the diagram,

Figure 2. Various semi-variogram curves.
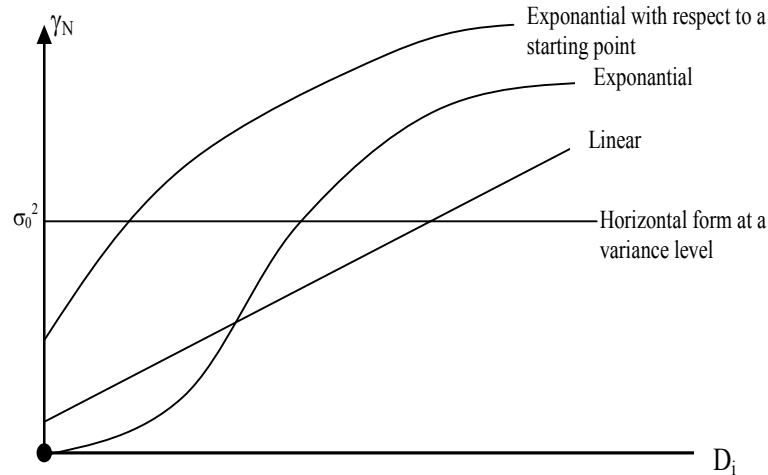
so that the semi-variance value between any two points whose variable values are not available can be obtained from the fitted curve.

2.2.2 Weight Determination by Punctual Kriging

The punctual application of Kriging is the most favorite sort of the method in a weight determination due to its simple mathematical model and ease solvability. Punctual Kriging is an ideal method for surface modeling by intervals with equivalent distance and low slope (GEDIKOGLU, 2000; DAVIS, 1986).

In Punctual Kriging, the relationship between semi-variances and the weights of observations at control points to be used for computing the variable at a point p is given by (λ),

$$\sum_{i=1}^{n} P_i \gamma(s_{ji}) + \lambda = \gamma(s_{jp}) \qquad (j = 1, 2, 3 \ldots, n) \qquad (8)$$

Here we will not attempt how to derive this formula; its further discussion is contained in (CLARK, 1979) and (OLEA, 1975). This formula yield n number of equations for n number of control points:

$$P_1\gamma(s_{11}) + P_2\gamma(s_{12}) + P_3\gamma(s_{13}) + \ldots + P_n\gamma(s_{1n}) + \lambda = \gamma(s_{1p})$$
$$P_1\gamma(s_{21}) + P_2\gamma(s_{22}) + P_3\gamma(s_{23}) + \ldots + P_n\gamma(s_{2n}) + \lambda = \gamma(s_{2p})$$
$$P_1\gamma(s_{31}) + P_2\gamma(s_{32}) + P_3\gamma(s_{33}) + \ldots + P_n\gamma(s_{3n}) + \lambda = \gamma(s_{3p}) \qquad (9)$$
$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots$$
$$P_1\gamma(s_{n1}) + P_2\gamma(s_{n2}) + P_3\gamma(s_{n3}) + \ldots + P_n\gamma(s_{nn}) + \lambda = \gamma(s_{np})$$

For the unbiased solution of the weights, the equation coming from the criterion $\sum P_i = 1$ is added to the simultaneous equations above. In this way, the final simultaneous equations can be rearranged in matrix form as follows,

$$\underbrace{\begin{bmatrix} \gamma(s_{11}) & \gamma(s_{12}) & \gamma(s_{13}) & \cdots & \gamma(s_{1n}) & 1 \\ \gamma(s_{12}) & \gamma(s_{22}) & \gamma(s_{23}) & \cdots & \gamma(s_{2n}) & 1 \\ \gamma(s_{13}) & \gamma(s_{23}) & \gamma(s_{33}) & \cdots & \gamma(s_{3n}) & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \gamma(s_{1n}) & \gamma(s_{2n}) & \gamma(s_{3n}) & \cdots & \gamma(s_{nn}) & 1 \\ 1 & 1 & 1 & \cdots & 1 & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ \vdots \\ P_n \\ \lambda \end{bmatrix}}_{P} = \underbrace{\begin{bmatrix} \gamma(s_{1p}) \\ \gamma(s_{2p}) \\ \gamma(s_{3p}) \\ \vdots \\ \gamma(s_{np}) \\ 1 \end{bmatrix}}_{B} \qquad (10)$$

this equation can be represented in matrix form as,

$$\mathbf{Ap = B} \qquad (11)$$

the weights sought for the solution are obtained as follows,

$$\mathbf{p = A^{-1}B} \qquad (12)$$

By replacing these weights into Equation (6), the variable at the point p is estimated. The error variance of the prediction is computed by,

$$m_\varepsilon^2 = P_1\gamma(s_{1p}) + P_2\gamma(s_{2p}) + P_3\gamma(s_{3p}) + \ldots + P_n\gamma(s_{np}) + \lambda \qquad (13)$$

2.2.3 Weight Determination by Universal Kriging

If the variable to be predicted exists on an uneven surface, a linear estimation is no longer unbiased. In this case, a trend function should be introduced to the weight determination in Punctual Kriging. For the trend, a first-order or second-order polynomial might be used; in the case of the first polynomial, Equation (10) becomes as follows:

$$
\begin{bmatrix}
\gamma(s_{11}) & \gamma(s_{12}) & \gamma(s_{13}) & \cdots & \gamma(s_{1n}) & 1 & \Delta x_1 & \Delta y_1 \\
\gamma(s_{12}) & \gamma(s_{22}) & \gamma(s_{23}) & \cdots & \gamma(s_{2n}) & 1 & \Delta x_2 & \Delta y_2 \\
\gamma(s_{13}) & \gamma(s_{23}) & \gamma(s_{33}) & \cdots & \gamma(s_{3n}) & 1 & \Delta x_3 & \Delta y_3 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\gamma(s_{1n}) & \gamma(s_{2n}) & \gamma(s_{3n}) & \cdots & \gamma(s_{nn}) & 1 & \Delta x_n & \Delta y_n \\
1 & 1 & 1 & \cdots & 1 & 0 & 0 & 0 \\
\Delta x_1 & \Delta x_2 & \Delta x_3 & \cdots & \Delta x_n & 0 & 0 & 0 \\
\Delta y_1 & \Delta y_2 & \Delta y_3 & \cdots & \Delta y_n & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
P_1 \\ P_2 \\ P_3 \\ \vdots \\ P_n \\ \lambda \\ \alpha_1 \\ \alpha_2
\end{bmatrix}
=
\begin{bmatrix}
\gamma(s_{1p}) \\ \gamma(s_{2p}) \\ \gamma(s_{3p}) \\ \vdots \\ \gamma(s_{np}) \\ 1 \\ 0 \\ 0
\end{bmatrix}
\quad (14)
$$

where Δx's are the shifted coordinates of control points to the location of the variable to be predicted, $\alpha 1$ and $\alpha 2$ are the coefficients of the first-order polynomial.

This application of Kriging where the weights of control points are determined as in the equation above is called Universal Kriging. In order to determine weights by Universal Kriging, additional control points are required because there are more parameters than Punctual Kriging.

3. CASE STUDY

In order to investigate performance of neural network method against the Kriging Method for modeling local geoid, we used a test network established in Bursa Metropolitan Area in Marmara Region located in the North-Western of Turkey. The test network consist of 74 points. Five points of the network were chosen as prediction points, the rest of the points was utilized as control points in computing the geoid undulations at the prediction points (Figure 3).

Within the frame of Digital Map Production Project of Bursa, Trimble 4000SSE and Topcon Turbo SII GPS receivers were used in determination of the point coordinates with the accuracy of ±6mm, and the leveled heights of all the points were measured by geometrical leveling with the determined orthometric height accuracy of ±15mm/km. The leveled heights of the five points chosen as the prediction points were determined as 36.560, 36.619, 36.676, 36.587, and 36.802 in meter, respectively.

Figure 3. Test network for the case study.
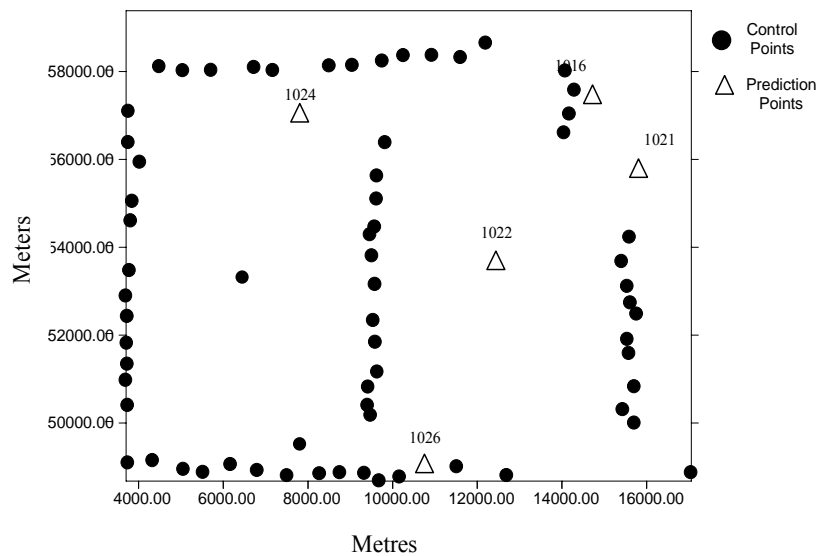
Using the leveled heights from geometrical leveling and the ellipsoidal heights from GPS, geoid heights were calculated, and a semi-variogram was drawn using the calculated undulations at the control points. A curve regarding the method of the two period moving average was fitted to this semi-variogram to obtain the semi-variances to be required for the weight determination; Figure 4 shows the semi-variogram and the fitted curve.

Figure 4. Semi-variogram from the control points and the fitted curve.
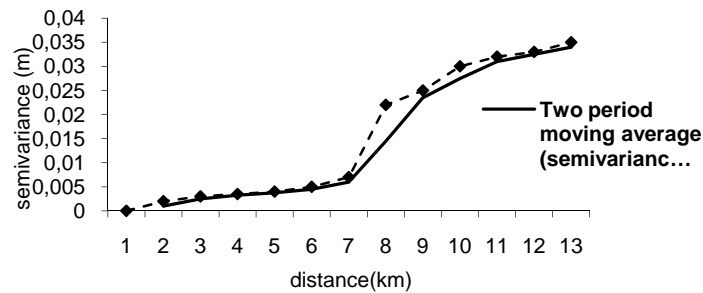
After fitting the semivariogram curve, we performed two different investigations for the prediction of the seven points. Firstly, we carried out the punctual application of Kriging with 4, 9, 18, 37 control points appropriately distributed around each prediction point and with all the 67 control points, and compared these predictions with the measurement values mentioned above. As a result of these comparisons, the absolute differences from the measurement values are visualized in Figure 5. Using the absolute differences, the root mean square (rms) values are obtained as 24.2, 45.9, 23.2, 13.9 and 35.3 in millimeter, respectively.

Figure 5. Absolute differences of the Punctual Kriging predictions with the varying number of control points from the measurement values.
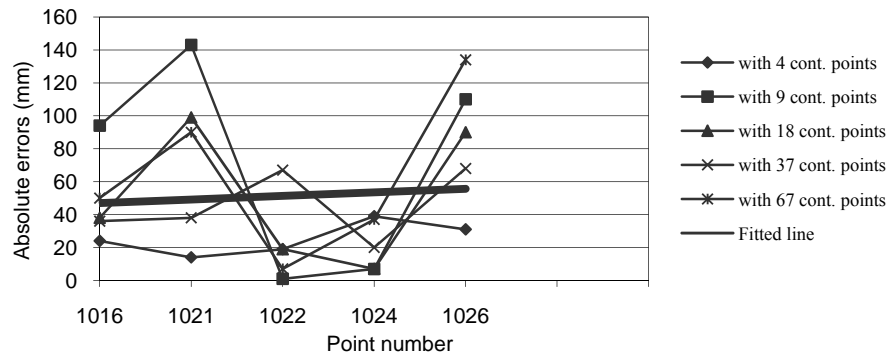
We subjected all the rms's to Cochran Equivalence Test given by $\overline{G}_{mak} > G_{mak}(f, m)$ with the degree of freedom f and the number of the applications m (Aytac, 1984). The test value of Cochran using the equation $\overline{G}_{max} = \sigma^2_{maximum} / \sum \sigma^2$ was obtained as 0.45, and the limit value for f=5 and m=5 was taken as 0.51 from the table of Cochran. As the test value is smaller than the limit value ($\overline{G}_{mak} < G_{mak}(f, m)$) in our problem, we concluded that all the predictions are equivalent to each other.

## 3.1 Implementation of ANN for Predicting Geoid Undulations

In theory, Backpropagation (BP) algorithm can solve any problem with one hidden layer. However, in practice the theory doesn't provide any sound tool to find optimal network structure and corresponding weights for the network, and there is also no specific algorithm or method of determining the number of neurons in a layer for a particular work; however, trial and error method is a one way of determining them as stated by Altun et al., (2007).

An artificial neural network was designed to learn the complex geoid surface using 251 geoidheights. The ANN consists of five layers, one input layer, one output layer, and three hidden layers having 12, 8, and 4 neurons respectively (Figure 6). In training phase learning rate was taken 0.02 and momentum factor 0.95 out of 4000 epochs. The training was based on the backpropagation algorithm described above. Transfer functions used were tangential sigmoid for the hidden layers and pure linear function for output layer.

The inputs are the normalized coordinates (Easting and Northing) and the output is the normalized geoid undulation.

The five prediction points were taken as input for the trained ANN and corresponding undulations were obtained as outputs. Figure 7 shows how the trained network fit to the data used for training.

Figure 6. Designed ANN for Modeling Geoid Undulation.
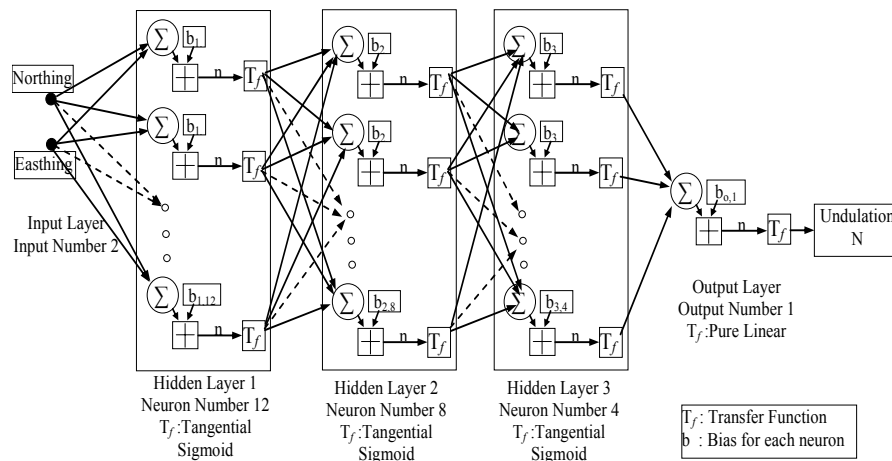
In Figure 7, the simple differences obtained from the results estimated using four control stations gave 'best' results as being closest to zero absolute error. This indicates that punctual Kriging methods did not provide better results as the control stations increased.

From the Figure 8, clearly Universal Kriging Method relatively gave better results in comparison to punctual one. However, ANN gave 'best' results as compared to the other two.

Figure 7. Trained network and Estimated geoid heights of Test Points.
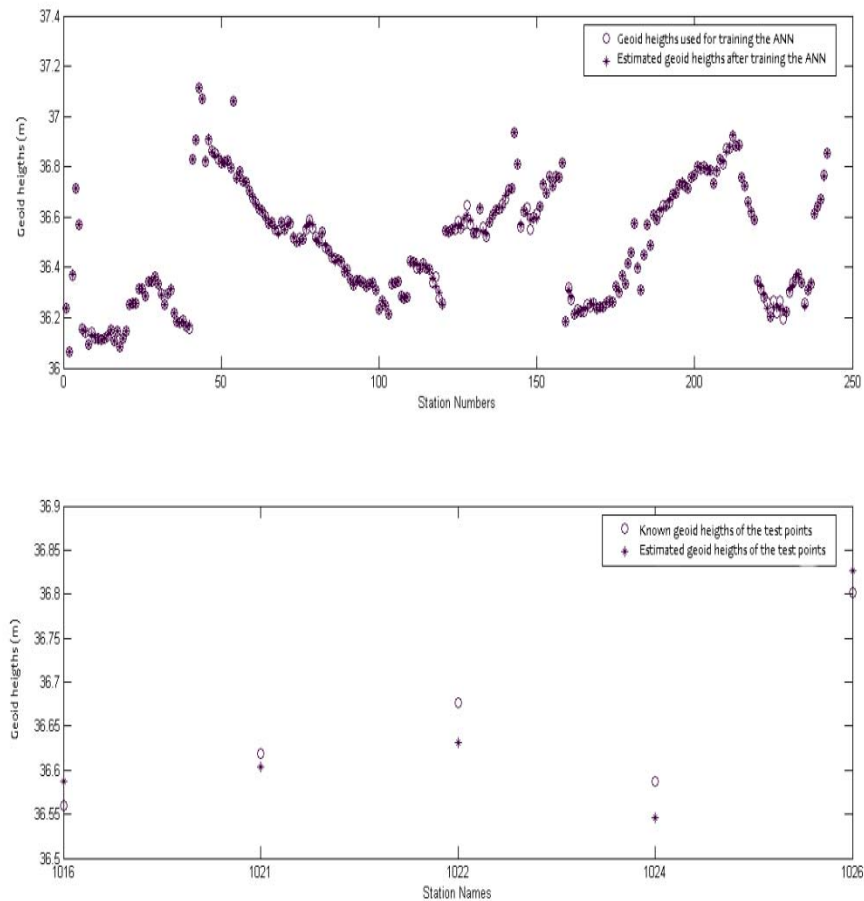
ANN results showed that test points 1016, 1021, and 1026 being laterally closer to the data used in training were better than those being laterally farther to data (1022 and 1024 from Figure 2). This indicates that ANN with more evenly distributed data over the area where a local geoid modeled may give better results than unequally distributed data. These results are compatible with the one given by Veronez et al, (2006) who used evenly distributed data from MAPGEO2004 software over the surface and suggested the ANN for modeling local or regional Geoid.

Figure 8. Absolute D of the Punctual, Universal Kriging and ANN P with all the C
P from the Measurement Values.
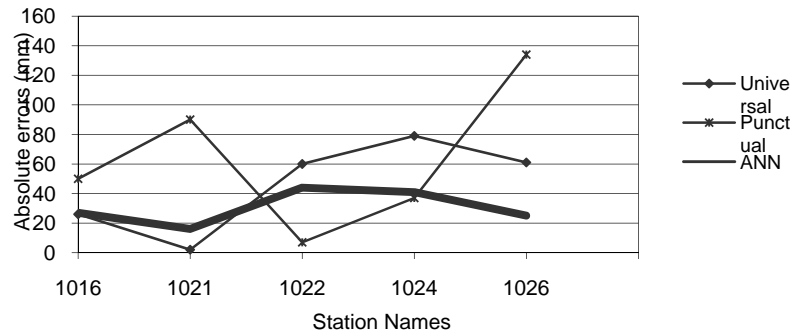
In our case, however training data were not evenly distributed over the surface but only surrounded the area as seen from the Figure 3.

4. CONCLUSION AND SUGGESTIONS

In this study, we performed comparisons of local geoid heights obtained from ANN to the heights estimated from Kriging prediction method over a test network in Bursa Metropolitan Area.

It was determined that the geoid height obtained from a trained ANN were better than those of the Kriging Method.

In general, an artificial neural network might be used in local or regional Geoid modeling.

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