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Article

MODEL FOR DETERMINATION OF THREE-DIMENSIONAL COORDINATES OF HIDDEN POINTS

Modelo para determinação de coordenadas tridimensionais de pontos ocultos

Érica Santos Matos

Alyne Raminelli Siguel Gemin

Pedro Luis Faggion

Universidade Federal do Paraná – UFPR.Programa de Pós-Graduação em Ciências Geodésicas – PPGCG.Centro Politécnico –Jardim das Américas –Curitiba–Paraná – Brazil. Email:ericamatos@ufpr.br; alynesiguel@gmail.com; faggion@ufpr.br.

Abstract:

One of the problems encountered during surveys is obstruction of points to be monitored, an example of this can be found in industrial environments where there are usually pipes and equipment that do not allow the establishment of straight lines;. An alternative to the application of topographic survey techniques is the use of plane mirrors to diversion of straight lines where the points of interest are observed indirectly. This study presents a new approach to the problem, whose analytical solution is based on the principles of Geometrical Optics and Surveying. The relative positional accuracy achieved in the preliminary tests is submillimetric order, and the absolute positional accuracy is millimetric order.

Keywords: Industrial Surveying; Hidden Points; Plane Mirror.

Resumo:

Um dos problemas encontrados durante levantamentos topográficos é a obstrução de pontos a serem monitorados, um exemplo deste empecilho é verificado em ambientes industriais, onde geralmente existem tubulações e equipamentos que não permitem o estabelecimento de linhas de visada direta. Uma alternativa para aplicação de técnicas topográficas é o uso de espelhos planos para o desvio de linhas de visada, onde os pontos de interesse são observados indiretamente. Neste estudo é apresentada uma nova abordagem para o problema, cuja solução analítica baseia-se nos princípios da ótica geométrica e da topografia. Os ensaios preliminares indicam a viabilidade do modelo proposto, nos quais a precisão posicional relativa alcançada é de ordem submilimétrica, enquanto no posicionamento absoluto do ponto são de ordem milimétrica.

Palavras-chave: Topografia Industrial; Pontos Ocultos; Espelho Plano.

1. Introduction

Industrial surveying has applicability in many areas of human activity where techniques and methodologies are used to development and monitoring of civil or mechanical projects, for example. Gonçalves (2009) cites some main applications as the location of structures, metric control of equipment and industrial products or the machine control shifts. Other activities are based on the integration of sensors and study of objects, focusing on modeling and temporal monitoring.

In this study, the objective is to determine coordinates of points in hard access environments, where industrial surveying is seen as an output to enable surveys to guarantee precision and accuracy, according to the necessity. In industrial environments, often the spaces are taken by pipes, thermal variability or operating equipment that make it difficult the establishment of straight lines to points of interest, avoiding the conventional surveying of object (Radovanovic and Teskey, 2003). In this sense, the aim of this work was to determine the coordinates of hidden spots where positions where the line of sight set from the observer to the point of interest is blocked, and then, preventing its direct reading.

According to Antonopoulos (2005), Easa and Shaker (2010) there are two options for determining the coordinates of hidden points. The first is based on the observation points using a mirror, which is the focus of this study however with limited literature, and the second through the use of the bar to hidden points. The procedure for the use of such bar is to position it on the point of interest and to observe marks calibrated by setting a line in space, and from this, mathematically estimate the position of the observed point. The disadvantage of this alternative is the need to have access to the point of interest, which is often not possible. Thus, the application of straight lines reflected by plane mirror becomes advantageous since it is unnecessary to point occupation for any type of accessory.

Ahmed (1994) will hold a discussion on the use of straight lines reflected by plane mirrors, indicating advantage as less time spent for indirect surveys when compared to change equipment position to observe all the points, whether hidden or not. In this sense, Gonçalves (2009) is based on the design of Ahmed (1994), by presenting the definition of a mathematical model to determine coordinates of points indirectly, with a solution based on recurrent transformations of reference systems, whose propagation error has been studied by Pinto (2013).

Finally, this study presents an enhancement of the solution suggested by Gonçalves (2009). It is shown a new mathematical modeling in a unique reference system based on the principles of surveying and geometrical optics. Thus, it is expected that the new model avoids the degradation of data quality due to recurrent changes, while minimizing disability in determining the mirror attitude with robustness, which was previously identified.

2. Problem definition

The purpose of this study is the determination of hidden point coordinates, whose observation is impossible by the presence of obstruction in direct line of sight between the observer and the point of interest. The proposed solution consist by including a new element in the system, a plane mirror which will allow the visualization of the hidden point indirectly, by the virtual image observation (Figure 1). Thus, if the position and attitude of the mirror is determined and if it is possible to perform measurements in point considering the mirror reflection characteristics, then the system geometry is determined and the position of the hidden point is performed.

Although the plane mirror used in the reflection the simplest is one of the existing optical elements, however, it is necessary to identify and minimize the possible systematic errors of the solution.

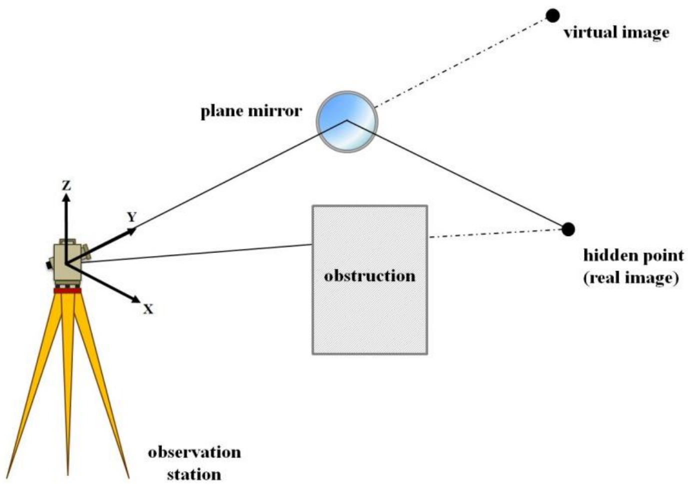

Figure 1: Proposed Solution. Source: Modified from Gonçalves (2009).

3. Mathematical Modeling

The proposed mathematical model considers that all solution elements are determined in a unique reference system avoiding transformations between systems over the calculation procedure, which can result in degradation of the quality of the initial observations. The proposed reference system is local, with origin arbitrated in the observation station (E_i), Y axis in the direction of the plane mirror, the Z axis coincident with the vertical line of the observer and the X axis completing a right-handed system (Figure 1). The mathematical development has been considered only versors in solution, in others words, vectors that have the same sense and direction of the original vector, but with unit module.

3.1. Mirror surface modeling

Initially, it is necessary to model the mirror surface where the interest object is reflected. In this case, the mirror surface is treated as a plan π, whose general equation is given by:

$$\pi: Ax + By + Cz + D = 0 \quad (1)$$

The solution of the unknowns, the terms, A, B, C and D of the general equation of the plane is based on using of three-dimensional coordinates of the marks observed on the mirror surface by irradiation. The observation of these points allows the determination of the plan by the method of least squares, using the adjustment of observations by the combined stochastic model. In solution, the term D is fixed as 1, in a way that the adjustment converges for a single plane equation among the infinite possibilities, avoiding the trivial solution which results in the fact that all terms are zero and then becoming the Equation 1 valid.

With the determination of Equation 1 it is possible to express the normal vector to the $\vec{n}$ plane π $(\vec{n})$, formed in accordance with the terms of the general equation of the plan:

$$\vec{n} = \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix} = \begin{bmatrix} A \\ B \\ C \end{bmatrix} \quad (2)$$

Where

n_x, n_y, n_z are the components of normal vector of the mirror surface.

So, the plan π is determined in the same coordinate system of the observation station E_i and then, begins the process of defining the coordinates of the point of interest, with the realization of virtual image observations of the hidden point P. For this, two lines will be determined in the solution this problem is that the incident ray and that representing the reflected rays.

3.2. Determination of straight line which represents the incident ray

The straight line r_i which represents the incident ray from observation station has parametric equation of the form:

$$r_i: \begin{cases} x = x_{Ei} + r_{i_x} d_i \\ y = y_{Ei} + r_{i_y} d_i \\ z = z_{Ei} + r_{i_y} d_i \end{cases} \quad (3)$$

Where

x_{Ei}, y_{Ei}, z_{Ei} are the coordinates of observation station E_i;

$r_{i_x}, r_{i_y}, r_{i_z}$ are the components of director vector $\vec{r_i}$, determined according to the horizontal α_i and zenith Z_i angles, which defined the direction of incident ray; and d_i is the slope distance, or else, the parameter adopted in the straight line equations.

Thus, an important parameter of this solution, the director vector of straight line which represents the incident ray, is extract from Equation 3, given by:

$$\vec{r}_i = \begin{bmatrix} r_{i_x} \\ r_{i_y} \\ r_{i_z} \end{bmatrix} \tag{4}$$

With

$\vec{r}_i$ is the director vector of straight line equation which represents the incident ray from observation station E_i.

3.3. Intersection of incident rays on the mirror surface

The intersection of incident ray on the mirror surface can be treated as an intersection of the line that materializes the incident ray and the plane that represents the mirror surface. The analytical solution is given by determining the three-dimensional coordinates of a point (x_i^I, y_i^I, z_i^I) which simultaneously belong to the straight line r_i (Equation 3) and the plane π (Equation 1), for each of the objects observed indirectly through the mirror.

3.4. Determination of incident angle

In addition to the position where the incident ray intersects the plane π, it is necessary to set the angle of incidence of the ray in the mirror. The angle of incidence (θ) is the angle between the direction of the incident ray and the normal vector to the mirror plane, as shown in Figure 2.

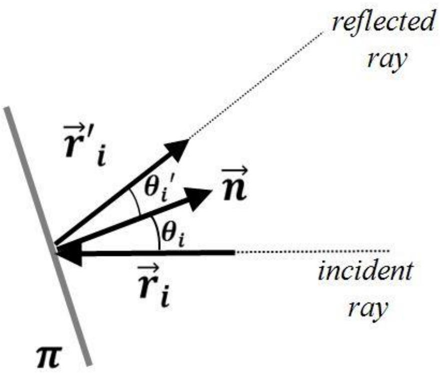

Figure 2: Angles of incidence θ and of reflection θ'. Source: Adapted from Nussenzveig (1998).

The angle of incidence θ is formed between the opposite of director vector of the incident ray (-$\vec{r}_i$) from the observation station and the normal vector of mirror plane $(\vec{n})$. Then, the following relationship is true:

$$cos(\theta) = \frac{-\vec{r}_i \cdot \vec{n}}{|-\vec{r}_i||\vec{n}|} \tag{5}$$

3.5. Determination of straight line which represents the reflected ray

Next step consists in determining the straight line which represents the reflected ray. For this, we considered the configuration showed in Figure 2, such that the direction of reflected ray is defined according the law of reflection (Nussenzveig, 1998). So, it is possible conditioned that:

(1) The director vector of incident ray ($\vec{r}_i$), the normal vector of the mirror surface $(\vec{n})$ and the director vector of reflected ray $(\vec{r}'_i)$ are coplanar, i.e., they belong to the same plane of incidence.

(2) The angle of reflection θ' is formed between the normal vector of the mirror surface $(\vec{n})$ and the director vector of reflected ray $(\vec{r}'_i)$;

(3) The angle between the opposite of the director vector of incident ray (-$\vec{r}_i$) and the director vector of reflected ray $(\vec{r}'_i)$ is given by the sum of incidence θ and reflection θ' angles, which are the same by law of reflection;

These conditions form a nonlinear system whose solution correspond to the components of the vector director of the reflected ray, given by $(r'_{i\,x}, r'_{i\,y}, r'_{i\,z})$. Finally, the line of the reflected ray

(r_i') is defined by containing as the origin the intersection point of the incident ray on the mirror surface and takes the direction of the reflected ray, such as:

$$r_i': \begin{cases} x = x_i^I + r'_{i_x} d \\ y = y_i^I + r'_{i_y} d \\ z = z_i^I + r'_{i_z} d \end{cases} \quad (6)$$

3.6. Determination of coordinates of hidden point

Information that must be collected in the field, through the mirror reflection is the measured distance between the observation station and the hidden point. For this, corrections must be applied in order to minimize the atmospheric effects. Then, in solution, the total measured distance (d_t) is expressed as a sum of two parts:

(1) The distance covered by the incident ray (d_{inc}), equal to the distance between the station and the mirror; and

(2) The distance covered by the reflected ray (d_{ref}), equal to the distance between the mirror and the hidden point.

At last, coordinates of the hidden point (x_p, y_p, z_p) can be determined by using the distance covered by the reflected ray as a parameter:

$$\begin{cases} x_p = x_i^I + r'_{i_x} d_{ref} \\ y_p = y_i^I + r'_{i_y} d_{ref} \\ z_p = z_i^I + r'_{i_z} d_{ref} \end{cases} \quad (7)$$

4. Validation of the proposed mathematical model

To validate the proposed mathematical model in section 3, several controlled tests were performed in which were used the following equipment and accessories:

- Simulated hidden points by two mini prism positioned side by side (Figure 3a), spaced about 17 cm with similar heights;
- A plane mirror with a diameter of 22 cm with high reflectivity and frontal mirroring property, constructed with nickel coating, that minimizes the effects of refraction of the light rays (Gonçalves, 2009); and

– A total station TS15 Leica Geosystems, whose angular nominal precision is 1 " and nominal linear accuracy is ± 1 mm + 1.5 ppm or ± 2 mm + 2 ppm to measures with reflectors prisms and reflectorless measures respectively, for distances of less than fifty meters.

In performing the tests, the total station was placed in a configuration that allows the viewing of marks recorded on the surface of the mirror (Figure 3b) and the hidden points directly and indirectly (Figure 3c) by the mirror reflection. The environmental conditions were also monitored and used to correct the distances obtained electronically.

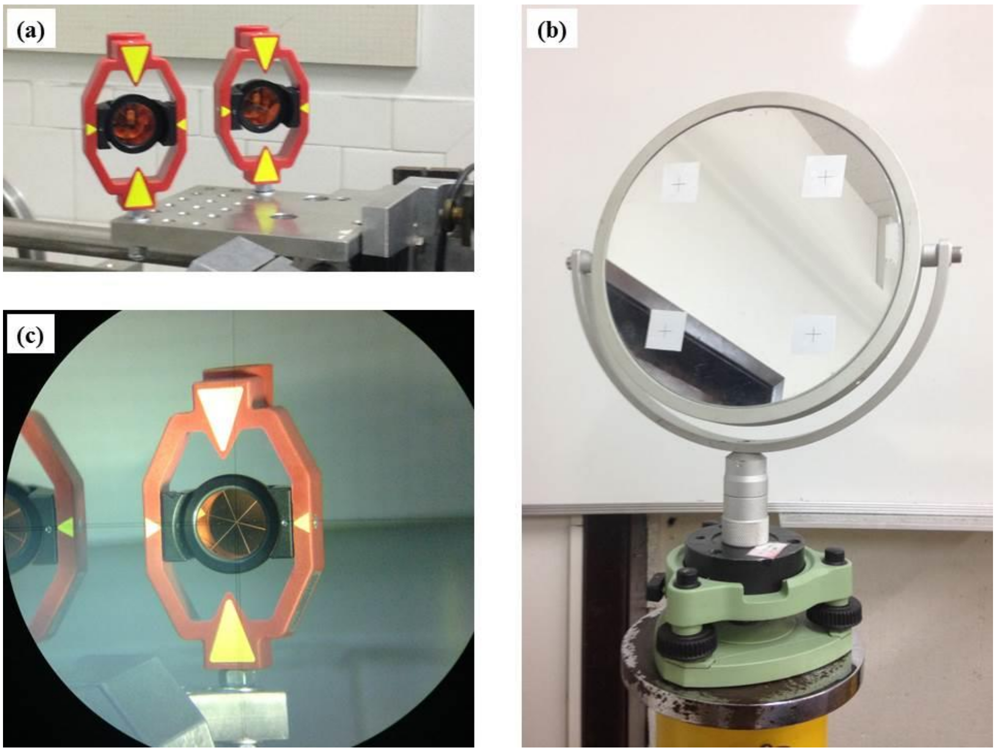

Figure 3: (a) simulated hidden points, (b) plane mirror and (c) view of the indirect observation through the telescope of the total station.

The direct and indirect observation of the hidden points was a condition imposed for model validation only. After testing, it was verified that:

a) Submillimetric precision in relative position of points observed indirectly; and

b) Systematic errors in the absolute position of points observed indirectly, with differences in the coordinates of centimetric order. These differences are directly proportional to the distance between the observed point and the mirror and perpendicular to the line of sight which is evidence of some angular effect.

To identify the source of systematic error, we considered the observed coordinates directly, and from these, in a reverse process, were determined which would be the ideal reflected rays from the mirror.

Figure 4 shows the incident vectors $\vec{r}(\vec{r})$ and reflected ($\vec{r}'$) to the two points observed indirectly, and the normal vector $(\vec{n})$ of the mirror surface. The red vector indicates the ideal direction that the reflected vector ($\vec{r}'$) should take according to the position determined directly.

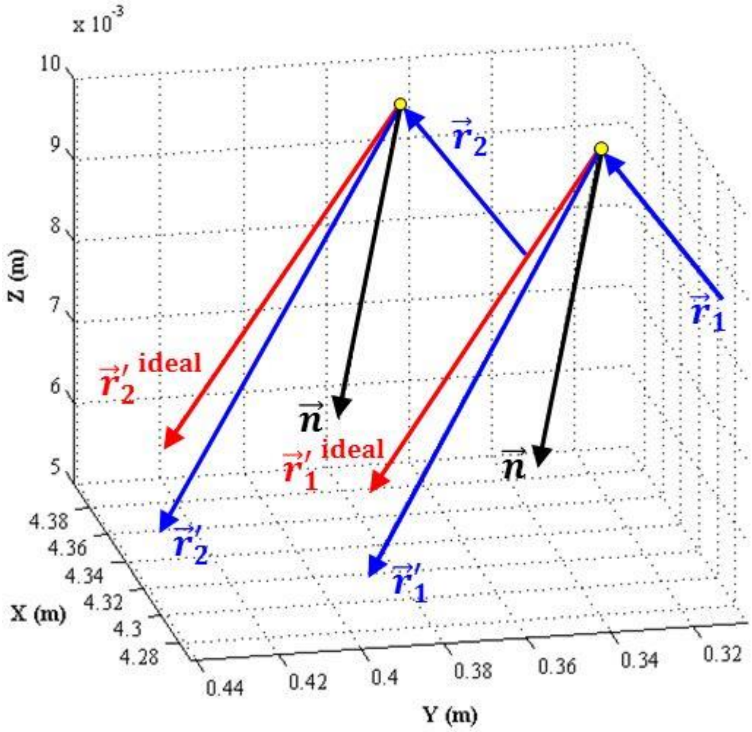

Figure 4: incident and reflect vector for two points observed on validation test.

In Figure 4 it is possible to identify the reflected ray calculated by the model is different compared to the ideal reflected ray, obtained by the reference coordinates of the points observed directly. It was also found that this effect is similar for all points measured at each mirror attitude. Then, when the mirror position is changed, therefore the distortion in the point coordinates is changed. This fact supports the assumption that mirror the inherent characteristics are conditions for systematic effects observed. In this reasoning, two hypotheses were formulated and tested for the possible cause of distortion of rays:

- Hypothesis I: There are constructive problems of the plane mirror, resulting in no homogeneity of its reflecting surface, so that the mirror distorts the rays reflected according to the incidence angle;

- Hypothesis II: The modeling of the surface plane is deficient and not representative of reality, which would cause a systematic error to all observations, since the normal vector is critical parameter for determining the reflected rays.

In order to investigate the flatness of the mirror and thereby verify the first hypothesis, mirror surface profiles were developed using an electronic level based on body suspended which with associated precision is 0.2 arc seconds. The feature of this level is to provide the relative slope between the sensors at each end of its contact surface. The profiles were generated for horizontal and vertical directions of the central mirror region (Figure 5). In the profiles indicate that the differences in relation to a reference plane, not exceeding 0.05 mm of magnitude which is a negligible value when compared to the precision of the observations of a higher order.

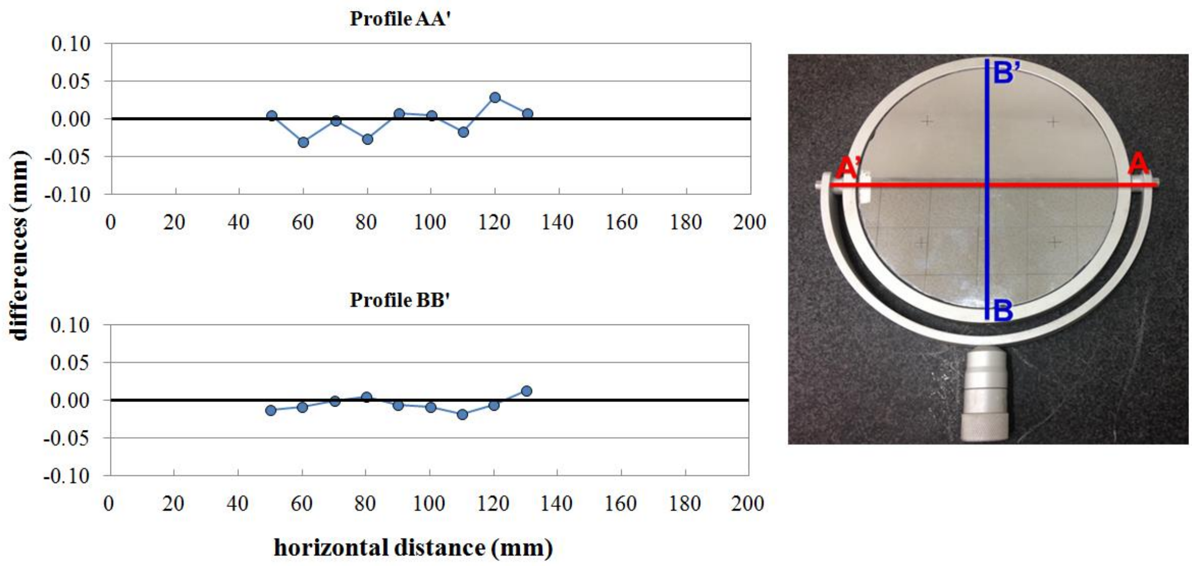

Figure 5: Central mirror region profiles.

The test results demonstrated that objects reflected by this mirror are shown in full scale, besides maintaining their shapes without deformation which is characteristic of a plane mirror. In a reverse process of calculation, it was established that the angles of incidence and reflection are equal. Therefore, the adoption plan model for the reflective surface is consistent, and that constructive irregularities were not found then the hypothesis I was rejected.

Regarding the second hypothesis, analyzing the waste from mirror marks coordinates using the adjustment from the plane model is values averaged 1 mm, which is consistent with the quality of the observations. However, the spread of the errors of the marks taken as a reference for determining the plans influences the quality of determining the normal vector used in the proposed model. Pinto (2013) states that to obtain coordinates of points observed indirectly with millimeter accuracy, it is necessary that attitude plan, represented by its normal vector has been determined with five seconds of arc or better. Then, hypothesis II was accepted.

In initial testing, quality data was not enough to achieve the rigidity in determining plan attitude. The accuracy of the values found is the order of minutes which justifies those found systematic errors, since the normal vector does not represent the reality concisely. The ideal solution would be to improve the determination of the coordinates of the marks of a micrometer precision, which makes it not feasible to adopt the proposed solution.

Alternatively, the information of a homologous point observed directly and indirectly was inserted into the mathematical model. This point does not need to be close to the hidden point, but its observation has been treated as a correction factor in plan modeling. In addition, the distances between the marks on the mirror materialized were fixed as they were previously calibrated by interferometry. The answer to this approach was positive, which minimized the systematic error identified.

5. Results

The adjustments made in the proposed mathematical model with the inclusion of new observations, we carried out a new test to check the consistency of the solution. A new test configuration with the same materials was carried out. A homologous point was inserted into the model which was possible observe it directly and indirectly, the coordinates of which have been integrated into the adjustment. In this test, in addition to this new point, four pairs of points positioned at different distances from the mirror as simulated hidden points, as shown in Figure 6.

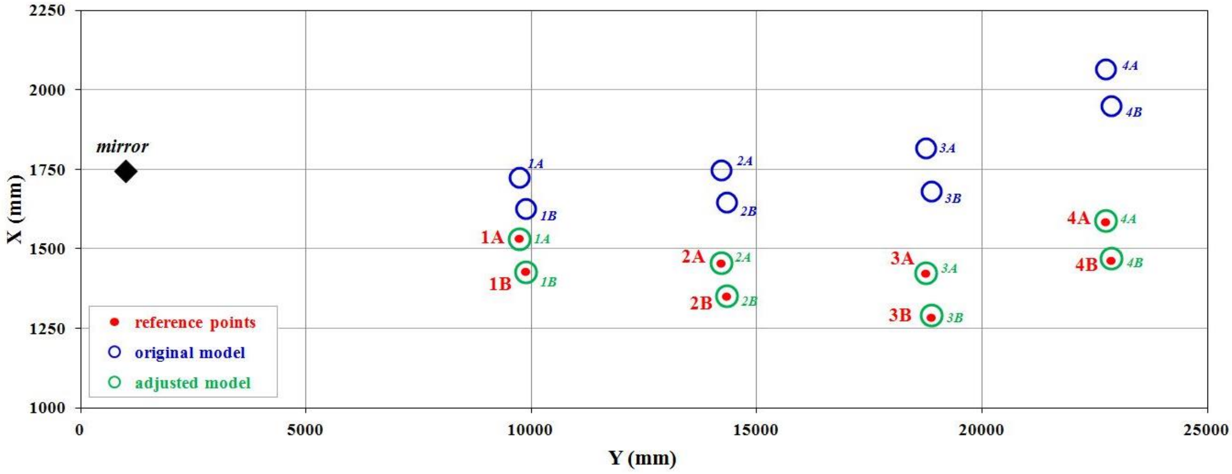

Figure 6: New test results with adjusted model.

In Figure 6, the results from the original model are displayed in blue while the results of the adjusted model display in green and the red points correspond to the actual position of the points that have been calculated directly and adopted as reference values. The first finding as mentioned earlier is that the coordinates of points calculated by the original model has errors in their absolute positions of greater magnitude than the adjusted model. Such errors are of order centimeter and are directly proportional to the distance between the object and the mirror as seen in Figure 6, since the effect of the angular uncertainty is amplified as the point departs from the mirror position. In the second group from the adjusted model, the results showed significant improvements in presenting to the millimeter order in absolute position. Table 1 displays the values found.

Table 1: Absolute position comparison of the simulated hidden points.

Point	Distance from mirror (m)	Original Model (mm)			Reference points (mm)			Adjusted Model (mm)			Differences (mm)			
		X	Y	Z	X	Y	Z	X	Y	Z	ΔX	ΔY	ΔZ	total
1A	8.7	1725.6	9730.1	-92.4	1530.7	9728.6	-48.7	1530.7	9728.6	-48.7	0.0	0.0	0.0	0.0
1B	8.9	1627.1	9870.4	-94.3	1429.1	9866.7	-49.0	1429.3	9866.9	-48.7	-0.2	-0.2	-0.3	0.4
2A	13.2	1747.1	14199.4	-112.5	1453.3	14197.3	-46.6	1456.1	14197.4	-46.5	-2.8	-0.1	-0.1	2.8
2B	13.4	1646.5	14339.0	-115.8	1349.6	14334.7	-48.4	1352.4	14334.2	-48.2	-2.8	0.5	-0.2	2.9
3A	17.7	1817.5	18755.1	-260.2	1421.7	18754.1	-172.2	1426.6	18754.2	-172.4	-4.9	-0.1	0.2	4.9
3B	17.9	1683.2	18868.1	-262.2	1286.5	18864.2	-172.4	1291.1	18864.3	-172.7	-4.6	-0.1	0.3	4.6
4A	21.7	2067.2	22739.5	-275.5	1583.1	22743.1	-169.8	1590.4	22743.3	-170.8	-7.2	-0.2	1.0	7.3
4B	21.8	1949.9	22865.7	-278.7	1463.0	22866.7	-171.4	1470.1	22867.6	-171.5	-7.0	-0.9	0.1	7.1

The point 1A was adopted as corrections to be as close to the mirror so its position was fixed in the solution, resulting in the absence of errors in its indirect determination by the adjusted model. Points that are closest to the mirror and simultaneously away from the hidden point should be used as references in surveys. The other differences presented millimeter tenths of values for the components Y and Z. The X component absorbed large portion of the existing error in the solution. After correcting model angular problems, the main source of error in the model are the distances measured, positioned in the X direction according to the composition of the reference system adopted. Note also that the coordinate differences observed are greater when the point moves away from the point 1A. In addition, similar distances contain errors in the absolute position also similar and with the same magnitude hence a residue as result of distance between the mirror and object.

In the relative position, the variations determined by the proposed model are presented in Table 2.

Table 2: Relative position comparison of the simulated hidden points.

Line	Adjusted Model (mm)				Reference points (mm)				Differences (mm)			
	δx	δy	δz	total	δx	δy	δz	total	Δ δx	Δ δy	Δ δz	total
1A-1B	-101.4	138.3	0.0	171.5	-101.6	138.1	-0.3	171.4	0.2	0.2	0.3	0.1
2A-2B	-103.7	136.8	-1.7	171.7	-103.7	137.4	-1.8	172.2	0.0	-0.6	0.1	-0.5
3A-3B	-135.5	110.1	-0.3	174.6	-135.2	110.1	-0.2	174.4	-0.3	0.0	-0.1	0.2
4A-4B	-120.3	124.3	-0.7	173.0	-120.1	123.6	-1.6	172.3	-0.2	0.7	0.9	0.6

Differences in submillimeter order indicated in Table 2 show that the attitude and dimensions between the points calculated by the proposed model are preserved, without deformation or scale changes. However, the relative position of submillimeter order is ensured only between points having similar distances, which in turn imply the minimization of the error residues shown in Table 1.

The relative position of submillimeter order is ensured only between points having similar distances that involve similar magnitude errors. This characteristic has not been observed when working with significantly different distances.

6. Conclusions

Over the tests performed it was possible to verify the feasibility of the proposed model for determining three-dimensional coordinates of points hidden when the straight line is blocked. We emphasize that the focus of this study were industrial scenarios or areas of difficult access, which restrict the use of other existing techniques, which explains the importance of this approach. Other application for the proposed model is modeling the shape and volume of objects, which avoids the occlusion information, observing the object as a whole.

Regarding the precision achieved, it was found that the absolute positioning coordinates of the points were estimated accuracies of millimeter order, however, this quality is only achieved by inserting a reference point in the adjustment model, which is observed directly and indirectly. In the relative positioning, the differences between coordinates are superior to the millimeter, this quality that allows controlling and monitoring of temporal variation between points rigorously.

Based on results and model validation, study prospects were identified in this work. First, it is necessary to research new methods for the mirror plane modeling and, if possible, propose ways to define their attitude by indirect measures that is external to the mirror. This eliminates the observations of surface marks, which have a poor geometry, due to its proximity and number not conducive to adjustment. Secondly, we need to investigate the quality of measurements of reflected distances and its effects. It is recommended to apply the solution with a superabundance of observations, for better estimation of observed points, considering for example two distinct observation points of the same virtual image of the hidden point.

Finally, this study showed a new approach that makes it possible to determine relative positions with quality submillimeter order, regardless of use point for correction or not. Thus, the mathematical model presented is an alternative to survey problems in unfavorable conditions to the application of conventional topographic techniques.

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