PRODUÇÃO

Monitoring process mean with a new EWMA control chart

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Abstract

In practice, sometimes the process data did not come from a known population distribution. So the commonly used Shewhart variables control charts are not suitable since their performance could not be properly evaluated. In this paper, we propose a new EWMA Control Chart based on a simple statistic to monitor the small mean shifts in the process with non-normal or unknown distributions. The sampling properties of the new monitoring statistic are explored and the average run lengths of the proposed chart are examined. Furthermore, an Arcsine EWMA Chart is proposed since the average run lengths of the Arcsine EWMA Chart are more reasonable than those of the new EWMA Chart. The Arcsine EWMA Chart is recommended if we are concerned with the proper values of the average run length.

1. Introduction

Control charts are commonly used tools in quality improvement, such as the X-bar and R charts, EWMA and CUSUM charts for variables data; p and C charts for attributes data. However in order to properly construct the chart, we need to know the sampling properties of the monitoring statistic, study the chart's behavior and compare its performance with other existing charts. In most cases, normality is assumed for variables data, some with distributions of known form. Hence the question is – what if we had no knowledge of the underlying distribution or the known distribution doesn't help us derive the necessary sampling properties? Using a nonparametric approach seems to be a good choice. Only limited researches were done in this area, like Ferrell (1953); Bakir and Reynolds Junior (1979); Amin, Reynolds and Baker (1995); Chakraborti, Van der Lann and Van de Wiel (2001); Altukife (2003a, 2003b); Bakir (2004, 2006); Chakraborti and Eryilmaz (2007); Chakraborti and

Graham (2007); Chakraborti and Van der Wiel (2008) and Das and Bhattacharya (2008).

A major drawback of the Shewhart variables charts is that they are not effective in detecting small shifts, so EWMA and/or CUSUM charts are used to achieve this purpose. In this paper, we propose a new chart for variables data to monitor small shifts of the process mean.

2. The new EWMA chart

A random sample of size n, X_1, X_2, ..., X_n is taken from a process and let μ be the process mean. Define

$$Y_j = X_j - \mu, j = 1, 2, ..., n \tag{1}$$

and

$$I_j = \begin{cases} 1, & if\ Y_j > 0, \\ 0, & otherwise. \end{cases} \tag{2}$$

218 | PRODUÇÃO

Yang, S. F. et al.
Monitoring process ... new EWMA control chart. Produção, v. 21, n. 2, p. 217-222, abr./jun. 2011

Let S be the total number of $Y_j > 0$, then

$$S = \sum_{j=1}^{n} I_j \qquad (3)$$

would follow a Binomial distribution with parameters (n, p) for an in-control process, where $p = P(Y_j > 0)$.

Note that, although the resulting chart is a np chart, this is a new chart in that the binomial variable is not the count of nonconforming units in the sample but rather the number of X_j values in a sample that are above the in-control process mean.

To monitor the small shifts of the process mean quickly and effectively, we will apply a New EWMA chart. Hence we define the EWMA statistic as:

$$EWMA_{S_i} = \lambda S_i + (1 - \lambda) EWMA_{S_{i-1}} \qquad 0 < \lambda \le 1 \qquad (4)$$

where S_i represents the value of S (according to (3)) in the ith sample (or time). Adopt the starting value, $EWMA_{S_0}$, as the mean of S; that is $EWMA_{S_0} = np$, the mean and variance are then $E(EWMA_{S_i}) = np$ and

$$Var(EWMA_{S_i}) = \frac{\lambda \left[1 - (1-\lambda)^{2i} \right]}{2 - \lambda} \left[np(1-p) \right].$$ If time is infinite then $Var(EWMA_{S_i}) = \frac{\lambda}{2 - \lambda} \left[np(1-p) \right]$.

The control limits of the EWMA chart are often based on the asymptotic standard deviation of the control statistic. Hence we could construct the New EWMA chart as follows:

$$UCL_{EWMA_S} = np + k\sqrt{\frac{\lambda}{2-\lambda} \left[np(1-p) \right]}$$

$$CL_{EWMA_S} = np \qquad (5)$$

$$LCL_{EWMA_S} = np - k\sqrt{\frac{\lambda}{2-\lambda} \left[np(1-p) \right]}$$

and plot $EWMA_{S_i}$.

The two parameters, k and λ, are chosen to satisfy certain required average run length (ARL).

3. In-control average run lengths of the new EWMA chart

We will use the ARL as a measure of performance of the proposed chart. Following Lucas and Saccucci (1990), the ARLs of the New EWMA chart are evaluated by Markov chain approach. The procedure to calculate ARL is described below.

Step 1. Divide the interval (LCL, UCL) into $(N - 1)$ subintervals each with an equal width.

Step 2. Denote the 1st interval (or State) as $(-\infty, LCL]$; the $(N + 1)^{th}$ interval (State) as (UCL, ∞); the $(N - 1)$ subintervals of the interval (LCL, UCL) are denoted as State 2, State 3, ..., State N.

Step 3. Since State 1 and $(N + 1)$ are the action regions of the New EWMA chart, it is considered the absorption states for the Markov chain. States 2, 3, ..., and N are transient states.

Step 4. Let P_{ij} be the transition probability that the statistic $EWMA_{S_t}$ reaches State j at time t, given that $EWMA_{S_{t-1}}$ was in State i at time $(t - 1)$, $i, j = 2,..., N$.

Step 5. Let $\mathbf{b}$ be a $(N - 1)$-vector of the initial probabilities that the process started in State 2, ..., N. We have in our study that

$\mathbf{b} = (b_2, b_3, ..., b_N)'$ with $b_{N+1} = 1$ and $b_i = 0$ for $i \ne \frac{N+1}{2}$.

Step 6. Let $\mathbf{P} = ||P_{ij}||$ be a $(N - 1) \times (N - 1)$ transition probability matrix, $i, j = 2,..., N$. The ARL is thus computed by

$ARL = \mathbf{b}'(\mathbf{I} - \mathbf{P})^{-1}\mathbf{1}$,

where $\mathbf{1}' = (1, 1, 1, ..., 1)$ is a $(N - 1)$-vector with element 1 in State 2, 3,..., N.

Step 7. To calculate the in-control and out-of-control ARLs, let p be in-control proportion, then $ARL = ARL_0$, the in-control ARL. If p is the out-of-control proportion, then $ARL = ARL_1$, the out-of-control ARL.

Table 1 and 2 list the ARLs under various combinations of (n, p), for n ranging from 9 to 20 and for a number of values of p between 0.25 and 0.75, by adopting the respective combination ($\lambda = 0.2$, $k = 2.84$) and ($\lambda = 0.05$, $k = 2.84$) in the New EWMA chart for the in-control process, i.e. ARL_0.

4. Out-of-control average run lengths of the new EWMA chart

The ARLs of the New EWMA chart are function of (n, k, λ). Adopting in-control process proportion $p = 0.613$, $n = 10$, $ARL_0 = 374.0$ with $\lambda = 0.2$ and $k = 2.84$, the ARLs of the EWMA chart for n ranging from 9 to 20 and the values of p between 0.25 and 0.95 when the process is out-of-control, i.e. ARL_1, are listed in Table 3.

5. Example

We will use an example from Montgomery (2009) to illustrate the new chart.

The fill volume of soft-drink beverage bottles is an important quality characteristic. The volume is measured (approximately) by placing a gauge over the crown and comparing the height of the liquid in the neck of the bottle against a coded scale. On this scale, a reading of zero corresponds to the correct fill height. Fifteen samples of size $n = 10$ have been analyzed, and the fill heights are shown in Table 4.

Yang, S. F. et al.
Monitoring process ... new EWMA control chart. Produção, v. 21, n. 2, p. 217-222, abr./jun. 2011

PRODUÇÃO | 219

Table 1. The ARL_0 of the new EWMA control chart ($\lambda = 0.2$, k = 2.84).

n	p										
	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75
9	137.4	191.4	238.7	290.8	312.6	356.1	350.2	381.1	374.9	375.1	367.9
10	171.6	217.4	280.4	313.6	340.0	363.4	367.3	373.4	387.9	364.3	373.8
11	191.1	245.0	295.5	328.1	356.7	383.2	374.9	371.3	367.0	359.8	360.2
12	216.0	275.0	330.2	337.6	371.2	371.0	382.1	363.5	386.0	367.1	359.4
13	235.4	295.9	337.5	344.9	364.6	367.8	370.7	360.2	374.1	371.4	362.0
14	274.4	316.8	332.6	352.4	366.1	356.3	369.7	362.2	356.4	375.4	376.7
15	281.7	321.0	348.7	381.5	370.1	378.0	372.3	389.3	365.8	363.7	365.9
16	292.5	334.2	342.8	356.1	363.1	371.1	364.1	360.2	354.9	367.4	360.6
17	309.3	341.8	358.3	360.9	359.6	361.8	360.3	363.4	367.1	364.6	367.4
18	304.0	342.0	356.8	366.2	362.2	368.7	362.4	367.6	361.9	358.5	341.6
19	329.4	354.7	358.9	357.3	361.3	355.6	361.7	358.2	362.3	367.2	364.7
20	328.2	362.5	367.1	355.2	374.7	373.1	374.8	355.7	369.4	372.3	354.2

Table 2. The ARL_0 of the EWMA control chart ($\lambda = 0.05$, k = 2.84).

n	p										
	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75
9	344.9	516.4	678.0	750.8	829.3	854.4	886.6	886.4	925.5	881.7	891.3
10	440.6	626.0	734.8	799.7	834.3	877.2	869.0	880.9	897.6	920.7	889.9
11	534.0	687.2	796.4	847.7	869.7	871.7	889.7	901.9	901.0	887.8	898.0
12	593.6	766.6	814.7	866.3	879.3	879.4	890.4	899.0	891.5	920.9	873.2
13	664.8	773.8	838.5	863.8	883.0	912.3	890.0	882.9	886.6	887.7	876.9
14	716.1	836.2	855.3	879.6	878.6	867.4	882.2	891.2	889.9	914.9	900.1
15	763.9	832.6	861.7	834.7	892.0	899.1	894.1	840.8	882.9	889.0	903.8
16	779.4	862.7	874.3	864.7	866.7	901.2	867.8	869.3	889.2	902.0	877.4
17	811.7	889.9	913.6	897.4	878.3	900.3	879.0	900.4	923.3	920.4	895.4
18	844.6	845.3	880.3	887.0	899.7	882.7	900.1	888.6	886.0	865.5	905.3
19	853.8	880.5	870.2	889.7	880.6	897.2	880.8	890.8	874.3	895.1	900.4
20	874.7	870.9	866.9	885.0	901.3	892.3	901.2	884.7	868.2	879.2	908.2

Table 3. The ARL_1 of the EWMA chart ($\lambda = 0.2$, k = 2.84).

n	p									
	0.25	0.35	0.45	0.55	0.613	0.65	0.75	0.85	0.95
9	3.1	4.6	9.6	53.6	385.7	172.0	14.7	5.3	3.3
10	2.9	4.3	8.6	47.7	374.0	154.8	14.3	4.9	3.2
11	2.7	4.0	8.0	43.1	372.9	143.1	13.1	4.6	2.8
12	2.6	3.8	7.5	40.9	391.2	142.2	11.6	4.4	2.8
13	2.5	3.7	7.1	38.0	370.8	122.8	10.6	4.1	2.4
14	2.4	3.5	6.7	35.6	368.6	114.4	9.7	3.9	2.4
15	2.4	3.4	6.4	33.2	373.9	111.4	9.6	3.8	2.3
16	2.3	3.3	6.1	31.6	372.0	102.4	8.8	3.4	2.3
17	2.2	3.1	5.8	29.9	362.9	93.1	8.6	3.4	2.2
18	2.2	3.0	5.6	28.1	363.4	90.3	8.0	3.3	2.2
19	2.1	2.9	5.4	26.7	362.7	86.2	7.9	3.3	2.1
20	2.1	2.8	5.2	25.9	368.7	81.3	7.5	3.1	2.1

220 | PRODUÇÃO

Yang, S. F. et al.
Monitoring process ... new EWMA control chart. Produção, v. 21, n. 2, p. 217-222, abr./jun. 2011

Table 4. The data and the calculated values of statistics.

m	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	$\bar{X}$	S	$EWMA_S$
1	2.5	0.5	2	−1	1	−1	0.5	1.5	0.5	−1.5	0.5	7	6.307
2	0	0	0.5	1	1.5	1	−1	1	1.5	−1	0.45	8	7.200
3	1.5	1	1	−1	0	−1.5	−1	−1	1	−1	−0.1	5	7.400
4	0	0.5	−2	0	−1	1.5	−1.5	0	−2	−1.5	−0.6	5	5.000
5	0	0	0	−0.5	0.5	1	−0.5	−0.5	0	0	0	7	5.400
6	1	−0.5	0	0	0	0.5	−1	1	−2	1	0	7	7.000
7	1	−1	−1	−1	0	1.5	0	1	0	0	0.05	7	7.000
8	0	−1.5	−0.5	1.5	0	0	0	−1	0.5	−0.5	−0.15	6	6.800
9	−2	−1.5	1.5	1.5	0	0	0.5	1	0	1	0.2	8	6.400
10	−0.5	3.5	0	−1	−1.5	−1.5	−1	−1	1	0.5	−0.15	4	7.200
11	0	1.5	0	0	2	−1.5	0.5	−0.5	2	−1	0.3	7	4.600
12	0	−2	−0.5	0	−0.5	2	1.5	0	0.5	−1	0	6	6.800
13	−1	−0.5	−0.5	−1	0	0.5	0.5	−1.5	−1	−1	−0.55	3	5.400
14	0.5	1	−1	−0.5	−2	−1	−1.5	0	1.5	1.5	−0.15	5	3.400
15	1	0	1.5	1.5	1	−1	0	1	−2	−1.5	0.15	7	5.400
$\bar{\bar{X}}$	-	-	-	-	-	-	-	-	-	-	−0.0033	-	-

Here, sample size = 10, number of samples = 15, $\bar{\bar{X}}$ = −0.0033, $\bar{p}$ = 0.613, adopting ARL_0 = 374.0 with λ = 0.2 and k = 2.84 based on Table 1. S = Sum of positive differences $(X_j + 0.0033)$, i = 1, 2, ..., 15.

The New EWMA chart is plotted in Figure 1.

The chart shows that the process seems to be in-control.

The $\bar{X}$ chart (MONTGOMERY, 2009) shown in Figure 2 gave the same conclusion.

6. The ARL of the $EWMA_Y$ chart

Table 1 shows that the $ARL_0 s$ of EWMA chart with (λ = 0.2, k = 2.84) when the in-control process proportion p is between 0.25 and 0.75, but ARL_0 values vary irregularly. The reason for this is that the binomial distribution is asymmetric when $p \neq 0.5$ for small and moderate values of n. To rectify this problem, we would apply the arcsine transformation (MOSTELLER, YOUTZ, 1961). That is, let $Y = sin^{-1}\left(\sqrt{\dfrac{S}{n}}\right)$ and then the distribution of Y would be approximately normal with mean $sin^{-1}\left(\sqrt{p}\right)$ and variance $1/(4n)$. A revised $EWMA_Y$ chart, the Arcsine EWMA chart is thus constructed as follows:

$$EWMA_{Y_i} = \lambda Y_i + (1-\lambda)EWMA_{Y_{i-1}} \qquad 0 < \lambda \leq 1 \qquad (6)$$

Let the mean of Y be the starting value, $EWMA_{Y_0}$; i.e. $EWMA_{Y_0} = sin^{-1}\sqrt{p}$ for in-control process. The mean and variance of $EWMA_{Y_i}$ are $E\left(EWMA_{Y_i}\right) = sin^{-1}\sqrt{p}$ and

$$Var\left(EWMA_{Y_i}\right) = \frac{\lambda\left[1-(1-\lambda)^{2i}\right]}{2-\lambda}(1/4n)$$ If time gets sufficiently large then $Var\left(EWMA_{Y_i}\right) = \dfrac{\lambda}{2-\lambda}(1/4n)$.

The control limits and the center line of the Arcsine $EWMA_Y$ chart are:

$$UCL = sin^{-1}\left(\sqrt{p}\right) + k\sqrt{\frac{\lambda}{4n(2-\lambda)}}$$

$$CL = sin^{-1}\left(\sqrt{p}\right) \qquad\qquad (7)$$

$$LCL = sin^{-1}\left(\sqrt{p}\right) - k\sqrt{\frac{\lambda}{4n(2-\lambda)}}$$

and plot $EWMA_{Y_i}$.

Let us use the same example in section 5 and plot the Arcsine $EWMA_Y$ chart in Figure 3.

This chart also showed that the process is in-control, just as the $EWMA_S$ chart.

To evaluate the performance of this new chart, we calculated the chart's $ARLs$. Table 5 shows that the $ARL_0 s$ of the Arcsine EWMA chart with (λ = 0.2, k = 2.84) gave the same value 350 for all in-control values of process proportion p between 0.25 and 0.75.

Adopting in-control process proportion p = 0.613, ARL_0 = 350 with λ = 0.2 and k = 2.84, the $ARL_1 s$ of the $EWMA_Y$ chart for n from 9 to 20 and the values of p between 0.25 to 0.95 when the process is out-of-control are listed in Table 6.

Now the $ARL_0 s$ are all 350 but not those of the EWMA chart. However the values of $ARL_1 s$ are smaller for EWMA chart when the values of p are

Yang, S. F. et al.
Monitoring process ... new EWMA control chart. Produção, v. 21, n. 2, p. 217-222, abr./jun. 2011
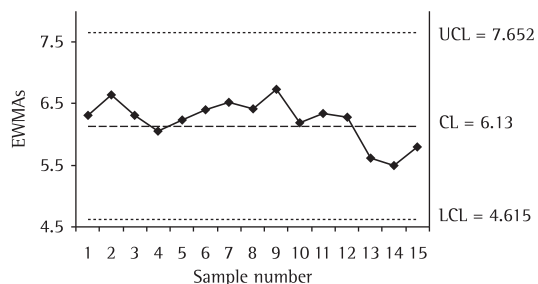
PRODUÇÃO | 221

Figure 1. The new $EWMA_S$ chart.

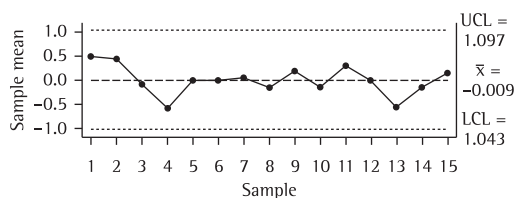Figure 2. The X-bar chart.

small, but it is reversed for large values of p. Overall we feel that the detection ability of the Arcsine $EWMA_\gamma$ chart is better than that of the EWMA chart. Hence, we would recommend the Arcsine $EWMA_\gamma$ chart if we were concerned with the proper ARL_0 values.
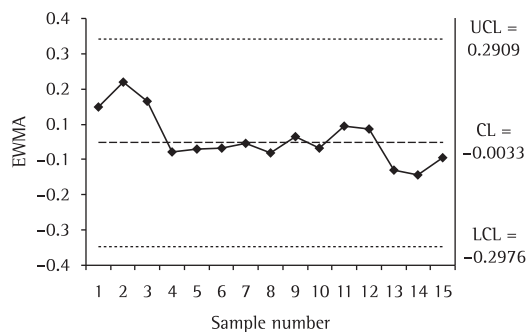

Figure 3. The Arcsine $EWMA_\gamma$ chart ($\lambda = 0.2$, $k = 2.84$).

Table 5. The ARL_0 of Arcsine $EWMA_\gamma$ chart ($\lambda = 0.2$, k = 2.84).

n	p										
	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75
9	350.0	350.0	350.0	350.0	350.0	350.0	350.0	350.0	350.0	350.0	350.0
10	350.0	350.0	350.0	350.0	350.0	350.0	350.0	350.0	350.0	350.0	350.0
11	350.0	350.0	350.0	350.0	350.0	350.0	350.0	350.0	350.0	350.0	350.0
12	350.0	350.0	350.0	350.0	350.0	350.0	350.0	350.0	350.0	350.0	350.0
13	350.0	350.0	350.0	350.0	350.0	350.0	350.0	350.0	350.0	350.0	350.0
14	350.0	350.0	350.0	350.0	350.0	350.0	350.0	350.0	350.0	350.0	350.0
15	350.0	350.0	350.0	350.0	350.0	350.0	350.0	350.0	350.0	350.0	350.0
16	350.0	350.0	350.0	350.0	350.0	350.0	350.0	350.0	350.0	350.0	350.0
17	350.0	350.0	350.0	350.0	350.0	350.0	350.0	350.0	350.0	350.0	350.0
18	350.0	350.0	350.0	350.0	350.0	350.0	350.0	350.0	350.0	350.0	350.0
19	350.0	350.0	350.0	350.0	350.0	350.0	350.0	350.0	350.0	350.0	350.0
20	350.0	350.0	350.0	350.0	350.0	350.0	350.0	350.0	350.0	350.0	350.0

Table 6. The ARL_1 of the Arcsine $EWMA_\gamma$ chart ($\lambda = 0.2$, k = 2.84).

n	p								
	0.25	0.35	0.45	0.55	0.613	0.65	0.75	0.85	0.95
9	6.0	10.5	25.9	132.0	350.0	153.8	15.1	3.4	1.5
10	5.6	9.6	23.4	123.0	350.0	146.8	13.3	3.1	1.4
11	5.2	8.9	21.3	115.1	350.0	140.3	11.8	2.8	1.4
12	4.9	8.2	19.6	108.1	350.0	134.3	10.6	2.6	1.4
13	4.7	7.3	17.0	101.8	350.0	128.8	9.6	2.4	1.3
14	4.4	7.3	17.0	96.2	350.0	123.6	8.7	2.3	1.3
15	4.2	6.9	15.9	91.1	350.0	118.8	8.0	2.1	1.2
16	4.1	6.6	15.0	86.5	350.0	114.3	7.4	2.0	1.2
17	3.9	6.3	14.1	82.4	350.0	106.1	6.4	1.9	1.1
18	3.8	6.0	13.4	78.5	350.0	102.4	6.0	1.8	1.1
19	3.6	5.8	12.8	75.1	350.0	99.0	6.0	1.8	1.1
20	3.5	5.6	12.2	71.8	350.0	96.1	5.4	1.7	1.1

222

PRODUÇÃO

Yang, S. F. et al.
Monitoring process ... new EWMA control chart. Produção, v. 21, n. 2, p. 217-222, abr./jun. 2011

7. Conclusion

A new chart, the EWMA chart, to monitor the process mean for variables data is proposed. It provides an alternative when the underlying distribution is unknown or non-normal. We have shown that it performs quite well. However, we would recommend a modified version, the Arcsine EWMA$_Y$ chart if we are concerned with attaining the proper ARL_0 values.

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Um novo gráfico de controle EWMA para monitoramento da média de processo

Resumo

Na prática a distribuição de probabilidade de muitas variáveis não é conhecida e sabe-se que não é proveniente de uma distribuição normal. Segue que o uso dos gráficos de controle Shewhart não é conveniente e daí há necessidade de procurar outros gráficos de controle alternativos. Neste artigo um novo gráfico de controle do tipo EWMA é proposto. Ele utiliza uma estatística não paramétrica para monitorar a média de um processo e observou-se que é ágil para detectar pequenos desvios da média. Propriedades amostrais da estatística são exploradas e um exemplo ilustra a nova proposta. Além disto, outro gráfico do tipo EWMA é apresentado utilizando como estatística o arco-seno da estatística não paramétrica. Os valores de ARL's deste gráfico apresentaram melhor desempenho do que a proposta anterior. Desta forma o gráfico Arco-Seno EWMA é recomendado se o critério do ARL for empregado.

Palavras-chave

Gráfico de controle EWMA. Monitoramento de média de processo. Distribuição binomial. Transformada arco-seno.