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Economic-statistical design of variable parameters non-central chi-square control chart

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Abstract

Production processes are monitored by control charts since their inception by Shewhart (1924). This surveillance is useful in improving the production process due to increased stabilization of the process, and consequently standardization of the output. Control charts keep track of a few key quality characteristics of the outcome of the production process. This is done by means of univariate or multivariate charts. Small improvements in control chart methodology can have significant economic impact in the production process. In this investigation, we propose the monitoring of a single variable by means of a variable parameter non-central chi-square control chart. The design of the chart is accomplished by means of optimizing a cost function. We use here a simulated annealing optimization tool, due to the difficulty of classical gradient based optimization techniques to handle the optimization of the cost function. The results show some of the drawbacks of using this model.

Keywords

Statistical process control. Economic design. Chi-square control chart. Variable parameters. Simulated annealing.

1. Introduction

Control chart, a largely known and used tool of Statistics Process Control due to its operational simplicity and efficiency in the monitoring of quality characteristics of production processes, was introduced by Shewhart (1924). Since then, several control charts have been devised. There are two distinct control charts classes: control charts for variables and for attributes. Control charts for variables require that measurements of the quality characteristics of the process are undertaken, as, for example, in the monitoring of means or individual observations. Control charts for attributes do not require measurements of the quality characteristics, and are used when such measurements are impossible or when, even if possible, one considers more advantageous to avoid them, as for instance when an accept/do not accept decision is made instead of the measurement of the diameter of a piece.

In the search of more efficient models, with reduced adjusted average time signal (AATS), new techniques or improvements are being developed for the control of processes. Control charts are being constructed with added flexibility in its operation through the variation of its design parameters (sample size, time interval between samples, coefficient of control limits), these schemes are called adaptives.

Initially, the investigation of adaptive control schemes focused in the study of the variation of sampling interval (REYNOLDS; ARNOLD,, 1989; RUNGER; PIGNATIELLO, 1991; RUNGER; MONTGOMERY, 1993; AMIN; MILLER, 1993; REYNOLDS, 1996; REYNOLDS; ARNOLD; BAIK, 1996) and the variation of the sample size (PRABHU; RUNGER; KEATS, 1993; COSTA, 1994). As a natural outgrow, further investigation combined variation in the sampling interval and the sample size (PRABHU; MONTGOMERY; RUNGER, 1994; COSTA, 1997), and also control charts with all its parameters varying (COSTA, 1998, 1999; DE MAGALHÃES; EPPRECHT; COSTA, 2001; DE MAGALHÃES; COSTA; EPPRECHT, 2002).

Joint charts for the mean and the variance have been studied as well (JONES; CASE, 1981; SANIGA, 1989; RAHIM, 1989; COSTA, 1993, 1998; RAHIM;

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COSTA, 2000; OHTA; KIMURA; RAHIM, 2002). Further work has been done by De Magalhães and Moura Neto (2005) and De Magalhães, Costa and Moura Neto (2006). Reynolds and Stoumbos (2001) investigated the use of three control charts for the simultaneous monitoring of the mean and the variability of normally distributed quality characteristics (X and MR chart, X and EWMA chart and two simultaneous EWMA charts). Although such studies demonstrate that none of the proposed charts have the ability of identifying the nature of the special cause that affects the process, such models have operational advantages related with the reduction of *AATS.* Costa and Rahim (2006) proposes a chi-square chart with two stages of sampling to monitor simultaneously both the mean and the variance.

Chen, Cheng and Xie (2001) combined two EWMA charts in one model, which proved to be an effective simultaneous control chart. Costa and Rahim (2004) proposed a fixed parameter control chart for the simultaneous monitoring of the mean and variability, also using the reading of only one chart, by means of a simple control statistics for the monitoring of the process, the non-central chi-square statistics (NCS). Such chart has proven more effective in the detection of special causes that affect the mean and/or the increase in the variability of the process when compared to joint Xbar and R charts.

In practice, a purely statistical model can be considered operationally disadvantageous depending on the costs incurred by the high frequency of sampling required for the optimal project (determination of the parameters which lead to minimum AATS) and/or for the high frequency of false alarms which appears when one diminishes unnecessarily the control limits of the chart. In this context, when one develops a chart for a production process some statistical and economic constraints have to be handled while determining optimal design of control charts.

The main trust of this work is to develop the economic-statistical design of a non-central chi-square control chart with variable parameters for the monitoring of variables, and to determine their optimal parameters for certain ranges of mean and variability perturbations of the process.

2. The NCS chart with variable parameters

The quality characteristic of interest, denoted by X, is normally distributed with mean μ and standard deviation σ. When the random variable X is in-control the value of μ is equal to its target value μ_0 and the value of σ equals σ_0, its target value. The aim of monitoring the process is the detection of a special cause that affects the value of μ, changing it from μ_0 to $\mu_1 = \mu_0 \pm \delta\sigma_0$, where $\delta \neq 0$, and/or affects the value of σ, altering it from σ_0 to $\sigma_1 = \gamma\sigma_0$, with $\gamma \neq 1$.

Consider X_{lj}, $l = 1, 2, 3, \ldots$, j = 1, 2, 3, ..., n_i the measurements of variable X presents in groups of size n_1 or n_2, $n_i > 1$, and varying with l, the index of the number of subgroup or sample. Let $\bar{x}_l = (x_{l1} + \ldots + x_{li})/n_i$ denote the l-th sample mean, and $e_l = (\bar{x}_l - \mu_0)$ the difference between the l-th sample mean and the target value of the mean of the process.

Define the function ξ_l which depends on the value of the error e_l:

$$\xi_l = \begin{cases} d \ \ if\ e_l \geq 0 \\ -d\ if\ e_l < 0 \end{cases}$$

the non-central chi-square (NCS) chart with variable parameters is based on the plotting of the statistics:

$$Y_l = \sum_{j=1}^{n_i} \left(x_{lj} - \mu_0 + \xi_l\sigma_0\right)^2, \ \ l = 1,\ 2,\ldots \qquad (1)$$

where i = 1, 2 indexes the size of the sample in the a-priori state; n_i is the variable sample size, and x_{lj}, $j = 1, 2, 3, \ldots, n_i$, are the measurements of the variable X grouped in samples of size $n_i > 1$.

While in the in-control period, Y_l/σ_0^2 is distributed as a non central chi-square with n_i degrees of freedom and non-centrality parameter $\lambda_{0i} = n_i d^2$. Then, the probability of false alarms is given by:

$$\alpha_i = P\left[Y_l > k_i\sigma_0^2\right] \qquad (2)$$

where $Y_l/\sigma_0^2 \sim \chi^2_{n_i}(n_i d^2)$, i = 1, 2 and k_i, i = 1, 2 are the coefficients of the control limits, respectively for strict and loose control.

During the out-of-control period, Y_l/σ^2_1 is distributed as a non-central chi-square distribution with n_i degrees of freedom and non-centrality parameter $\lambda_{1i} = n_i(\delta + \xi_l)^2/\gamma^2$. Therefore, the power of the control chart is given by:

$$1 - \beta = P\left[Y_l > k_i\sigma_1^2\right] \qquad (3)$$

where $Y_l/\sigma^2_1 \sim \chi^2_{n_i}(n_i(\delta + \xi_l)^2/\gamma^2$, i = 1, 2.

3. Characteristics of the chart

The interval (0, UCL) of the chart is partitioned in two distinct sub-regions: (0, WL) and (WL, UCL), where $0 < WL < UCL$. The region defined by (0, WL)

is called central region and the region (WL, UCL) is called warning region. The region above UCL is called action region of the chart. An out-of-control signal is produced when the sample falls in the action region.

The values of the parameters of the control chart in the sampling instant l (the sample size n_i, the sampling interval h_i and the coefficient of the control limit k_i) depend on the position of the statistics in the previous instant of time (l – 1). If there is an indication of a possible change in the mean and/or variability of the production process (when the statistics hits the warning region in instant (l – 1)) the control of the process changes from a loose state to a strict state of control by collecting samples with larger sizes after smaller time intervals, with stricter control limits in the chart. Otherwise, during the loose control state, one collects smaller samples, after larger time intervals and using control limits less strict. This strategy, in general, minimizes the mean time of detection of an out-of-control process. After an interval of time h_i the statistic Y_l is computed based in a sample of size n_i and is plotted in a control chart with warning limits given by

$$WL_i = w_i\sigma_0^2 \quad (4)$$

where w_i is the coefficient of the varying warning limit and with control limits are given by

$$UCL_i = k_i\sigma_0^2 \quad (5)$$

where k_i is aperture factor of the variable control limit and $w_i < k_i$, i = 1, 2. The values of the parameters change in the following way:

- **loose control** at instant l (n_1, h_1, w_1, k_1) if $0 < Y_{l-1} < WL_i$, i = 1, 2.
- **strict control** at instant l (n_2, h_2, w_2, k_2) if $WL_i < Y_{l-1} < UCL_i$, i = 1, 2.

Since all parameters are allowed to vary, a non-central chi-square chart is called a VP-NCS chart (*variable parameters*), illustrated in Figure 1. The chart with two scales is used for the simultaneous monitoring of the mean and the variance. The left side is used for values of Y with small samples (n_1) and for the right side one plots values of Y determined from large samples (n_2). Since the scale on the left is not a linear function of the scale of the right side, the control and warning limits for small and large samples do not coincide, making it difficult to plot and interpret the points of the control chart. To avoid this, the left scale is related piecewise linearly with the scale of the right in such a way that the warning and action limits for both sizes coincide.

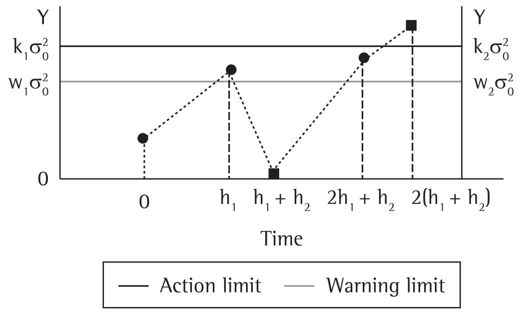

Figure 1. VP-NCS chart with two scales: circles mean that a loose control is performed and squares that a strict control is exerted (previous observation is in action region).

When the process is initiated or after the occurrence of a false alarm, the size of the first sample is chosen randomly. If the size of the sample is large (small), it should be collected after a small (large) time interval. During the in-control period, all the samples, including the first one, should have probability p_0 of being small, n_1, and a probability (1 – p_0) of being large, n_2, (COSTA, 1999),

$$p_0 = P\left[Y_l < w_i\sigma_0^2 \middle| Y_l < k_i\sigma_0^2\right], \quad i = 1,2 \quad (6)$$

where $Y_l/\sigma_0^2 \sim \chi^2_{n_i}(n_i d^2)$, i = 1, 2.

A non-central chi-square chart with fixed parameters ($h_1 = h_2$, $n_1 = n_2$ and $k_1 = k_2$) is known as FP-NCS (*fixed parameters*). If variation of sample sizes is allowed ($h_1 = h_2$, $n_2 > n_1$ and $k_1 = k_2$), it is known as VSS-NCS (*variable sample size*). When variation of the sampling interval is included ($h_1 > h_2$, $n_1 = n_2$ and $k_1 = k_2$), the chart is known as VSI-NCS (*variable sampling interval*). When sample size and sampling interval are allowed to vary ($h_1 > h_2$, $n_2 > n_1$ and $k_1 = k_2$) it is known as NCS–VSSI chart (v*ariable sample size and sampling interval*).

4. Computation of the transition probabilities

The time that the control chart takes to detect changes in the process is a measure of its statistical efficiency. Usually, the process initiates in an in-control state and a special cause occurs in a random time in the future introducing changes in the mean and/or in the variability of the process. This hypothesis is assumed in the development of the model. The adjusted mean time since the occurrence of a shift after a change of δ times the standard deviation in the mean and/or γ times the standard deviation in the variability until a signal is called AATS (*Adjusted Average Time to Signal*).

At each sampled value, one of the following transient states is reached depending on the position of the statistics in the chart (central region or warning region) and the status of the process (in or out-of-control).

The transient states of the Markov chain are classified in the following way (Table 1):

- **State 1:** The process is in-control ($\mu = \mu_0$ and $\sigma = \sigma_0$) and the control is loose ($n = n_1$, $h = h_1$, $w = w_1$, $k = k_1$);
- **State 2:** The process is in-control ($\mu = \mu_0$ and $\sigma = \sigma_0$) and the control is strict ($n = n_2$, $h = h_2$, $w = w_2$, $k = k_2$);
- **State 3:** The process is out-of-control ($\mu = \mu_1$ and $\sigma = \sigma_1$) and the control is loose ($n = n_1$, $h = h_1$, $w = w_1$, $k = k_1$);
- **State 4:** the process is out-of-control ($\mu = \mu_1$ and $\sigma = \sigma_1$) and the control is strict ($n = n_2$, $h = h_2$, $w = w_2$, $k = k_2$).

When the control chart produces a signal in the action region of the chart (the statistics falls out of the control limit) and the process is in state 1 or 2, this characterizes a false alarm. However, if the process is in states 3 or 4, the signal is a true alarm.

The transition matrix is given by:

$$\mathbf{P} = \left[\begin{array}{cccc:c} p_{11} & p_{12} & p_{13} & p_{14} & 0 \\ p_{21} & p_{22} & p_{23} & p_{24} & 0 \\ 0 & 0 & p_{33} & p_{34} & p_{35} \\ 0 & 0 & p_{43} & p_{44} & p_{45} \\ \hdashline 0 & 0 & 0 & 0 & p_{55} \end{array}\right]$$

where p_{ij} denotes the transition probability of the priori state i to the present state j.

Let T, the time the process is in-control, be exponentially distributed with parameter θ. Then, $P(T > h_i) = e^{-\theta h_i}$ is the probability of non-occurrence of a shift in μ and in σ during an interval of size h_i, that is, the probability that the process is in-control during the time interval h_i. The transition probabilities are given by

Table 1. States of Markov chain.

Sample $l-1$	Sample l	
Position of the statistics Y	Status of the process	State
Central	In-control	1
Warning	In-control	2
Central	Out-of-control	3
Warning	Out-of-control	4
Action	In or out-of-control	5

$$p_{11} = P\left(Y_l < w_1\sigma_0^2 \middle| Y_l < k_1\sigma_0^2\right)P\left(T > h_1\right) = P\left(Y_l < w_1\sigma_0^2 \middle| Y_l < k_1\sigma_0^2\right)e^{-\theta h_1} \quad (7)$$

$$p_{12} = P\left(w_1\sigma_0^2 < Y_l < k_1\sigma_0^2 \,|\, Y_l < k_1\sigma_0^2\right) P(T > h_1) = P\left(w_1\sigma_0^2 < Y_l < k_1\sigma_0^2 \,|\, Y_l < k_1\sigma_0^2\right) e^{-\theta h_1} \quad (8)$$

$$p_{13} = P\left(Y_l < w_1\sigma_0^2 \,|\, Y_l < k_1\sigma_0^2\right) P(T \le h_1) = P\left(Y_l < w_1\sigma_0^2 \,|\, Y_l < k_1\sigma_0^2\right)\left(1 - e^{-\theta h_1}\right) \quad (9)$$

$$p_{14} = P\left(w_1\sigma_0^2 < Y_l < k_1\sigma_0^2 \,|\, Y_l < k_1\sigma_0^2\right) P(T \le h_1) = P\left(w_1\sigma_0^2 < Y_l < k_1\sigma_0^2 \,|\, Y_l < k_1\sigma_0^2\right)\left(1 - e^{-\theta h_1}\right) \quad (10)$$

where, in Equations 7 to 10, $Y_l/\sigma_0^2 \sim \chi^2_{n_1}(n_1 d^2)$. Also,

$$p_{21} = P\left(Y_l < w_2\sigma_0^2 \,|\, Y_l < k_2\sigma_0^2\right) P(T > h_2) = P\left(Y_l < w_2\sigma_0^2 \,|\, Y_l < k_2\sigma_0^2\right) e^{-\theta h_2} \quad (11)$$

$$p_{22} = P\left(w_2\sigma_0^2 < Y_l < k_2\sigma_0^2 \,|\, Y_l < k_2\sigma_0^2\right) P(T > h_2) = P\left(w_2\sigma_0^2 < Y_l < k_2\sigma_0^2 \,|\, Y_l < k_2\sigma_0^2\right) e^{-\theta h_2} \quad (12)$$

$$p_{23} = P\left(Y_l < w_2\sigma_0^2 \,|\, Y_l < k_2\sigma_0^2\right) P(T \le h_2) = P\left(Y_l < w_2\sigma_0^2 \,|\, Y_l < k_2\sigma_0^2\right)\left(1 - e^{-\theta h_2}\right) \quad (13)$$

$$p_{24} = P\left(w_2\sigma_0^2 < Y_l < k_2\sigma_0^2 \,|\, Y_l < k_2\sigma_0^2\right) P(T \le h_2) = P\left(w_2\sigma_0^2 < Y_l < k_2\sigma_0^2 \,|\, Y_l < k_2\sigma_0^2\right)\left(1 - e^{-\theta h_2}\right) \quad (14)$$

where, in Equations 11 to 14, $Y_l/\sigma_0^2 \sim \chi^2_{n_2}(n_2 d^2)$. Moreover,

$$p_{33} = P\left(Y_l < w_1\sigma_0^2 \,|\, Y_l < k_1\sigma_0^2\right) \quad (15)$$

$$p_{34} = P\left(w_1\sigma_0^2 < Y_l < k_1\sigma_0^2 \,|\, Y_l < k_1\sigma_0^2\right) \quad (16)$$

$$p_{35} = P\left(Y_l > k_1\sigma_0^2\right) \quad (17)$$

where, in Equations 15 to 17, $Y_l/\sigma_1^2 \sim \chi^2_{n_1}(n_1(\delta+\xi)^2/\gamma^2)$. Finally,

$$p_{43} = P\left(Y_l < w_2\sigma_0^2 \,|\, Y_l < k_2\sigma_0^2\right) \quad (18)$$

$$p_{44} = P\left(w_2\sigma_0^2 < Y_l < k_2\sigma_0^2 \,|\, Y_l < k_2\sigma_0^2\right) \quad (19)$$

$$p_{45} = P\left(Y_l > k_2\sigma_0^2\right) \quad (20)$$

where, in Equations 18 to 20, $Y_l/\sigma_1^2 \sim \chi^2_{n_2}(n_2(\delta+\xi)^2/\gamma^2)$.

Each transient state has a probability bi of the process to be initiated from it. The initial probabilities of all transient state make up the vector of initial probabilities and are given by

State 1: $b_1 = p_0 e^{-\theta h_1}$ (21)

State 2: $b_2 = (1-p_0)e^{-\theta h_2}$ (22)

State 3: $b_3 = p_0\left(1-e^{-\theta h_1}\right)$ (23)

State 4: $b_4 = (1-p_0)\left(1-e^{-\theta h_2}\right)$ (24)

The number of expected visits to any transient of the Markov chain is determined by

$$\mathbf{b'}(\mathbf{I}-\mathbf{Q})^{-1} \quad (25)$$

where **b** is the vector of initial probabilities, **I** is the identity matrix of order 4, **Q** is the matrix of transition probabilities removing the column and line corresponding to the absorbing state.

5. Economic Design of the VP-NCS chart

The process is initiated in an in-control state with $\mu = \mu_0$ and standard deviation $\sigma = \sigma_0$, assuming independent and identically distributed samples. The occurrence of a special cause introduces a change in the mean and/or variability of the process. The process is non-self correcting (it up-helds the mean and/or the variability change until it is repaired) and it can continue to operate or not during the search for a special cause and/or during the repair of the process. The parameters μ, σ, δ and γ are assumed known and the unknown parameters are n_1, n_2, h_1, h_2, w_1, w_2, k_1 and k_2.

The operation of a production process under the monitoring of a control chart designed economically can be seen as a sequence of cycles. The production cycle is defined as the time between the beginning or re-initialization of the process until the elimination of the special cause. The production cycle consists of in-control periods (time since the beginning, or the re-start of the process until the occurrence of a special cause, including interruptions due to false alarms), out-of-control period (time since the change in the process until the signal), time for investigation (to find the special cause when it does exist), and repair (time for repairing the process, to bring it to the original in-control behavior).

The average number of false alarms per cycle is determined by

$$E(F) = \mathbf{b'}(\mathbf{I}-\mathbf{Q})^{-1}\boldsymbol{\alpha} \quad (26)$$

where $\boldsymbol{\alpha}' = [\alpha_1, \alpha_2, 0, 0]$ and $Y_i/\sigma^2_0 \sim \chi^2_{n_i}(n_i d^2)$.

The average number of observed items per production cycle is given by

$$E(M) = \mathbf{b'}(\mathbf{I}-\mathbf{Q})^{-1}\mathbf{m} \quad (27)$$

where $\mathbf{m}' = [n_1, n_2, n_1, n_2]$.

The average time to cycle (ATC) is the mean time from the beginning of the production until the first true signal of out-of-control, and is determined by

$$ATC = E(TC) = \mathbf{b'}(\mathbf{I}-\mathbf{Q})^{-1}\mathbf{h} \quad (28)$$

where $\mathbf{h}' = [h_1, h_2, h_1, h_2]$.

Computing the average time of cycle, the adjusted average time since the occurrence of a shift after an alteration of in the mean and/or in the variability until a signal (AATS) is determined by

$$AATS = E(TC) - E(T) = ATC - \frac{1}{\theta} \quad (29)$$

where T is the time the process remains in control. The average time from the beginning of the process until a false alarm, that is when the process is in-control (ATS_0) is determined by

$$ATS_0 = \mathbf{b'}(\mathbf{I}-\mathbf{Q})^{-1}\mathbf{t} \quad (30)$$

where $\mathbf{t}' = [h_1, h_2, 0, 0]$.

5.1. *Average cycle length*

The length of a production cycle is formed by the in-control period and by the out-of-control period.

The average time spent in a production cycle comprises the average time in which the process remains in control and the average time the process remains out-of-control,

$$E(TC) = E(T_{ic}) + E(T_{oc}) \quad (31)$$

5.1.1. Period in-control

$$E(T_{ic}) = \frac{1}{\theta} + (1-\delta_1)T_0 E(F) \quad (32)$$

where T_0 is the expected time in the search of a false alarm

$$\delta_1 = \begin{cases} 1, & \text{when the production continues during the search of false alarms} \\ 0, & \text{when the production ceases during the search of false alarms} \end{cases}$$

5.1.2. Period out-of-control time

$$E(T_{oc}) = AATS + T_{***} \quad (33)$$

where T_{***} is the expected time in the search and removal of the special cause.

Therefore, the expected length of the cycle is determined by

$$\begin{aligned} E(TC) &= \frac{1}{\theta} + (1-\delta_1)T_0E(F) + AATS + T_{***} \\ &= \frac{1}{\theta} + (1-\delta_1)T_0E(F) + ATC - \frac{1}{\theta} + T_{***} \\ &= ATC + (1-\delta_1)T_0E(F) + T_{***} \end{aligned} \quad (34)$$

5.2. *Expected cost per cycle*

Some costs are due to the maintenance of the quality control of a production process. The cost structure considered here includes costs of investigation of false alarms, of sampling and inspection, of removal of special cause, of production of non-conforming products while the process operates in-control and out-of-control.

The expected cost of a production cycle comprises the average cost of the process while in-control and by the average costs of the process operating out-of-control.

$$E(C) = E(C_{ic}) + E(C_{oc}) \quad (35)$$

5.2.1. Expected cost in the in-control state

$$E(C_{ic}) = \frac{C_0}{\theta} + A_0.E(F) \quad (36)$$

where C_0 is the expected cost per hour of production of non-conforming products during the in-control period, and A_0 is the expected cost per false alarm.

5.2.2. Expected cost when the process is out-of-control

$$E(C_{oc}) = C_1(AATS + \delta_2 T_{***}) + W + A_1E(M) \quad (37)$$

where C_1 is the expected cost per hour of production of non-conforming during the out-of-control period, W is the expected cost of repairing the process and A_1 is the expected cost per inspected item,

$$\delta_2 = \begin{cases} 1, & \text{when production continues during the search and repair of special cause} \\ 0, & \text{when production ceases during the search and repair of special cause} \end{cases}$$

Therefore, the expected cost in a cycle is determined by

$$E(C) = \frac{C_0}{\theta} + A_0.E(F) + C_1(AATS + \delta_2 T_{***}) + W + A_1E(M) \quad (38)$$

5.3. *The economic model*

An expression is obtained for the expected cost per unit of time (ECTU) in the monitoring of the process through a VP-NCS chart. Since the process considered is a renewal-reward process (ROSS, 1970), ECTU can be written as a rate of expected cost (E(C)) per expected time in a cycle (E(TC)).

$$ECTU = \frac{E(C)}{E(TC)} = \frac{C_0/\theta + A_0.E(F) + C_1(AAATS + \delta_2 T_{***}) + W + A_1E(M)}{ATC + (1-\delta_1)T_0E(F) + T_{***}} \quad (39)$$

The design parameters of the economic model for the VP-NCS chart are obtained by finding the solution of the following constrained optimization problem

$$Min\left(\frac{\frac{C_0}{\theta} + A_0.E(F) + C_1(AATS + \delta_2 T_{***}) + W + A_1E(M)}{ATC + (1-\delta_1)T_0E(F) + T_{***}}\right) \quad (40)$$

subjected to

$$p_0 = P\left[Y_l < w_1\sigma_0^2 | Y_l < k_1\sigma_0^2\right] = P\left[Y_l < w_2\sigma_0^2 | Y_l < k_2\sigma_0^2\right]$$

and

$d \leq 1, 2$

5.4. *The economic-statistical model*

In the determination of the minimum ECTU, one gets the optimal parameters of the VP-NCS chart. However, the economic design does not take into account any of the statistical properties not even the most relevant. The optimal design parameters for the economic model can lead to an excessive large number of false alarms and at the same time to the increase of the time of signaling the detection of out-of-control of the process.

Usually, the process is initiated in-control, $\mu = \mu_0$ and $\sigma = \sigma_0$, and afterwards an alteration in the mean and/or variability of the process occurs in a random instant in the future. When the process is in-control, it is desirable that the mean time from the beginning of the process until a signal (false alarm) be large, guaranteeing a reduced number of false alarms. This mean time is denoted by ATS_0. When the process is out-of-control, it is desirable that the mean time between the occurrence of the special cause and the signal detecting an out-of-control behavior of the process be short, guaranteeing a rapid detection of changes in the production process. This mean time is denoted by AATS (*Adjusted Average Time to Signal*).

In the search for a better statistic behavior of the economic design, constraints are adjoined to the minimization problem of ECTU providing restrictions on the average time until a signal when the process is in-control (ATS_0) and out-of-control (AATS).

The design parameters of the economic-statistic model for the VP-NCS chart are obtained by finding the solution of the following constrained optimization problem

$$Min\left(\frac{\frac{C_0}{\theta}+A_0.E(F)+C_1\left(AATS+\delta_2 T_{***}\right)+W+A_1E(M)}{ATC+(1-\delta_1)T_0E(F)+T_{***}}\right) \quad (41)$$

subjected to the following constraints

$$p_0=P\left[Y_l<w_1\sigma_0^2|Y_l<k_1\sigma_0^2\right]=P\left[Y_l<w_2\sigma_0^2|Y_l<k_2\sigma_0^2\right]$$

$d \le 1, 2$, $n_1 \le n_2$, $n_1 \ge 1$, $n_2 \ge 3$, $0.1 \le h_2 \le h_1$, $h_1 \ge 1$, $0.1 \le w_2 \le w_1$, $1 \le k_2 \le k_1$, $ATS_0 > L$, $AATS < U$

where L is the lower bound of ATS_0 and U is the upper bound of *AATS*.

6. Results and discussion

For the minimization of ECTU, we used simulated annealing, a stochastic-based optimization algorithm, because of the alternation between large flat plates and steep regions of the objective function which are difficult to handle with gradient based methods, see Figure 2.

We present the results of the optimization procedure in Tables 2 and 3, respectively for FP-NCS and VP-NCS charts. Several quantities are determined as functions of the shifts of the mean and variance (delta and gamma) of the process: ECTU, adjusted average to signal (AATS), average time in control (ATC), expected number of false alarms (E(F)), average number of itens (ANI), and ATS_0 for FP-NCS chart (Table 2) and VP-NCS (Table 3). A comparison between the two charts is presented in Figure 3. There, the percentage gain attained when using the VP-NCS chart instead of the FP-NCS chart, is exhibit. It is clear that the VP-NCS outperforms the FP-NCS chart and it may be worth using in some situations.

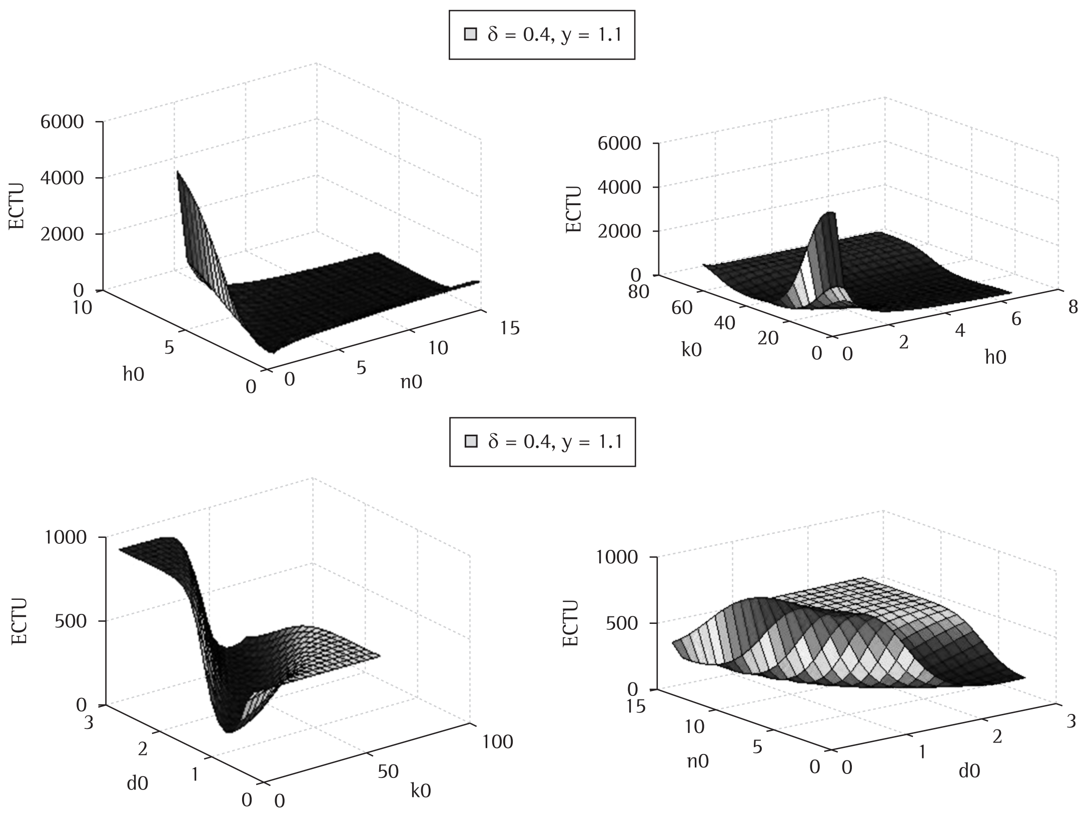

Figure 2. Partial views of the objective function ECTU for FP-NCS chart.

Table 2. ECTU, adjusted average to signal (AATS), average time in control (ATC), expected number of false alarms (E(F)), average number of itens (ANI), and ATS_0 for FP-NCS charts depending on the shift (δ and γ).

γ	δ	ECTU	AATS	n_0	h_0	k_0	d	ATC	E(F)	ANI	ATS_0
1	0	205.01	23.65	2	2.94	24.9	2.66	457.05	19.61	310.5	434.87
1	0.1	197.12	19.27	3	2.5	25.83	2.07	452.67	18.24	543.44	434.65
1	0.2	186.53	20.95	4	2.47	28.63	1.73	454.35	11.05	736.97	434.63
1	0.3	177.09	15.25	5	2.42	28.83	1.49	448.65	10.32	926.61	434.61
1	0.4	169.62	16.12	5	2.36	29.45	1.39	449.52	6.01	951.93	434.58
1	0.5	164.95	16.45	5	2.42	29.36	1.28	449.85	3.66	928.7	434.61
1	0.6	158.21	8.68	6	2.42	29.54	1.22	442.08	6.15	1094.52	434.61
1	0.7	157.82	9.79	3	0.88	26.87	1.4	443.19	3.22	1514.46	433.84
1	0.8	149.99	8.46	4	1.74	29.74	1.4	441.86	2.9	1018.12	434.27
1	0.9	146.51	7.95	5	2.35	29.38	1.17	441.35	2.07	937.97	434.58
1	1	144.1	6.6	5	2.32	29.8	1.19	440	2.06	946.5	434.56
1.1	0	235.84	48.18	3	3.84	22.97	2	481.58	15.84	376.69	435.32
1.1	0.1	216.32	23.06	5	3.86	23.69	1.61	456.46	24.25	591.11	435.33
1.1	0.2	205.69	32.4	5	3	23.81	1.31	465.8	11.12	775.55	434.9
1.1	0.3	191.12	17.4	6	2.7	23.93	1.19	450.8	14.87	1001.03	434.75
1.1	0.4	180.63	16.52	6	2.47	24.38	1.1	449.92	10.23	1094	434.63
1.1	0.5	172.59	14.2	6	2.37	24.5	1.04	447.6	8.15	1134.43	434.58
1.1	0.6	166.45	11.46	6	2.34	24.78	1.04	444.86	7.49	1141.64	434.57
1.1	0.7	160.58	12.73	6	2.46	25.63	0.96	446.13	3.96	1088.64	434.63
1.1	0.8	156.6	8.6	6	2.31	25.29	0.98	442	5.25	1147.58	434.56
1.1	0.9	152.59	7.9	6	2.36	25.95	0.97	441.3	4.1	1120.16	434.58
1.1	1	151.57	6.13	5	1.85	28.3	1.23	439.53	4.81	1186.13	434.33
1.2	0	249.08	38.45	5	3.73	23.89	1.67	471.85	28.53	632.58	435.27
1.2	0.1	232.4	30.45	6	3.46	24.06	1.41	463.85	25.48	804.6	435.13
1.2	0.2	219.37	35.9	6	3.21	23.92	1.23	469.3	14.67	878.03	435.01
1.2	0.3	205.36	29.59	6	2.63	23.82	1.12	462.99	11.89	1057.4	434.71
1.2	0.4	193.9	25.33	6	2.47	23.97	1.06	458.73	9.46	1113.04	434.64
1.2	0.5	183.75	20.79	6	2.28	24.16	1.01	454.19	7.84	1196.87	434.54
1.2	0.6	174.99	16.61	6	2.22	24.01	0.97	450.01	6.96	1218.51	434.51
1.2	0.7	169.31	16.82	6	2.35	24.23	0.9	450.22	4.43	1150.63	434.57
1.2	0.8	162.8	13.38	6	2.32	24.32	0.89	446.78	4.16	1156.91	434.56
1.2	0.9	157.83	10.64	6	2.26	24.17	0.87	444.04	4.01	1176.6	434.53
1.2	1	153.97	9.54	6	2.35	24.34	0.85	442.94	3.33	1131.98	434.57
1.3	0	268.57	42.28	6	3.38	23.67	1.49	475.68	34.55	845.12	435.09
1.3	0.1	252.1	45.07	6	3.54	23.92	1.4	478.47	24.42	810.82	435.17
1.3	0.2	236.69	42.15	6	3.11	23.81	1.27	475.55	18.37	917.82	434.96
1.3	0.3	221.15	36.21	6	2.51	23.66	1.14	469.61	14.24	1120.77	434.66
1.3	0.4	203.92	20.48	7	2.28	23.71	1.02	453.88	16.78	1395.92	434.54
1.3	0.5	195.26	25.2	6	2.06	23.52	0.99	458.6	9.23	1336.93	434.43
1.3	0.6	186.8	23.26	6	2.11	23.96	0.95	456.66	6.88	1298	434.46
1.3	0.7	176.21	12.89	7	2.15	23.95	0.89	446.29	9.5	1452.28	434.48
1.3	0.8	170.74	16.32	6	2.14	24.05	0.89	449.72	5	1263.81	434.47
1.3	0.9	164.72	13.57	6	2.13	24.17	0.88	446.97	4.44	1260.62	434.46
1.3	1	159.63	11.63	6	2.2	23.73	0.83	445.03	3.88	1211.61	434.5
1.4	0	290.62	52.04	6	3.29	23.67	1.52	485.44	38.71	886.59	435.04
1.4	0.1	268	34.7	7	3.43	23.64	1.38	468.1	39.19	954.35	435.12
1.4	0.2	252.95	46.58	6	3.47	23.34	1.36	479.98	23.66	829.65	435.14
1.4	0.3	240.59	46.48	6	2.41	23.88	1.17	479.88	15.89	1194.56	434.61
1.4	0.4	223.42	38.93	6	1.96	23.42	1.03	472.33	11.86	1448.61	434.38
1.4	0.5	208.6	30.98	6	1.86	23.47	0.99	464.38	10.46	1496.81	434.33
1.4	0.6	193.32	18.08	7	1.93	23.81	0.92	451.48	12.58	1634.69	434.37
1.4	0.7	186.77	19.88	6	1.8	23.69	0.95	453.28	8.62	1510.73	434.3
1.4	0.8	177.53	13.02	7	1.99	23.9	0.87	446.42	9.51	1566.69	434.4
1.4	0.9	171.03	11.64	7	2.05	24.07	0.84	445.04	7.81	1519.36	434.43
1.4	1	166.61	14.65	6	2.05	23.95	0.85	448.05	4.22	1311.22	434.43

Table 3. ECTU, adjusted average to signal (AATS), average time in control (ATC), expected number of false alarms (E(F)), average number of itens (ANI), and ATS_0 for VP-NCS charts depending on the shift (δ and γ).

γ	δ	ECTU	AATS	n_1	n_2	h_1	h_2	w_1	w_2	k_1	k_2	d	p_0	ATC	E(F)	ANI	ATS_0
1	0	207.36	25.69	4	7	4.21	1.93	18.52	18.12	24.03	19.04	1.63	0.86	459.09	18.4	482.51	435.43
1	0.1	192.78	21.35	3	7	3.49	0.1	9.77	15.65	26.54	21.04	1.18	0.68	454.75	13.19	852.06	435.12
1	0.2	177.92	16.83	3	6	3.57	0.1	6.26	12.5	26.3	21.39	1.01	0.58	450.23	9.19	983.72	435.15
1	0.3	178.81	8	5	7	2.93	0.49	7.97	10.97	25.27	20.05	0.89	0.51	441.4	14.37	1509.69	434.7
1	0.4	164.08	8	5	7	3.07	0.44	8.49	11.28	25.16	20.13	0.82	0.58	441.4	8.45	1314.05	434.82
1	0.5	153.02	8	5	7	3.26	0.1	8.09	11.31	25.4	20.28	0.77	0.65	441.4	4.29	1103.38	435
1	0.6	146.29	7.68	3	6	3.29	0.1	5.92	10	26.63	21.39	0.78	0.62	441.08	2.19	939.29	435.02
1	0.7	142.8	6.68	4	6	3.25	0.1	6.6	10.01	26.92	21.71	0.78	0.65	440.08	1.55	910.31	435
1	0.8	140.02	5.94	3	5	3.05	0.1	6.58	10	26.38	21.72	0.76	0.69	439.34	1.12	861.31	434.91
1	0.9	137.9	5.33	3	5	2.85	0.1	6.74	10.52	26.36	21.94	0.73	0.73	438.73	0.8	828.89	434.81
1	1	136.58	4.98	3	5	2.78	0.1	7.24	10.69	26.22	21.56	0.69	0.77	438.38	0.6	805.08	434.78
1.1	0	225.63	30.53	3	8	4.06	2.09	4.68	11.34	23.82	18.5	0.87	0.44	463.93	21.36	978.39	435.04
1.1	0.1	208.62	26.35	4	8	3.94	1.96	4.82	11.34	23.94	18.86	0.82	0.47	459.75	16.35	970.93	435.02
1.1	0.2	192.49	21.26	3	8	4.15	0.1	4.79	11.23	23.58	18.59	0.7	0.55	454.66	11.97	1116.37	435.44
1.1	0.3	178.68	16.93	4	9	3.99	0.1	4.87	11.82	25.23	20.57	0.69	0.55	450.33	8.6	1181.61	435.36
1.1	0.4	183.68	8	5	7	3.11	0.48	8.43	11.3	25.05	19.94	0.93	0.51	441.4	16.85	1455.92	434.78
1.1	0.5	166.81	8	5	7	3.02	0.43	8.42	11.45	25.15	20.01	0.85	0.58	441.4	9.7	1315.92	434.79
1.1	0.6	154.76	8	5	7	3.21	0.1	8.17	11.11	25.29	20.33	0.76	0.63	441.4	4.73	1184.94	434.98
1.1	0.7	149.14	8	5	7	3.24	0.1	8.42	11.25	25.27	20.22	0.68	0.69	441.4	2.66	1070.08	435
1.1	0.8	143.66	6.92	3	6	3.11	0.1	4.91	10.15	26.56	21.69	0.67	0.64	440.32	1.6	941.84	434.93
1.1	0.9	141.07	6.21	3	6	3.04	0.1	5.62	10.12	26.52	21.64	0.66	0.68	439.61	1.16	908.8	434.9
1.1	1	138.87	5.61	3	6	2.9	0.1	5.86	10.24	26.32	21.8	0.65	0.71	439.01	0.85	866.07	434.83
1.2	0	234.69	32.73	4	8	2.18	2.19	0.03	0.88	23.9	19.1	0.65	0	466.13	20.66	1766.04	434.49
1.2	0.1	220.37	28.59	4	8	3.89	1.87	1.72	5.96	24.02	18.84	0.63	0.15	461.99	17.48	1613.82	434.61
1.2	0.2	204.2	24.76	4	9	4.29	1.62	3.22	8.25	23.88	18.51	0.63	0.32	458.16	14.01	1300.23	434.96
1.2	0.3	188.94	19.93	3	9	4.2	0.1	3.72	9.81	23.39	18.63	0.56	0.49	453.33	10.36	1338.83	435.45
1.2	0.4	176.91	16.42	4	9	4.02	0.1	4.01	10.33	23.91	19.26	0.55	0.53	449.82	7.84	1258.63	435.37
1.2	0.5	167.03	13.47	4	8	4.04	0.1	4.1	10.38	25.47	20.57	0.58	0.52	446.87	5.67	1245.04	435.38
1.2	0.6	168	8	5	7	3.05	0.42	7.59	10.55	25.25	20.05	0.81	0.53	441.4	9.72	1436.14	434.78
1.2	0.7	156.89	8	5	7	3.08	0.1	8.47	11.82	25.27	20.18	0.79	0.65	441.4	5.69	1190.05	434.92
1.2	0.8	150.2	8	5	7	3.14	0.1	8.37	11.42	25.34	20.37	0.7	0.69	441.4	3.01	1101.81	434.95
1.2	0.9	144.57	7.15	3	7	3.13	0.1	4.58	10.46	26.65	21.65	0.57	0.65	440.55	1.61	992.68	434.94
1.2	1	142.07	6.48	4	7	3.12	0.1	5.44	10.4	26.57	21.66	0.57	0.68	439.88	1.17	960.41	434.94
1.3	0	240.24	35.45	4	9	2.18	1.9	0.02	0.68	24.62	18.71	0.55	0	468.85	19.89	2101.85	434.35
1.3	0.1	226.72	30.54	4	8	2.57	1.94	0.02	0.53	24.65	18.6	0.54	0	463.94	17.42	1992.38	434.37
1.3	0.2	212.96	27.1	4	8	4.13	1.76	1.6	5.78	23.9	18.41	0.55	0.16	460.5	14.93	1639.08	434.65
1.3	0.3	199.15	22.81	4	8	4.5	0.15	2.88	8.29	24.03	18.83	0.53	0.38	456.21	11.43	1683.67	435.54
1.3	0.4	185.38	18.99	3	8	4.3	0.1	3.25	8.72	23.74	18.35	0.51	0.45	452.39	9.07	1418.78	435.49
1.3	0.5	174.65	15.73	4	9	4.18	0.1	3.65	9.47	23.77	18.73	0.48	0.5	449.13	7.16	1297.21	435.44
1.3	0.6	166.12	13.2	4	8	4.07	0.1	3.85	9.51	24.1	19.6	0.5	0.51	446.6	5.35	1272.03	435.39
1.3	0.7	171.4	8	5	7	3.01	0.46	7.7	10.63	25.13	20.15	0.85	0.52	441.4	11.22	1451.37	434.75
1.3	0.8	162.93	8	5	7	3.09	0.42	9.38	12.3	25.29	20.05	0.87	0.63	441.4	8.47	1188.18	434.85
1.3	0.9	150.81	8	5	7	3.25	0.1	7.43	10.08	25.41	20.31	0.66	0.63	441.4	2.92	1185.25	435
1.3	1	145.28	7.38	3	8	3.2	0.1	4.56	10.63	26.51	21.51	0.5	0.65	440.78	1.6	1025.35	434.98
1.4	0	247.32	38.51	4	8	1.55	1.68	0.05	1.01	24.32	18.43	0.55	0	471.91	20.3	2289.41	434.24
1.4	0.1	233.92	34.1	4	8	1.84	1.9	0.02	0.54	24.64	18.66	0.56	0	467.5	18.12	2026.04	434.35
1.4	0.2	219.88	28.68	4	8	3.83	1.87	0.54	3.04	23.84	18.43	0.49	0.03	462.08	15.57	1978.54	434.39
1.4	0.3	206.74	25.63	4	9	4.19	1.45	1.88	6.26	23.6	18.41	0.48	0.21	459.03	12.74	1715.39	434.73
1.4	0.4	193.34	21.24	3	8	4.27	0.1	2.75	7.93	23.58	18.4	0.46	0.39	454.64	9.8	1705.32	435.47
1.4	0.5	182.01	17.97	4	8	4.37	0.1	3	8.24	23.54	18.6	0.46	0.42	451.37	7.88	1514.62	435.52
1.4	0.6	172.29	14.97	3	8	4.1	0.1	2.82	8.78	24.04	18.37	0.43	0.48	448.37	6.62	1311.25	435.4
1.4	0.7	164.81	12.85	4	9	4.18	0.1	3.73	9.33	24.14	19.44	0.44	0.51	446.25	4.96	1286.05	435.44
1.4	0.8	173.36	8	5	7	2.97	0.48	7.73	10.59	25.24	20.02	0.84	0.51	441.4	11.92	1496.36	434.72
1.4	0.9	159.94	8	5	7	2.95	0.4	6.9	10.02	25.29	20.09	0.73	0.56	441.4	6.25	1381.55	434.75
1.4	1	152.71	8	5	7	3.14	0.1	8.41	11.44	25.2	20.21	0.72	0.67	441.4	3.91	1158.18	434.95

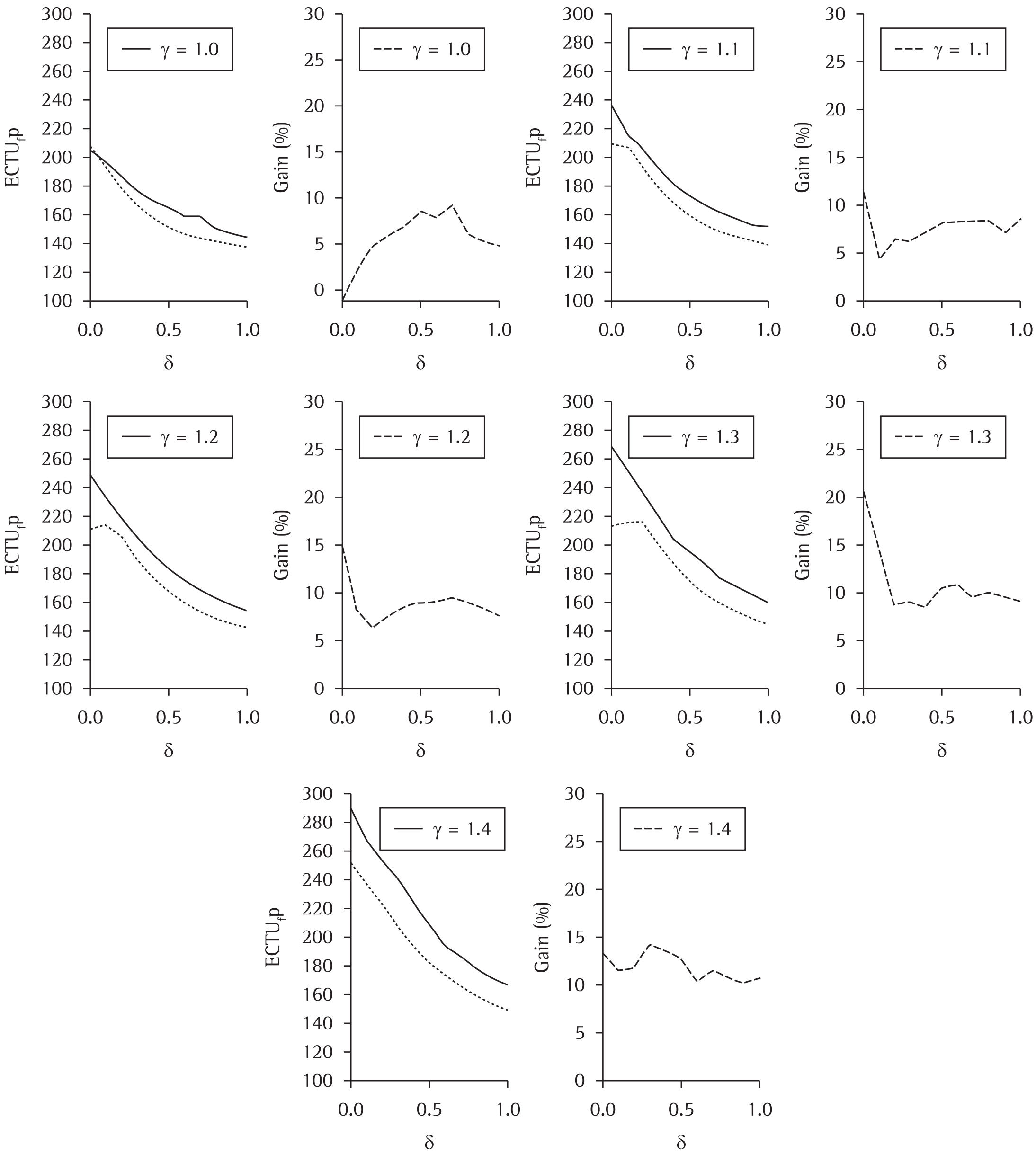

Figure 3. These graphs show the value of ECTU for FP-NCS and VP-NCS charts for different values of the shifts (delta and gamma). The curves bellow are the ECTU for VP. In their rights it is shown the percentage gain when the VP chart is used instead of the FP chart.

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Projeto econômico-estatístico de gráficos de controle qui-quadrado não-central com parâmetros variáveis

Resumo

Processos de produção são monitorados por gráficos de controle desde a sua introdução por Shewhart (1924). Este monitoramento é útil na melhoria do processo de produção devido à crescente estabilização do processo, e consequentemente, padronização do produto. Gráficos de controle mantêm vigilância de características de qualidade de um processo de produção. Isto é feito por intermédio de gráficos univariados ou multivariados. Melhorias na metodologia de gráficos de controle podem levar a um impacto econômico significativo no processo de produção. Neste artigo, propomos um gráfico de controle de parâmetros variáveis baseado na estatística qui-quadrado não-central para monitorar uma característica de qualidade de interesse. O projeto do gráfico é realizado através da otimização de uma função custo. O algoritmo *simulated annealing* é usado devido à dificuldade dos métodos clássicos de otimização baseados no gradiente, de lidarem com a otimização da função custo. Os resultados mostram algumas das dificuldades de se usar este modelo.

Palavras-chave

Controle estatístico de processos. Projeto econômico. Gráfico de controle qui-quadrado. Parâmetros variáveis. *Simulated annealing*.

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Appendix 1. Notation.

Symbol	Object
x_{lj}	measurements of the variable X at instant of time l grouped in samples of size n_i, j =1, ..., n_i
μ	mean of the process
μ_0	mean of the in-control process
μ_1	mean of the out-of-control process
σ	standard deviation of the process (measure of the variability of the process)
σ_0	standard deviation of the in-control process
σ_1	standard deviation of the out-of-control
ξ	function of the sample mean
δ	shift factor in the mean of the process
γ	shift factor in the standard deviation of the process
VP	Variable Parameters
NCS	Non-central chi-square
Y	control statistics
α	type I error probability
β	type II error probability
λ	non-centrality parameter of the non-central chi-square distribution
θ	parameter of the negative exponential distribution
d	constant parameter used in the definition of the function ξ
n_1	smaller sample size (loose control)
n_2	larger sample size (strict control)
h_1	larger sampling interval (loose control)
h_2	smaller sampling interval (strict control)
k_1	larger coefficient factor of the control limit (loose control)
k_2	smaller coefficient factor of the control limit (strict control)
w_1	coefficient of the warning limit for loose control
w_2	Coefficient of the warning limit for strict control
WL_i	variable warning limit
UCL_i	variable upper control limit
p_{ij}	transition probability from a-priori state i to the current state j
b	vector of initial probabilities of the markovian states
Q	transition probability matrix without row and column corresponding to the absorbing state
$E(F)$	average number of false alarms per cycle
$E(M)$	average number of observed items per cycle
ATC	average time to cycle
TS	time to signal
ATS	average time to signal
$AATS$	adjusted average time to signal
$E(TC)$	expected cycle length
$E(T_{ic})$	expected time in-control
$E(T_{0c})$	expected time out-of-control
T_{***}	expected time in the search and removal of a special cause
δ_1	indicator binary variable of continuity of production during the search for false alarms
T_0	expected time in the search of a false alarm
$E(C)$	expected cost per cycle
$E(C_{ic})$	expected cost in-control
$E(C_{0c})$	expected cost out-of-control
C_0	expected cost per hour of production of non-conforming products during the in-control period
A_0	cost per detected false alarm
W	expected cost per hour of production of non-conforming products during the out-of-control period
C_1	expected repair cost in the process
A_1	expected cost per item inspected