The weighted average cost of capital (wacc) for firm valuation calculations: a reply[1]

El costo promedio ponderado de capital (wacc) para los cálculos de valuación de empresas: una respuesta

O custo médio ponderado de capital (wacc) para os cálculos de valorização de empresas: uma resposta

Ignacio Vélez-Pareja. [2]

Abstract

Llano-Ferro (2009) proposes a solution to avoid "significant errors" when the Weighted Average Cost of Capital (WACC) "obtained by the standard formula leads to significant errors in Net Present Value of the Firm calculations; particularly in those that apply to perpetual cash flow series". In this paper we show that there are not "significant errors" but a wrong use of the formula and improper calculations of values.

The Weighted Average Cost of Capital (WACC) is used in finance for several applications, including Capital Budgeting analysis, EVA® calculations, and firm valuation. WACC obtained by the standard formula leads to significant errors in Net Present Value of the Firm calculations; particularly in those that apply perpetual cash flow series. The present paper identifies the problem, and provides alternative, and accurate formulas to obtain WACC for Firm Valuation calculations.

Keywords: Weighted Average, Cost of Capital, WACC, firm valuation, capital budgeting, equity cost of capital. JEL codes D61, G31, H43

Resumen

Llano-Ferro (2009) propone una solución para evitar "errores significativos" cuando el Costo Promedio Ponderado de Capital (WACC) "obtenido por la fórmula general conduce a errores significativos en el valor presente neto de los cálculos de la empresa, particularmente en aquellas que se aplican a perpetuo flujo de efectivo serie". En este trabajo se muestra que no hay "errores significativos", pero sí un mal uso de la fórmula y el cálculo incorrecto de los valores.

El Costo Promedio Ponderado de Capital (WACC) se utiliza en la financiación de varias aplicaciones, incluyendo el análisis del presupuesto de capital, los cálculos de EVA®, y la valoración de empresas. WACC obtenido por la fórmula general conduce a errores significativos en el valor presente neto de los cálculos de la firma, particularmente en las que se aplican series perpetuas de flujo de caja. El presente documento identifica el problema, y ofrece alternativas, y las fórmulas precisas para obtener el WACC para el cálculo de valuación de empresas.

Palabras clave: Promedio Ponderado de Costo de Capital, WACC, valoración de empresas, el presupuesto de capital, costo del capital propio.

Resumo

Llano-Ferro (2009) propõe uma solução para evitar "erros significativos" quando o Custo Médio Ponderado de Capital (WACC) "obtido pela fórmula geral leva a erros significativos no valor presente neto dos cálculos da empresa, particularmente naquelas que se aplicam ao perpétuo fluxo de efetivo série". Neste trabalho se mostra que não existem "erros significativos", mas sim um mau uso da fórmula e o cálculo incorreto dos valores.

O Custo Médio Ponderado de Capital (WACC) se utiliza no financiamento de várias aplicações, incluindo a análise do orçamento de capital, os cálculos de EVA® e a valorização de empresas. O WACC obtido pela fórmula geral leva a erros significativos no valor presente neto dos cálculos da firma, particularmente nas que se aplicam séries perpétuas de fluxo de caixa. O presente documento identifica o problema e oferece alternativas e as fórmulas exatas para obter o WACC para o cálculo de valorização de empresas.

Palavras-chave: Média Ponderada de Custo de Capital, WACC, valorização de empresas, o orçamento de capital, custo do capital próprio.

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Introduction

Apart from the fact that the author calls Net Present Value to the present value of future cash flows (the value of the debt, equity or the firm) Llano-Ferro (2009) proposes a strange definition of the Weighted Average Cost of Capital, WACC. The typical textbook definition of WACC is

$$WACC_t = Kd\times(1-T)\times D_{t-1}/V_{t-1} + Ke_t\times E_{t-1}/V_{t-1} \quad (1)$$

where WACC is the weighted average cost of capital, Kd is the cost of debt, T is the tax rate, D_{t-1} is the debt value at t-1, Ke_t is the cost of levered equity, E_{t-1} is the market value of equity, and V_{t-1} is the market value of the firm.. This is a standard formulation "and it is known by heart by teachers, students and practitioners around the world" (Llano-Ferro (2009, p. 1). See any typical corporate finance textbook, for instance, Brealey et al. 2006, cited by the author.

In contrast, Llano-Ferro (2009) says (his equation (1))

$$WACC_1 = E\times i_E + D\times(1-T)\times i_D \;/\; E+D \quad (2)$$

Where: "**E = Annual Free Cash Flow to Equity**, i_E = Annual cost of equity, **D = Annual interest payments (before taxes)**, i_D = Annual cost of debt, T = tax rate"

Note the difference on the elements that weigh the cost of debt and equity. What weighs the cost of debt and cost of equity is not the cash flows, but the market value of debt and equity. This is a fundamental and conceptual mistake.

An interpretation is that the paper by Llano-Ferro (2009) has some typos and instead of meaning **E = Annual Free Cash Flow to Equity,** he meant **E = market value of equity etc.**

Llano-Ferro (2009) claims that traditional textbook formula for WACC is inconsistent and wrong and proposes two alternate formulations that according to the paper are correct and consistent.

Examining the Proposed Example with a Correct Approach

Llano-Ferro (2009) in section III, Numerical Example of Difference of the Firm Calculations, illustrates his argument with an example. Not discussing where the Ke = 12% came from and not discussing what the discount rate for the tax savings DRTS is, let us rework the example proposed by the author.

The example is a perpetuity with the following input data

CFE	100
CFD = Kd×Debt	30
Ke = i_E	12%
Kd = i_D	6%
T	40%

Table 1. Input data for example

CFE is the cash flow to equity (E in Llano-Ferro (2009)), CFD is the cash flow to debt, assuming constant debt on perpetuity and hence, CFD is only interest charges, (D in Llano-Ferro (2009)), Ke is the cost of equity, Kd is the cost of debt and T is corporate tax rate.

The value of CFE, Equity, in perpetuity is

$$Equity = CFE / Ke \quad (3)$$

this is, 100/0.12 = 833.33.

The value of debt in perpetuity is

$$Debt = CFD / Kd \quad (4)$$

this is 30/0.06 = 500

The total value of the firm is

$$V = Debt + Equity \quad (5)$$

This is 500 + 833.33 = 1,333.33.

Given Ke and Kd and the two values of Debt and Equity, we can calculate the cost of unlevered equity, Ku as

$$Kd\times Debt/V + Ke\times Equity / V \quad (6)$$

and in this case Ku is 6.00%×500/1,333.33 + 12%×833.33/1,333.33 = 9.75%.

The Capital Cash Flow CCF, is

$$CCF = CFD + CFE \quad (7)$$

This is CCF = 100 + 30 = 130

and the value of the firm with the CCF assuming that the discount rate for the tax savings DRTS is Ku, can be calculated as

$$V = CCF/Ku \quad (8)$$

This is, 130/9.75% = 1,333.33, which is identical to the value calculated as Debt + Equity. This means that the implicit assumption in Ke is that DRTS is Ku, the unlevered cost of equity(see Velez-Pareja and Burbano, (2008) and Tham and Velez-Pareja (2002 and 2004)).

According to Modigliani and Miller (1958)

$$FCF + TS = CFD + CFE \quad (9a)$$

and hence

$$FCF = CFD + CFE - TS \quad (9b)$$

Where TS is tax savings.

Also,

$$TS = Kd \times Debt \times T \quad (10)$$

This is 6.00% × 500 × 40% = 12.

Hence FCF from 9b is 100 + 30 – 12 = 118

WACC according to (1) above, is

WACC = 6.00%×(1-40%)×500/1,333.33 + 12%× 833.33/1,333.33 = 8.85%

Firm value V is

$$V = FCF/WACC \quad (11)$$

This is, 118/8.85% = 1,333.33 which is identical to the two previous values calculated above.

Using the APV and assuming DRTS is Ku

$$APV = PV(FCF \text{ at } Ku) + PV(TS \text{ at } DRTS) \quad (12a)$$

Where DRTS is the discount rate for the TS.

$$V = FCF/Ku + TS/Ku \quad (12b)$$

This is, using the numbers from the example,

118/9.75% + 12/9.75% = 1,210.25641 + 123.0769231 = 1,333.33

This means that again, the DRTS implicit in the example is Ku, the cost of unlevered equity, as concluded above. As can be seen when using the proper formulation for WACC and for FCF the four methods coincide and are identical.

If the DRTS is Ku, then the expression for Ke in perpetuity is

$$Ke = Ku + (Ku - Kd) \times D_{t-1} / E_{t-1} \quad (13)$$

This is,

Ke = 9.75% + (9.75% - 6.00%)×500/833.33 = 12%

This result is consistent with the assumed Ke = 12% in the example.

In this section we have shown that the traditional textbook formula for WACC[1] is correct and gives consistent results when calculations are properly and correctly done.

Replicating Results from the Proposed Example

Llano-Ferro (2009) calculates debt value as

Debt = Interest payment × (1-T)/Kd = D×(1-T) = D - D×T (14)

This is

Debt = 30×60%/6% = 300.

According to Modigliani and Miller (1958) the value of the tax savings in perpetuity is D×T when DRTS is Kd, the cost of debt. What the author has calculated is a measure of "after tax" value of debt in perpetuity when we assume DRTS equal to Kd. However, it has to be said that the value of debt is the value of debt for the debt owner. For the firm we find the value of tax shields. It makes no sense to subtract the value of the tax savings (that is earned by the firm for the shareholder) from the value of debt for the debt holder. This is equivalent to subtract pears from apples. In addition, we have to remind that we calculated that the implicit DRTS was Ku, hence, this is an inconsistency.

Llano-Ferro (2009) calculates equity value using (3), this is, Equity is 833.33.

According to this Llano-Ferro (2009) considers that the correct value for the firm is

V = Debt + Equity = 300 + 833.33 = 1,133.33

We assume that the paper has another typo because in the example the author uses a FCF equal to 112. In addition, it is not clear where the author obtained a WACC of 11.08%[2]. However, when we calculate

1. There are some restrictions to the use of the traditional textbook formula for WACC, but it is beyond the scope of this Reply.

2. At page 2, author says: "The WACC for our example, obtained by formula (1), is 0.1108. If we use this WACC to obtain the Net Present Value of the Firm, with formula (2), with the corresponding cash flows for the firm, the result is NPVF =112/0.1108 = $1065"

112/11.08% we do not get $1065 but 1010.83. However, when we calculate 118/11.08% we find 1,065. Using his equation (1) in Llano-Ferro (2009), we obtain a WACC of 10.06% and the V based on the FCF would be 1,172.78 (118/10.06%) instead of 1,065 as claimed by the author. This does not match with the 1,133.33 proposed by the author neither with the announced 1,065 mentioned in page 2.

Note that the author is destroying a straw man because at the start it is clear that the formulation for the traditional textbook formula is wrong. The correct one that can be verified in any corporate finance book is (1).

For this discrepancy the author proposes the following formulation for WACC

$$WACC_2 = (i_D \times i_E\ [D \times (1-T)+E]\ /\ i_E \times D \times (1-T)+i_D \times E \qquad (15)$$

Calculating $WACC_2$ we find 10.41% and value is calculated with (11) and V is V=118/10.41% = 1,133.33

This matches with his "correct" value calculated above.

When we use **his** equation ((1) in his paper, (2) in this Reply) for WACC and assuming that the definition of D and E in (2) is a typo, this is, interpreting D and E not as said by Llano-Ferro (2009), "Annual Free Cash Flow to Equity" and "Annual interest payments (before taxes)" but as values (the values of 833.33 for equity and 300 for debt, wrong value as said above) we obtain a WACC of 9.78% and a value of 1,206.98 and this does not match with his value of 1,133.33.

However, when calculating the after tax value of debt, the author assumes DRTS is Kd. In that case the formulation to be used is

$$Ke = Ku + (Ku-Kd) \times (1-T) \times D_{t-1}/E_{t-1} \qquad (14)$$

Ku is

Ku = 300×6.00% + 833.33×12.00% = 10.41%

And Ke is

Ke = 10.41%+ (10.41%- 6%)×(1-40%)×300/833.33 = 11.36%

And this is not consistent with the initial input data where Ke is 12%.

If we assume that DRTS is Kd as it is implied from the calculation of after tax value of debt and an interest charge as indicated in the example, then APV should be based on a TS of 12 (500×6%×40%). In that case,

APV = 118/10.41% + 12/6.00% = 1,133.33 + 120 = 1,253.33

Note that the unlevered value (FCF/Ku) is exactly the value V calculated by Llano-Ferro (2009) and obviously, the APV does not match with the value calculated by the author. This means that in any case, assuming that V calculated by the author is correct (1,133.33), APV overestimates value and does not match with the "correct" value proposed by the author (1,133.33).

Finally, the author presents a new $WACC_3$(equation (7) in his paper) as

$$WACC_3 = \frac{1}{n} Ln\left[\frac{\varepsilon^{n \times i_E} \times \varepsilon^{n \times i_D}\,(CFE_j + CFD_j \times (1-T))}{\varepsilon^{n \times i_D} \times CFE_j + \varepsilon^{n \times i_E} \times CFD_j \times (1-T)}\right]$$

Where [CFEj] "is the Free Cash Flow to Equity in period j, and [CFDj] is the Free Cash Flow to Debt, before taxes, in period j" and ε is the continuous interest for Kd and Ke.

The interesting thing about this equation is that the author starts deducing it for non growing perpetuities and suddenly, this simplified and compact equation is suitable for a WACC that "is not constant. It varies from period to period. It decreases exponentially as a function of time" Llano-Ferro (2009, p. 3). It is hard to believe that such an equation could be of any practical application in calculating a perpetuity.

Concluding Remarks

Concluding, there are several mistakes and inconsistencies:

1. The author departs from a wrong calculation of market value of debt (Using an after tax cash flow to debt, CFD, discounted with a before tax cost of debt).
2. The author consider the value of Debt as the net between the present value (net present value, says the author) and the value of tax savings assuming that DRTS is Kd. The value of debt is for the debt holder and the value of TS is for the equity holder. They cannot be subtracted.
3. The author constructs a straw man to argument against, namely the traditional, popular and well known formulation for WACC. Equation (1) in this work is the standard definition of WACC and equation (2) (this is equation (1) in author's paper) is the "new", creative definition of WACC. This alternative formulation is strange and is wrong.
4. The author apparently picks from the thin air a value for WACC calculated with equation (1)

of this work (11.08% that is not clear where it comes from and how it is calculated).

5. The author is inconsistent because in some places of his argument he makes some assumption (implicitly) regarding the DRTS. In some instances DRTS is Ku and in others Kd.
6. The last version of creative WACC ($WACC_3$) is inconsistent. He starts explaining the formula for non growing perpetuities and ends with a FCF and WACC that change every period.

This paper is an anthological example of a conundrum. This means that Llano-Ferro (2009) has concluded the correctness of his formulation based on a wrong formula, the matching of some numbers, but this includes wrong values of debt. In the way of calculating the numbers for his example, he mixes assumptions regarding the DRTS and finds wrong values that match and hence, claims consistency and correctness of his proposal. Consistency with some methods is not enough to declare the correctness of a formulation. Consistency should be found with all methods and we have shown that the procedure used by Llano-Ferro (2009) is not consistent with APV. The formulation and the calculations have to be conceptually correct and consistent.

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