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# Mathematical formulation of restrictions for the design of low voltage bandgap references

## Formulación matemática de restricciones para el diseño de referencias de banda prohibida de bajo voltaje

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### Abstract

Nowadays the qualities more looked for in the design of bandgap references are the low potential of the reference and the low value of its coefficient of temperature, being these parameters more favorable in curvature corrected bandgap references.

The design parameters of bandgap references depend on the value of the potential of reference $V_{REF}$; nevertheless, in the reported designs there are not paid attention to conditions that limit the attainable value of $V_{REF}$.

In this work the mathematical formulation of the restrictions that really limit the attainable value of the reference potential is presented in order that the design of a curvature corrected bandgap reference can be realizable.

----- *Keywords:* curvature correction, low temperature coefficient, low voltage bandgap references

### Resumen

Actualmente las cualidades más buscadas de las referencias bandgap son el bajo potencial de referencia y el bajo valor del coeficiente de temperatura, siendo estos parámetros mas favorables en las referencias de bandgap con corrección de curvatura. Por otra parte y aunque claramente los parámetros de diseño dependen del valor del potencial de referencia $V_{REF}$. Habitualmente en los diseños reportados no se analizan las condiciones que limitan el valor de $V_{REF}$ alcanzable.

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En este trabajo se presenta la formulación matemática de las restricciones que realmente limitan el valor alcanzable del potencial de referencia para que el diseño de una fuente de referencia de bandgap con corrección de curvatura sea realizable.

----- *Palabras clave:* bajo coeficiente de temperatura, corrección de curvatura, referencia bandgap de bajo potencial

## Introduction

Bandgap voltage references are frequently used in analog circuits as voltage regulators and as such in A/D and D/A converters [1]. At present the qualities more looked for in the design of bandgap references are the low values of voltage and temperature coefficient [2,3].

Although the dependencies of design parameters on the wished value of reference voltage $V_{REF}$ are known, in the designs reported in the literature [3,4] the attainable value of $V_{REF}$ is rarely analyzed or its importance is neglected.

In this work the mathematical formulation of the restrictions that really limit the attainable $V_{REF}$ value and make a design of a curvature corrected bandgap reference realizable, are obtained. This value is verified experimentally through the design and measurement of bandgap type circuits, based on the sum of two base-emitter voltages.

### *Analysis of design equations*

The block diagram of a low voltage bandgap reference, based on the sum of two base-emitter voltages is shown in figure 1 [2].
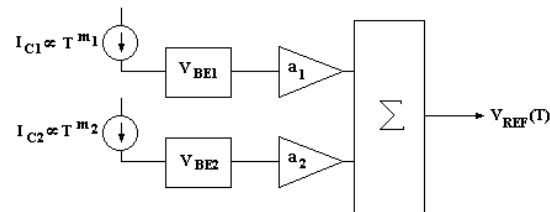


**Figure 1** Block diagram of a bandgap reference source with collector current $I_C \mu T^m$

$V_{BE1}$ and $V_{BE2}$ stand for the expressions of the base-emitter voltages of the transistors and m1

and m2 determine the bias current temperature dependences [2,4,5].

The expression of $V_{REF}(T)$ for the circuit of figure 1 is given by equation (1):

$$V_{REF}(T) = a_1 V_{BE1}(T) + a_2 V_{BE2}(T) \quad (1)$$

Using the linear approximation of bandgap voltage $V_G(T)$ [4,6,7], the expressions of $V_{BE1}(T)$ and $V_{BE2}(T)$ can be written via equations (2) and (3):

$$V_{BE1}(T)$$
$$= V_{G0}(1 - \frac{T}{T_r}) + \frac{T}{T_r}V_{BE1}(T_r) - (\eta - m_1)\frac{k}{q}(T \ln \frac{T}{T_r}) \quad (2)$$

$$V_{BE2}(T)$$
$$= V_{G0}(1 - \frac{T}{T_r}) + \frac{T}{T_r}V_{BE2}(T_r) - (\eta - m_2)\frac{k}{q}(T \ln \frac{T}{T_r}) \quad (3)$$

where $V_{G0}$ is the bandgap voltage extrapolated at zero Kelvin and $\eta$ is a parameter related to the temperature dependence of the mobility of minority carriers in the base region [8]. Their values are obtained from experimental measurements of $V_{BE2}(T_r)$ [4,8].

For the design of bandgap reference the parameters a1, a2, and VBE1(Tr) of expression (1) should be determined, taking into account the known values $m_1$, $m_2$ and $V_{BE2}(T_r)$.

In order to reduce the temperature dependence of the reference voltage, that is $V_{REF}(T)=V_{REF,}$ the Taylor criterion will be applied.

Using Taylor criterion at $T = T_r$, all possible derivates are zero around $T_r$, so the equations (4), (5) and (6) result:

$$V_{REF}(T_r) = V_{REF} \qquad (4)$$

$$V'_{REF}(T_r) = 0 \qquad (5)$$

$$V''_{REF}(T_r) = 0 \qquad (6)$$

Substituting (2) and (3) in (1), the expression (7) is obtained:

$$V_{REF}(T) = V_{G0}(1 - \tfrac{T}{T_r})(a_1 + a_2)$$
$$+ \tfrac{T}{T_r}[a_1 V_{BE1}(T_r) + a_2 V_{BE2}(T_r)] \qquad (7)$$
$$- [a_1(\eta - m_1) + a_2(\eta - m_2)]\tfrac{k}{q} T \, ln \tfrac{T}{T_r}$$

From derivation of (7), equations (8) and (9) result:

$$V'_{REF}(T) = a_1 \left[ -\frac{V_{G0}}{T_r} + \frac{V_{BE1}(T_r)}{T_r} - (\eta - m_1)\frac{k}{q} ln\frac{T}{T_r} - (\eta - m_1)\frac{k}{q} \right]$$
$$+ a_2 \left[ -\frac{V_{G0}}{T_r} + \frac{V_{BE2}(T_r)}{T_r} - (\eta - m_2)\frac{k}{q} ln\frac{T}{T_r} - (\eta - m_2)\frac{k}{q} \right] \qquad (8)$$

$$V''_{REF}(T) = -[a_1(\eta - m_1) + a_2(\eta - m_2)]\tfrac{k}{q}\tfrac{1}{T} \qquad (9)$$

The use of condition (6) in (9) allows obtaining equation (10):

$$a_1(\eta - m_1) + a_2(\eta - m_2) = 0 \qquad (10)$$

This condition is valid for all the temperature range.

Considering (10) in (7), equation (11) results:

$$V_{REF}(T)$$
$$= V_{G0}(1 - \tfrac{T}{T_r})(a_1 + a_2) + \tfrac{T}{T_r}[a_1 V_{BE1}(T_r) \qquad (11)$$
$$+ a_2 V_{BE2}(T_r)]$$

From where the equation (12) is obtained:

$$a_1 + a_2 = \frac{V_{REF}}{V_{G0}} \qquad (12)$$

Expressions (1), (10) and (12) form an equation system which allows to obtain the parameters $a_1$, $a_2$ and $V_{BE1}(T_r)$ as written in (13), (14) and (15):
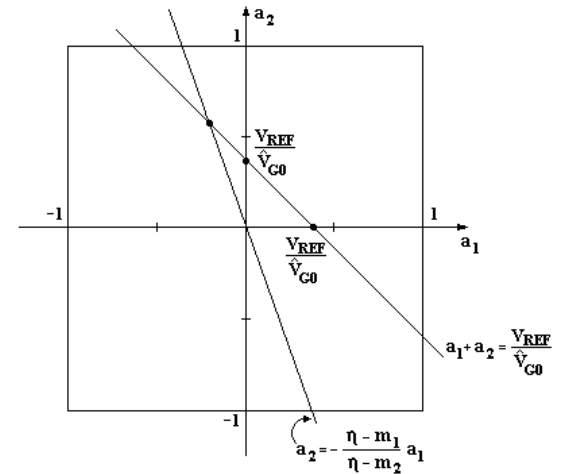
$$a_1 = \left( \frac{\eta - m_2}{m_1 - m_2} \right) \frac{V_{REF}}{V_{G0}} \qquad (13)$$

$$a_2 = \left( \frac{\eta - m_1}{m_1 - m_2} \right) \frac{V_{REF}}{V_{G0}} \qquad (14)$$

$$V_{BE1}(T_r) = \tfrac{1}{a_1}(V_{REF} - a_2 V_{BE2}(T_r)) \qquad (15)$$

### Analysis of the design restrictions

Equations (13) and (14) allow a convenient visualization of the design conditions when they are represented as lines in a1, a2 planes, as shown in figure 2 The solution for the design will be given by the intersection of both lines.



**Figure 2** Geometrical representation of equations (13) and (14)

Since only the interval [-1, 1] is admitted for $a_1$ and $a_2$ values, the design conditions given by (16), (17) and (18) have to be satisfy:

$$\frac{V_{REF}}{V_{G0}} \langle 1 \Rightarrow V_{REF} \langle V_{G0} \qquad (16)$$

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$$\left|\frac{\eta - m_2}{m_1 - m_2}\frac{V_{REF}}{V_{G0}}\right|\langle 1 \Rightarrow \frac{V_{REF}}{V_{G0}}\langle\left|\frac{m_1 - m_2}{\eta - m_2}\right| \quad (17)$$

$$\left|\frac{\eta - m_1}{m_1 - m_2}\frac{V_{REF}}{V_{G0}}\right|\langle 1 \Rightarrow \frac{V_{REF}}{V_{G0}}\langle\left|\frac{m_1 - m_2}{\eta - m_1}\right| \quad (18)$$

For this reason $V_{REF}$ has to be less than the minor of the three equations given by (16), (17) and (18). This means that for a determined design, where $m_1$ and $m_2$ values are given by the temperature dependence of the bias currents, and the values of h and $V_{G0}$ depend on the transistor used, $V_{REF}$ can not be greater than certain value that here will be denominated $V_{REFMAX}$.

### *Use of the design restrictions in a practical case*

Criterion expressed in equations (16), (17) and (18)) was assessed experimentally through the design and measurements of the bandgap type circuit shown in figure 3, using constants (m1=0) and IPTAT (m2=1) bias currents.
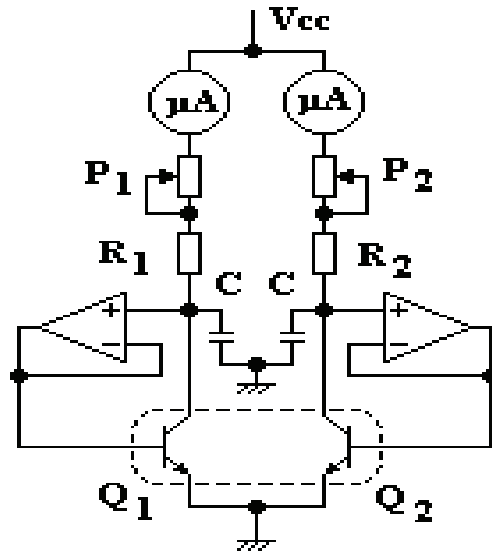


**Figure 3** Bandgap type circuit where Q1 and Q2 are MAT01 transistors

A mathematical program was developed as a tool in order to:

Calculate the values of η and $V_{G0}$ for the transistors MAT01 used (based in experimental measurements of $V_{BE}$ at three different temperatures) and $V_{REFMAX}$, for $m_1=0$ and $m_2=1$.

Calculate the design parameters $a_1$, $a_2$ and $V_{BE1}(Tr)$ for the wished $V_{REF}$.

The requirements needed in the measurement of temperature, potential and current to obtain a mistake less than 0.25 ppm/°C in the range 20 to 100 °C were found by means of analysis of sensibility as:

a) Temperature: Resolution better than 0.01 °C in the work range

b) Potential: Resolution better than 1 μV

c) The stability of the potentials $V_{BE1}(T)$ and $V_{BE2}(T)$ better than 0.2 μV respectively

For the measurement the following instruments were used:

- Astandard platinum resistor PT1000 DIN EN60751 as temperature sensor. The resistance of the PT1000 was measured with a Keithley digital electrometer model 614.

- Base-emitter voltages $V_{BE1}(T)$ and $V_{BE2}(T)$ were measured with the Keithley nanovoltmeter model 2182 with 100 nV resolution in a 1 V scale.

For the used MAT01 transistors the following values were obtained: η=5.88818419, $V_{G0}$=1.08109259 V and $V_{REFMAX}$=0.18360373 V. These results were used to calculate the design parameters in two cases (The screen shots of the mathematical program are shown in figures 4 and 5 respectively) :

a.) $V_{REF}$=160 mV < $V_{REFMAX}$ (figure 4)

b.) $V_{REF}$=200 mV > $V_{REFmax}$ (figure 5)

In screen shot of figure 5 the value of $a_1 > 1$ can be observed. This value is out of [-1,1] interval (see figure 2) affecting the $a_2$ value too in equation

(12). , confirming that a design for $V_{REF} > 0.18360$ V $= V_{REFMAX}$ is not possible.



**Figure 4** Screen shot of the mathematical program for $V_{REF} = 160$ mV $< V_{REFMAX}$

The reference voltages were calculated in both cases, using in equation (1) the experimentally measured values of $V_{BE1}(T)$ and $V_{BE2}(T)$ together with the values $a_1$ and $a_2$ obtained by the mathematical program. The experimental measurements and calculated $V_{REF}$ values are shown in table 1.



**Figure 5** Screen shot of the mathematical program for $V_{REF} = 200$ mV $> V_{REFMAX}$

**Table 1** Experimental data and $V_{REF}$ obtained

| Temp. ($^oC$) | $V_{BE2}$ (V) | $V_{BE1}$ (V) | $V_{REF} < V_{REFMAX}$ 160 (mV) ($a_1$=0.90694819 $a_2$=-0.81708358) | $V_{REF} > V_{REFMAX}$ 200 (mV) ($a_1$=1.08930248 $a_2$=-0.90430445) |
|---|---|---|---|---|
| 20.657 | 0.563238 | 0.651186 | 160.00043 | 130.37998 |
| 33.996 | 0.532382 | 0.625571 | 159.99993 | 132.35925 |
| 40.579 | 0.517263 | 0.613343 | 159.99936 | 133.62281 |
| 49.781 | 0.496143 | 0.595485 | 159.99999 | 134.68400 |
| 62.274 | 0.467055 | 0.571339 | 160.00067 | 136.55175 |
| 76.523 | 0.433315 | 0.543328 | 160.00066 | 138.71659 |
| 83.186 | 0.417628 | 0.530306 | 160.00050 | 139.72294 |
| 91.005 | 0.399010 | 0.514849 | 159.99970 | 140.91671 |
| 99.888 | 0.378010 | 0.497117 | 160.00045 | 141.99338 |

In table 1 the values of $V_{REF}$ were obtained using equation (1). For $V_{REF} > V_{REFMAX}$ case, $V_{REF}$ values result far away from the design one (200 mV) due to incorrect $a_1$ and $a_2$ values, showing that a target design value greater than $V_{REFMAX}$ is not attainable. Besides their increase with temperature invalids its use as a reference.

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In figures 6 and 7 the variations of $V_{REF}$ with temperature are shown for both cases.

For $V_{REF} > V_{REFMAX}$ case, $V_{REF}$ values result far away from the design one (200 mV) due to incorrect $a_1$ and $a_2$ values.
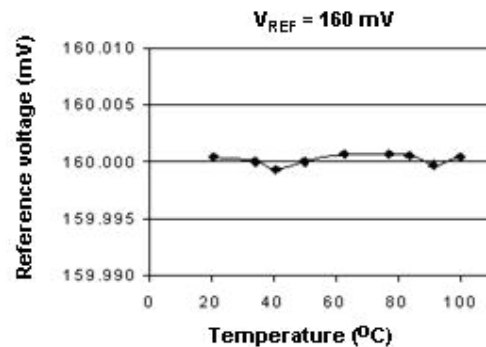


**Figure 6** Variation of $V_{REF}(T)$ with temperature for $V_{REF} = 160$ mV $< V_{REFMAX}$
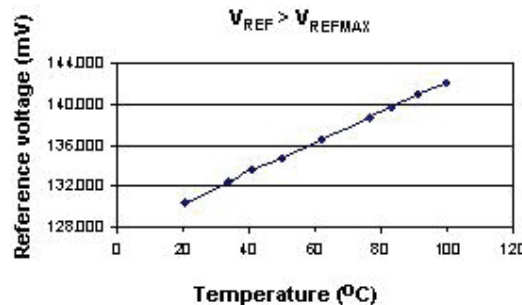


**Figure 7** Variation of $V_{REF}(T)$ with temperature for $V_{REF} = 200$ mV $> V_{REFMAX}$

The reference voltage in figure 6, designed to 160 mV has an average temperature coefficient of 0.101 ppm/°C, while the one of the figure 7, designed to 200 mV shows an increase with temperature that invalids its use as a reference.

## Conclusions

The analysis and the visualization of the design equations of a bandgap voltage reference based on the sum of two base-emitter voltages lead to obtain three equations that really limit the attainable value of the reference voltage $V_{REF}$.

This means that $V_{REF}$ has to be less than the minor of the three quotas and can not overpass the value of $V_{REFMAX}$.

This result was assessed experimentally through the design and measurement of a bandgap type circuit. The results show that the reference voltage designed to 160 mV $< V_{REFMAX}$ has an average temperature coefficient of 0.101 ppm/°C, while that designed to 200 mV $> V_{REFMAX}$ results far away from the design one, increasing with temperature. This behavior invalids its use as a reference. This fact confirms the importance of knowing restrictions that really limit the attainable value of reference voltages, aspect not studied up to now.

## References

1. Analog Devices Inc. *Data Converter Refernce Manual* Vol. 1. 1992. pp. 2-1141.

2. A. Van Staveren. *Structures electronic design of high-performance low-voltage low-power references*. PH.D. Thesis. Delft University of Technology. The Netherlands. 1997. pp. 155-180.

3. A. Pletersek. "A Compensated Bandgap Voltage Reference with Sub-1-V Supply Voltage". *Analog Integrated Circuits and Signal Processing*. Vol. 44. 2005. pp. 5-15.

4. G. C. M. Meijer, P. C Schmale, K. Van Zalinge. "A new curvature-corrected bandgap reference". *IEEE Journal of Solid-State Circuits*. Vol. 17. 1982. pp. 1139-1143.

5. C. Popa. "Optimal superior-order curvature-corrected voltage reference based on the weight difference of gate-source voltages". *Analog Integrated Circuits and Signal Processing*. Vol. 54. 2008. pp. 1-6.

6. A. Polanco, A. Nagy. M. Alvarez. "Physical design to verify theoretical 0.1 ppm/°C stability in a bandgap type circuit". *Energía y Computación*. Vol. 13. 2005. pp. 29-32.

7. R. Amador, A. Polanco, A. Nagy, M. Alvarez. "Accurate design of low-voltage curvature-corrected bandgap references". DCIS´2000. XV Design of Circuits and Integrated Systems Conferences. Le Corum. Montpellier. France. Nov. 21-24. 2000.

8. R. Amador, A. Polanco, A. Nagy. "The spread of η and $V_{G0}$ and its influence on the sensitivity of a bipolar I.C Celsius sensor". *Sensors and Actuators*. Vol. 77. 1999. pp. 9-13.

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