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## **Achievable transmission rate in an IEEE 802.11 Manet link**

### **Tasa de transmisión alcanzable en un enlace de una red Manet IEEE 802.11**

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#### **Abstract**

For management purposes, it is very important to estimate the available bandwidth for each link in a MANET, in an accurate, timely and efficient way. In this paper we show analytical results on the probability distribution function of the bandwidth of a link in a MANET based on IEEE 802.11, that take into account transmission errors. We also show some analytical results on the fraction of time the channel is available for a given virtual link, so the effects of other transmitting nodes can also be taken into account. Together, these results can be usefully exploited in an efficient, accurate and distributed available bandwidth estimation mechanism.

----- **Keywords:** MANET, IEEE 802.11, bandwidth, RTS/CTS, packet length dependency, busy period

#### **Resumen**

Para propósitos de gestión, es muy importante estimar el ancho de banda disponible en cada enlace de una red MANET, de una manera precisa, oportuna y eficiente. En este artículo mostramos resultados analíticos sobre la función de distribución de probabilidad del ancho de banda de un enlace en una red MANET basada en IEEE 802.11, los cuales tienen en cuenta

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errores de transmisión. También mostramos algunos resultados analíticos sobre la fracción de tiempo en que el canal está disponible para un enlace virtual dado, de tal manera que los efectos de las transmisiones de otros nodos puedan tenerse también en cuenta. Conjuntamente, estos resultados pueden ser explotados en un mecanismo de estimación de ancho de banda disponible que puede resultar eficiente, preciso y distribuido.

----- *Palabras clave:* MANET, IEEE 802.11, Ancho de banda, RTS/CTS, dependencia de la longitud del paquete, período de ocupación

## Introduction

In a Mobile Ad Hoc NETWORK (MANET), the nodes are connected through wireless links without any communication infrastructure. Since some nodes can be out of range of some other nodes, these networks require a multi-hop communication mechanism. Furthermore, since the nodes are allowed to move randomly, this mechanism must self-organize to the dynamic varying topology [1]. Under these circumstances, the operation of a MANET is a very challenging task, so it becomes important for the nodes within the network to cooperate among them, finding in a distributed manner globally optimal operation conditions [2]. For example, each node should be able to probe the network in order to infer from its measurements the environmental conditions it is facing, such as the achievable transmission rate to each of its one-hop neighbors [3]. In effect, such information could be useful for traffic sources to adjust their transmission rates [4], for ingress nodes to control the admission of new flows [5], for routing algorithms to take optimal routing decisions [6], etc. Unfortunately, an accurate, timely and efficient estimation of such an important parameter has been proved to be highly difficult in the context of MANETS [7]. In this paper we consider three important theoretical results to be applied in the estimation of the unused capacity of a single link in an IEEE 802.11 MANET.

The contribution of this paper is threefold. First, we demonstrate a result on the bandwidth of a link with no errors. This result, which was presented without proof in [3], describes the probability density function of the bandwidth, which is highly dependent on packet length. Second, we extend

that result with an analytical study of the effect of transmission errors, taking into account the details of the medium access protocol. And third, we consider analytically the interaction between local estimations of the utilization factor at both ends of a link. Together, these results could be used in estimating the available bandwidth of a one-hop link and, with appropriate extensions, the available bandwidth of a multi-hop path. Although we do not explore this possibility here, we refer to [7] for some alternatives. Finally, it is worth mentioning that we use the case of 802.11b for numerical examples, although the results are still valid for other versions of the standard, even for 802.11n.

## Related work

Two pioneering theoretical models of capacity in wireless networks are those of Bianchi [8] and Gupta and Kumar [9]. Bianchi computed the saturation throughput of a single DCF IEEE 802.11 cell with a finite number of nodes, under ideal channel conditions. He considers a bi-dimensional Markov chain to model both the back off window size and the number of consecutive collisions of each station, from where he derives the average time it takes a successful transmission among  $n$  saturated stations. Based on a very different approach, an information theoretic one, Gupta and Kumar [9] established some basic limits for the throughput of wireless networks. They introduce the concept of transport capacity as the maximum achievable bandwidth-distance product that a network can support, under the effect of multiple sources. Their asymptotic results assume optimal node positioning, optimal traffic pattern and optimal transmission range of each node. Both seminal works have inspired many

additional developments, but the ones based on [8] are valid only for saturated sources, and the ones based on [9] find asymptotically valid limits under a big number of nodes in the network, ignoring the multiple access overhead, which can be determinant on the true network capacity.

Because of the lack of appropriate theoretical models, most practical estimation methods are based on a highly simplified model: each node should measure the fraction of time it perceives the channel is busy,  $u$ , and assume that it has the possibility to transmit at a rate  $C(1 - u)$ , where  $C$  is the physical transmission capacity [10]. This is a very efficient and timely estimation, but far from accurate [3]. Indeed, even in a point-to-point dedicated link, where the assumption of a fixed capacity  $C$  is reasonable, the utilization  $u$  becomes a highly variable random process that cannot be changed by its average over a given period of time without serious consequences in accuracy and precision, due to the statistical characteristic of modern traffic [11]. However, this simple model has also been applied to MANETS (see [12-14], for example), where it is even worse due to the shared and unreliable nature of the transmission medium. Indeed, each link capacity is an ensemble effect of physical layer random behavior (fading, path loss, capture, etc.), complex CSMA-based MAC layer interactions, effects from multiple active sources, etc. [15].

Some other proposals go through more elaborate inference procedures based on active measurements [16-18], enhancing accuracy at the cost of efficiency or timeliness. However, it is clear that we need simple and accurate theoretical models in order to attain accuracy, timeliness and efficiency in ad hoc wireless networks bandwidth estimation. That is why we propose an analytical model that suggests a simple and accurate estimation based on a distributed estimation between the source and the destination of a link, by sharing the locally observed fraction of busy time of the medium. The assumption is that we can keep the simple  $C(1-u)$  model, but changing both the capacity  $C$  and the utilization  $u$ , with an analytically accurate definition of bandwidth

( $BW$ ) and a distributed estimation of the utilization of the physical channel around the source and destination nodes of the link, respectively. The  $BW$  parameter is characterized not only through its mean, but through its complete probability density function, with and without errors.

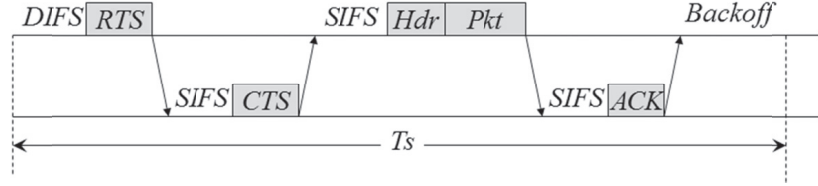
### **Achieved Bandwidth with no errors**

Assume an IEEE 802.11b node wants to send a large number of  $L$ -bits-long packets using a completely available wireless medium, at a bit transmission rate of  $C$  bps. The effective transmission time of a single packet in the RTS/CTS mode, is (see figure 1 and [19]):

$$T_s = \begin{array}{ll} \text{DIFS} & + \dots \text{ The transmitter waits a DIFS} \\ \text{RTS} & + \dots \text{ The transmitter sends a Request to Send} \\ T_p & + \dots \text{ The Request To Send arrives to the receiver} \\ \text{SIFS} & + \dots \text{ The receiver waits a SIFS} \\ \text{CTS} & + \dots \text{ The receiver sends a CTS} \\ T_p & + \dots \text{ The CTS arrives to the transmitter} \\ \text{SIFS} & + \dots \text{ The transmitter waits a SIFS} \\ \text{Hdr} & + \dots \text{ The transmitter sends the PHY and MAC headers} \\ L/C & + \dots \text{ The transmitter sends the frame payload at } C \text{ bps} \\ T_p & + \dots \text{ The frame arrives to the receiver} \\ \text{SIFS} & + \dots \text{ The receiver waits a SIFS} \\ \text{ACK} & + \dots \text{ The receiver sends an ACK} \\ T_p & + \dots \text{ The ACK arrives to the transmitter} \\ T_{backoff} & \dots \text{ The transmitter waits a random backoff before sending the following frame} \end{array}$$

So, the effective transmission time of an  $L$ -bit long packet is

$$T_s = L/C + \text{DIFS} + 3\text{SIFS} + \text{RTS} + \text{CTS} + \text{Hdr} + \text{ACK} + 4T_p + T_{\text{backoff}} \quad (1)$$



**Figure 1** Time diagram for a packet transmission

Similarly, in the basic mode, where the RTS/CTS mechanism is omitted, the effective transmission time is

$$T_s = L/C + \text{DIFS} + \text{SIFS} + \text{Hdr} + \text{ACK} + 2T_p + T_{\text{backoff}} \quad (2)$$

Both expressions take the general form

$$T_s = L/C + T_{\text{oh}} + B_o \sigma \quad (3)$$

where  $L/C$  is the packet length dependent transmission time,  $T_{\text{oh}}$  is a deterministic overhead delay and  $B_o \sigma$  is the additional random backoff time during which the node is still “busy”, where  $\sigma$  is the contention slot and  $B_o$  is a discrete random variable uniformly distributed in the closed interval  $[0, W-1]$ , where  $W$  is the minimum backoff window.

In IEEE 802.11b, DIFS is 50  $\mu\text{s}$ , SIFS is 10  $\mu\text{s}$ ,  $\sigma$  is 20  $\mu\text{s}$ ,  $W$  is 32 and the propagation time is less than 1  $\mu\text{s}$  (since the distances between neighbors are usually less than 300 m). Because of rate adaptation, every card support a basic rate set, and all control frames (RTS, CTS and ACK) must be sent at one of the rates within the basic rate. Indeed, the PLCP preamble and header of every frame must be sent at 1 Mbps, but these fields themselves can be long (192 bits) or short (96 bits). Similarly,

the control frames are usually sent at 1 Mbps or 2 Mbps, while the data frames are transmitted at the rate selected by the rate control system. These differences in the transmission rate between control and data information affect the evaluation of Equation (3). In this paper, we assume that the control frames are sent at the same rate of the data frames, which is the assumption made by the selected simulation tool, Qualnet® [20]. So, the deterministic overhead delay is  $T_{\text{oh}} = T_0 + L_0/C$ , where  $T_0$  is a constant delay (propagation time, control timers, and PLCP transmission times), and  $L_0$  is the length of the overhead control information transmitted at the data rate.

According to the discussion above, in RTS/CTS mode (in which we are going to concentrate from now on), the time to acquire and release the transmission medium is  $T = T_0 + L_0/C + B_o \sigma$ . If  $T$  is approximated as a continuous random variable uniformly distributed in  $[T_0 + L_0/C, T_0 + L_0/C + (W - 1)\sigma]$ , the following distribution for the link bandwidth,  $BW(L)$ , can be obtained [3]:

$$f_{BW(L)}(b) = \begin{cases} \frac{L}{b^2 \sigma (W-1)} & b \in I_b \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

$$\text{where } I_b = \left[ \frac{CL}{L + L_0 + C(T_0 + \sigma(W - 1))}, \frac{CL}{L + L_0 + CT_0} \right]$$

Effectively,  $BW(L)$  becomes a function of the random variable  $T$ ,

$$g(T) = \frac{L}{\frac{L}{c} + T} \quad (5)$$

as shown in figure 2. Since  $g(T)$  is a monotonically decreasing function of  $T$ , for  $T \geq 0$ , for each realization of  $T$ ,  $t$ , there is a unique realization of  $BW(L)$ ,  $b=g(t)$ . For a negative increment on  $t$ ,  $\Delta t < 0$ , it can be readily notice that

$$\Pr[b < BW(L) \leq b + \Delta b] = \Pr[t + \Delta t \leq T < t] \quad (6)$$

which, for a small value of  $|\Delta t|$ , can be interpreted in terms of the corresponding probability density functions (*pdf*),

$$f_{BW(L)}(b)\Delta b \approx f_T(t + \Delta t)|\Delta t| \quad (7)$$

where the equality becomes exact in the limit when  $\Delta t \rightarrow 0$ . Dividing by  $\Delta b$  and taking the limit,

$$f_{BW(L)}(B) = \lim_{|\Delta t| \rightarrow 0} \frac{1}{|\frac{\Delta b}{\Delta t}|} f_T(t + \Delta t) = \frac{1}{|g'(t)|} f_T(t) \quad (8)$$

where, from (5),

$$t = g^{-1}(b) = L \left( \frac{1}{b} - \frac{1}{c} \right) \quad (9)$$

and

$$|g'(t)| = \frac{L}{\left(\frac{L}{c} + t\right)^2} = \frac{b^2}{L} \quad (10)$$

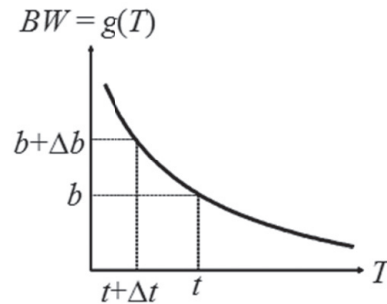
Replacing (9) and (10) in (8) leads to

$$f_{BW(L)}(b) = \frac{L}{b^2} f_T \left( L \cdot \left( \frac{1}{b} - \frac{1}{c} \right) \right) \quad (11)$$

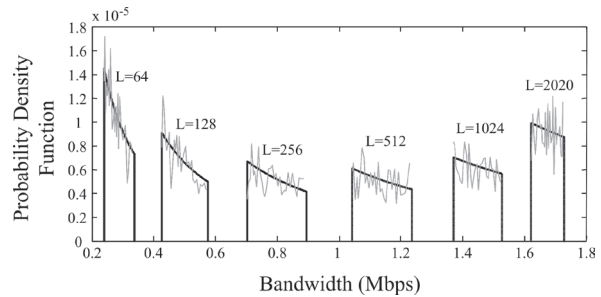
When  $T$  is uniformly distributed in the interval  $[T_0 + L_0/C, T_0 + L_0/C + (W - 1)\sigma]$ , Equation (11) becomes Equation (4).

The *pdf* expressed in equation (4) is shown in figure 3 for a 2 Mbps link and different packet lengths, along with the corresponding histogram

based *pdf* estimations, obtained from Qualnet® [20] simulations. The range of available bandwidths for each packet length and the corresponding normalized relative frequencies validate our theoretical results.



**Figure 2** The random variable  $BW$  is a function  $g(T)$  of the random variable  $T$  ( $\Delta t < 0$ )



**Figure 3** Bandwidth distribution of a 2 Mbps IEEE 802.11b link (from [7])

By direct integration, the average link bandwidth becomes

$$E[BW^{link}(L)] = \frac{L}{(W-1)\sigma} \log \left( 1 + \frac{c(W-1)\sigma}{cT_0 + L + L_0} \right) \quad (12)$$

which can be well approximated as

$$E[BW^{link}(L)] \approx \frac{L}{\frac{L_0}{c} + \left( \frac{L_0}{c} + T_0 + \frac{W-1}{2}\sigma \right)} \quad (13)$$

In effect, the Taylor series of both  $\log(1+x)$  and  $x/(1+x/2)$  is  $x(1-x/2) + o(x^3)$ , so both functions tend to be equal as  $x$  gets smaller,



$$\log(1+x) \approx \frac{x}{1+\frac{x}{2}} \approx x \left(1 - \frac{x}{2}\right) \text{ as } x \rightarrow 0 \quad (14)$$

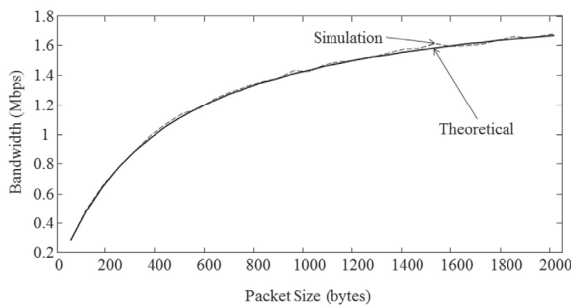
Replacing  $x$  with  $C(W-1)\sigma/(L+L_0+CT_0)$  and multiplying by  $L/(\sigma(W-1))$  the approximation from Equation (12) to Equation (13) is obtained.

With Qualnet® [20] it is possible to measure (i) the time at which the  $n^{\text{th}}$  packet is moved from the queue to the transmission buffer (or the time at

which a new packet arrives from the network layer to the transmission buffer if the queue is empty),  $T_M(n)$ , (ii) the time at which the corresponding ACK is received,  $T_A(n)$ , and (iii) the backoff timer established at  $T_A(n)$ ,  $B_O(n)$ . The timer will expire at  $T_B(n) = T_A(n) + B_O(n)$ . However, if the  $n^{\text{th}}$  packet was waiting in the queue at  $T_A(n-1)$ , it is moved to the transmission buffer at  $T_A(n-1)$  and not at  $T_B(n-1)$ . Consequently, the complete “service time” of the  $n^{\text{th}}$  packet should be computed in Qualnet® [20] as

$$T_s(n) = T_A(n) + B_O(n) - \max(T_M(n), T_A(n-1) + B_O(n-1)) \quad (15)$$

which will give the  $n^{\text{th}}$  bandwidth measurement,  $BW_n(L_n) = L_n/T_s(n)$ , where  $L_n$  is the length of the  $n^{\text{th}}$  packet. Figure 4 compares simulation results with Equation (13). 32 groups of 20 equal-length packets are transmitted, each group with a fixed size that ranges from 64 bytes to 2048 bytes in steps of 64 bytes. The dotted line of figure 4 shows the average rate of each group and the thick continuous line shows the theoretical result of Equation (13). The nodes were located very close to each other to ensure there were no transmission errors. Clearly, the simulation results validate the theoretical expression, since the difference is small even for a very small number of samples during the simulation.



**Figure 4** Simulation and theoretical results of the mean total bandwidth perceived by individual packets

### **Achieved Bandwidth with transmission errors**

Previous analysis omitted the effects of transmission errors to find the probability density

function of the bandwidth  $BW(L)$  of an IEEE 802.11b link, given in Equation (4). Of course, the assumption of perfect transmission is far from reality. To consider imperfections, the bit error rate (BER) is taken as the parameter that summarizes the physical impairments of the link. The Qualnet® [20] simulations assume a statistical propagation model with free space path loss for near sight and flat earth reflection for far sight, 4 dB of shadowing mean, no fading, 290 K of temperature, noise factor of 10, 1.5 m high omnidirectional antennas with 0.3 dB of mismatch losses and an efficiency 0.8, 15 dBm of transmission power and a receiver sensitivity of -89 dBm. These conditions allow us to compute the Signal-to-Noise-Ratio, SNR, as a function of distance, with which the BER is computed for a 2 Mbps IEEE 802.11b link using DQPSK, which is the main parameter for our analytical results.

A transmission can be aborted because either the Wait\_For\_CTS timer expires,  $T_{cts}$ , or the Wait\_For\_ACK expires,  $T_{ack}$ . In the first case, there was an error on the RTS and/or the CTS frames while, in the second case, both RTS and CTS arrived with no errors to their destinations, but the data frame or the ACK frame experienced errors. Assuming independence among bit errors, the first event will happen with probability  $p_1 = 1 - (1 - \text{BER})^{\text{RTS}+\text{CTS}}$  and the second one will occur with probability  $p_2 = 1 - (1 - \text{BER})^{\text{Hdr}+L+\text{ACK}}$ , given there were no errors in RTS nor CTS (recall the assumption that control frames and data frames

are sent at the same transmission rate). In the first case, the wasted time will be  $DIFS + RTS + T_{cts} + DIFS + n\sigma$ , where the last two terms correspond to the time it takes the sender to recover,  $n$  is an integer number uniformly distributed between 0 and  $2^{k-1}W-1$ , and  $k$  is the number of consecutive

transmission failures. In the second case, the wasted time will be  $DIFS + RTS + T_p + SIFS + CTS + T_p + SIFS + Hdr + L/C + T_{ack} + DIFS + n\sigma$ . In general, with  $k_1$  errors of the first type and  $k_2$  errors of the second type, the total “service time” will be

$$T_s = k_1 \left( 2DIFS + \frac{RTS}{C} + T_{cts} \right) + \dots + k_2 \left( 2DIFS + 2SIFS + 2T_p + \frac{RTS + CTS + Hdr + L}{C} + T_{ack} \right) + \dots + \left( 2DIFS + 3SIFS + 4T_p + \frac{RTS+CTS+Hdr+L+ACK}{C} \right) + \sigma(n_0 + \sum_{k=0}^{k_1+k_2-1} n_k) \quad (16)$$

where  $n_k$  is uniformly distributed in the range  $[0, 1, 2, \dots, \min(2^k W - 1, 1023)]$ , and  $k_1$  and  $k_2$  are jointly distributed as

$$P[k_1, k_2] = \binom{k_1 + k_2}{k_1} p_1^{k_1} (p_2(1 - p_1))^{k_2} (1 - p_1)(1 - p_2) \quad (17)$$

Next assume that the distribution of  $X = n_0 + \sum_{k=0}^{k_1+k_2-1} n_k$  in (16) is continuous uniform

when  $k_1+k_2=0$ , triangular when  $k_1+k_2=1$  and normal with appropriate mean and variance

when  $k_1+k_2>1$ . Then the corresponding pdfs can be weighted with Equation (17) for the corresponding values of  $(k_1, k_2)$ . Under this assumption, the corresponding distribution of the achievable transmission rate is

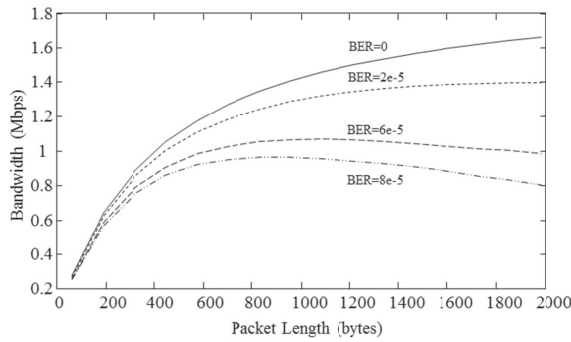
$$f_{BW(L)}(b) = \frac{L}{bw^2} \sum_{(k_1, k_2)} P[k_1, k_2] f_x \left( \frac{L}{b} - \left( T_0 + \frac{L}{C} \right) - k_1 T_1 - k_2 \left( T_2 + \frac{L}{C} \right) \mid (k_1, k_2) \right) \quad (18)$$

Figure 5 shows the average of the above pdf as a function of packet length for a given BER. The impact of noise as it interacts with the MAC protocol becomes evident.

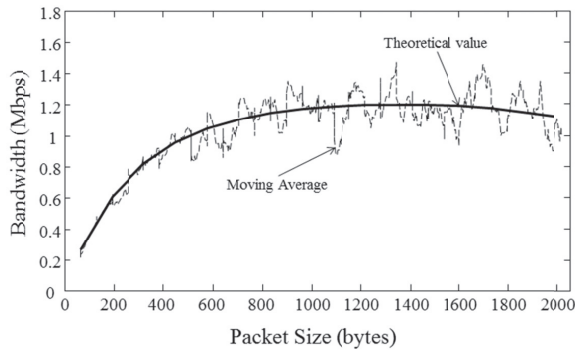
In order to validate the above results, we present two simulation results. Figure 6 shows a comparison of theoretical and simulation results of the transmission bandwidth when source and destination nodes are at 351 meters of distance, for a BER of  $4 \times 10^{-5}$ , with different packet lengths. Figure 7 shows the achieved bandwidth for a source node sending packets of 512 bytes

to a destination that is moving away from the source, increasing the BER. Simulation and theoretical results agree, though there is a high variance to consider. Although this variability can be reduced in the simulation with a higher number of simulation samples, it is clear that the experience of a node does not need to be too close to the expected mean. A simple analysis of equation (18) would give us the variance (and other higher moments) of the bandwidth if we want to take into account the dispersion around the mean.

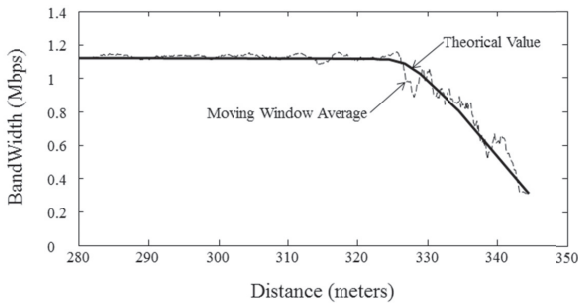




**Figure 5** Expected Value of  $BW$  for different packet lengths and BERs



**Figure 6** Simulation and theoretical results of achieved bandwidth when  $BER = 4 \times 10^{-5}$

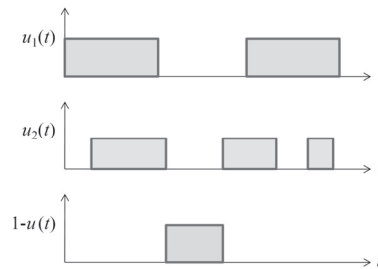


**Figure 7** Perceived bandwidth variation with distance between the nodes

### Effects of other source nodes

Sharing the channel among multiple transmitting nodes reduces the fraction of time the channel is available for each source node, since each of them can detect busy periods during which

it must refrain from transmitting. As said in the related work section, most QoS mechanisms for wireless ad hoc networks rely on a simple end-to-end available bandwidth estimation: If the  $i^{\text{th}}$  node of the path, that transmits at a rate  $C_i$  bps, perceives that the transmission medium is busy during a fraction  $u_i$  of the time, the path available bandwidth is estimated as  $ABW = \min_i ((1 - u_i)C_i)$ . There are several reasons why this could not be the case. For example, if the second node is a relaying node, not the final destination node, the forwarding transmission will make a single packet to occupy the medium twice (at least), because these two nodes cannot transmit simultaneously, so the available bandwidth will be (at most) half of that predicted by the oversimplified model. But, even considering a single link path, which is our studying case, there is a fundamental drawback in such assumption: The receiver node could be subject to the transmission of other nodes that the transmitter node is not aware of, and *vice versa*, so the busy fraction of time they measure could differ from one node to another. Figure 8 represents the busy periods perceived by the transmitter node,  $u_1(t)$  and by the receiver node,  $u_2(t)$ , along with the intersection of the corresponding available periods of time,  $1 - u(t) = \min\{1 - u_1(t), 1 - u_2(t)\}$ . In a general case,  $1 - u(t) \leq 1 - u_1(t)$  because the sender is exposed to a signal that is not perceived by the receiver, or *vice versa*, as shown in the figure 8.

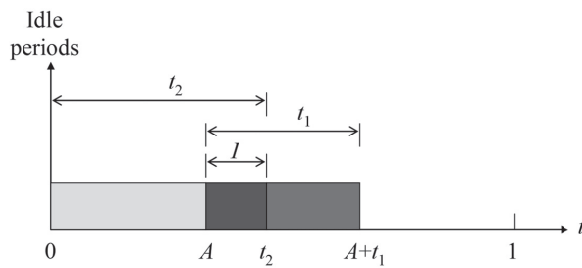


**Figure 8** Idle time available for the link between nodes 1 (Tx) and 2 (Rx)

To elaborate, let us say that the idle fractions of time during an observation interval are  $0 \leq t_1 \leq t_2 \leq 1$ , where node 1 was renamed as that with

the smaller idle period. The intersection between those two fractions is the true idle period for the link between them. According to figure 8, it is expected to be less than  $t_1$  because of possible non-simultaneous occupations of the medium at nodes 1 and 2.

Let us construct a graph like this: put together, on the left, the fractions of time during which node 1 senses the medium busy and node 2 senses it idle; immediately after, put together the fractions of time during which both nodes sense it idle; then put together the fractions of time during which node 1 senses it idle and node 2 senses it busy; and, finally, let us put together the fractions of time during which both nodes sense the medium busy, as shown in figure 9.

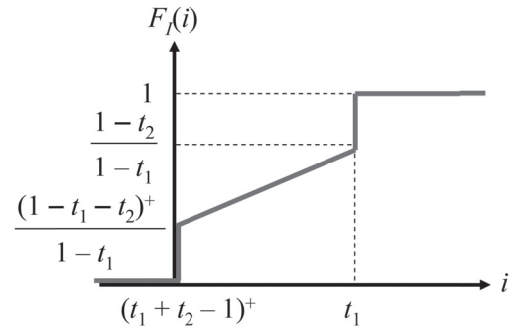


**Figure 9** Idle and busy periods as sensed by both nodes

In figure 9, the time instant  $A$  becomes a continuous random variable uniformly distributed in  $[0, 1-t_1]$ , in which case the intersection  $I$ , which is the true idle period, becomes

$$I = \begin{cases} 0 & t_2 \leq A \\ t_2 - A & t_2 - t_1 \leq A < t_2 \\ t_1 & A \leq t_2 - t_1 \end{cases} \quad (19)$$

From the distribution of  $A$ ,  $\Pr[I=t_1] = (t_2 - t_1)/(1 - t_1)$  and, if  $t_1+t_2 < 1$ ,  $\Pr[I=0] = (1 - (t_1 + t_2))/(1-t_1)$ . Otherwise ( $t_1+t_2 \geq 1$ ), the minimum value of the intersection is  $(t_1+t_2-1)$ . In both cases, the *pdf* of  $I$  is the constant value  $1/(1-t_1)$  within the open interval  $((t_1+t_2-1)^+, t_1)$ , where  $x^+ \equiv \max(0, x)$ . Consequently, the *CDF* of  $I$  is as shown in figure 10.

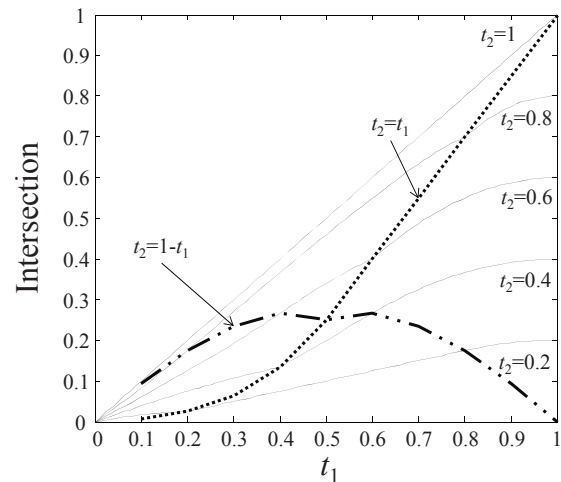


**Figure 10** CDF of the intersection between the perceived idle periods

The corresponding mean of the intersection of idle periods is

$$E[I|t_1, t_2] = \begin{cases} \frac{t_1(2t_2-t_1)}{2(1-t_1)} & t_1 + t_2 \leq 1 \\ t_1 - \frac{1-t_2(2-t_2)}{2(1-t_1)} & t_1 + t_2 > 1 \end{cases} \quad (20)$$

as depicted in figure 11, where  $t_1$  and  $t_2$  are redefined to be the idle periods perceived by the first and second nodes, independently of which one is smaller. By the way, notice how easy would be for the nodes to share their perceptions  $t_1$  and  $t_2$  in order to compute equation (20).



**Figure 11** Intersection between the idle periods  $t_1$  and  $t_2$

In the average, the fraction of time during which the transmission medium is available for the link is less than the minimum of the locally perceived fraction of available time, which invalidates the assumptions implicitly stated in the commonly used formula  $ABW = \min_i((1-u_i)C_i)$ . However we can recover this formula for a single link by considering the average intersection between idle periods instead of the factor  $(1-u_i)$ , and considering the average bandwidth for the selected packet length and received signal strength instead of the factor  $C_i$ . Nevertheless, the available bandwidth in a path will be much less than the minimum of the available bandwidth in each link, because each packet can occupy each physical channel several times, as described in [3,21], for example.

## Conclusions

We have conducted an accurate analytical description of the probability density function of the bandwidth of a link in a MANET based on the IEEE 802.11 physical and multiple access protocols. The analysis includes a result under ideal conditions (no errors and no sharing with other sources) that was previously presented without proof in [3], but extends it with new insights about the effects of transmission errors. The conclusion is that the bandwidth of a link, far from being a constant transmission capacity, is a highly variable random quantity whose mean can be easily computed as a function of the packet length and the signal to noise ratio at the receiver antenna.

Additionally, we have shown how to compute, in a distributed way, the availability of the transmission medium around the source/destination link. Again, we considered it a random variable whose expected value can be easily estimated in a distributed way between the nodes that form the link, using a typical local sensing procedure.

These analytical results could be used in an accurate, timely and efficient available bandwidth estimator for IEEE 802.11 MANETs.

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