Rev. Fac. Ing. Univ. Antioquia N.° 66 pp. 181-188. Marzo, 2013

Ricatti stochastic filter as an estimator

Filtro estocástico de Ricatti como estimador

J. Jesus Medel[1*], *M. Teresa Zagaceta A*[2]

[1] Computing Research Centre. Col. Nueva Industrial. Av. 100 Metres, St. Venus S/N, 07738.

[2] Mechanical and Electrical Engineering School. Av. De las Granjas N.- 682 Col. Santa Catarina.CP 02250.

(Recibido el 22 de mayo de 2012. Aceptado el 6 de febrero de 2013)

Abstract

The Ricatti stochastic digital filter as an estimator is based on a first differences order model with uncorrelated innovation properties bounding the spatial operation region with two auxiliary equations. This permits optimal results having the traditional inversion instead of the pseudo-inverse strategy. The stationary conditions and uncorrelated trajectories were the tools applied in adaptive estimation and identification integrated form. In spite of the black-box form observing the output system, the parametre and identification simulation achieved a great convergence rate in agreement with functional error. It was built considering the identification second probability moment defined as the difference between the desired signal and the output filter response. The parametres estimated were inside the unit circle and had a great advantage because the primitive solution depends on their values.

---------- *Keywords:* Instrumental variable, least squares method, second probability moment, Ricatti estimation, convergence

Resumen

El filtro estocástico de Ricatti como un estimador está basado en un modelo en diferencias de segundo grado, de primer orden con el proceso de innovación no correlacionado acotado por la región del espacio de operaciones a través de dos ecuaciones auxiliares que permiten tener resultados óptimos con la inversión tradicional en vez de usar la estrategia de la pseudo-inversa.

Las trayectorias no correlacionadas y las condiciones estacionarias fueron las herramientas aplicadas en la forma integrada adaptable del estimador con el identificador. A pesar de todas las restricciones del sistema de caja negra en la

* Autor de correspondencia: teléfono: + 01 + 52 + 572 960 00, ext. 56570, correo electrónico: jjmedelj@yahoo.com.mx (J. Medel)

recursividad, fue simulado con una gran relación de convergencia observada en el funcional. Este fue construido considerando el segundo momento de probabilidad del erro de identificación definido por la diferencia entre la señal deseada y la respuesta del filtro. Los parámetros estimados se encuentran dentro del círculo unitario y representa una gran ventaja para el sistema en diferencias ya que su primitiva depende de estos valores.

---------- *Palabras clave:* Variable instrumental, mínimos cuadrados, segundo momento de probabilidad, estimador de Ricatti, convergencia

Introduction

Adaptive Control Theory (ACT) in Digital Systems (DS) uses some mechanisms adjusting the gains into a controlled system according to a reference model answer. These mechanisms are known as Digital Filter Estimation (DFE) techniques and affect the control actions before the feedback ends, adapting its parametres set to a reference objective. For example, the discrete Proportional Integral and the Derivative (PID) control has three gains (K_p, K_i, K_d) and commonly use the Butterworth method in the calculus coefficients, without changes through the time never considering if the system has or has not variations. Without other options, this control is applied in real conditions adjusted experimentally, against the natural evolution system conditions.

The control law action in a digital system operates in finite differences where the order is represented by the output delays signal. The parametre filter estimation considers the output stage order as an element on its dimension [1], using some of the following tools: *Least Square Method* (LSM), *Instrumental Variable* (IV), *Forgetting Factor* (FF), or *Kalman Filter* (KF). In the black-box system scheme, the unobservable internal states and unknown parametres have one of the following conditions:

a) Stationary. The input-output signals rate is smooth or rigid [2] needing some tools in estimation techniques:

   a_1) Pseudo-inverse traditional form [3] applied LSM, IV, and FF techniques,

   a_2) Correlation inverse matrix used as a Kalman filter gain [4],

   a_3) Second probability moment and gradient based on linear stochastic model $Ay_k + \xi_k = u_k$, with uncertainty around the equilibrium solution [5], and $y_k, \xi_k, u_k \in \left(\Re_{[0,1]}, \Im(\Re), P_{[0,1)}\right)$, with $\{\xi_k\} \subseteq N(\mu_\xi, \sigma_\xi^2 < \infty)$, $\{y_k\} \subseteq N(\mu_y, \sigma_y^2 < \infty)$, and $\{u_k\} \subseteq N(\mu_u, \sigma_u^2 < \infty)$ has in common with black-box scheme, the unknown internal matrix parametres A,

   a_4) Output natural frequency system bounded all on-line techniques [6 - 8].

   a_5) Mean square error applied on adaptive filter convergence without losing its stability [9, 10].

The simulation results, in all cases showed that the functional error converges exponentially to an equilibrium point.

b) Non-stationary. The input-output signals rate has a different velocity set in each time interval, needing some tools in estimation techniques:

   b_1) Output matrix variances [11] applied into adaptive filter with Maximum Likelihood,

   b_2) Maximal output amplitude [12] into outlier M-estimator,

   b_3) Input-output covariance weights [13] used in quasi-maximum likelihood.

The input and output signals had high changes with marginal stable conditions; but in spite of, the estimations converge in a good distribution sense.

The filter estimation is used in:

1) Electrical rotor-position requiring specific velocity using a filter technique affecting the control law amplitude voltages.
2) Multi-Input Multi-Output Orthogonal Frequencies-Division and Multi-Access (MIMO-OFDMA) system had offset up-link non-stationary frequencies [6] dynamically adjusting their LSM gains [14, 15]. The results were limited to smooth conditions.
3) The Signal-to-Noise Ratio (SNR) is seen as a system answer having non-stationary conditions. The estimation results had good distribution convergences [7, 16]. The azimuth and elevation variations in short-range and low-flying air routes were predicted with the LSM Ricatti parametres estimation method [17].
4) The 3D affine motion with two cameras via observations as a single feature point estimates the nine rotational, three translational and 3D position parametres using the LSM Ricatti estimation method [18].

Different estimation techniques were resumed observing the output system with or without stationary conditions. The present paper considers the Auto Regressive Moving Average model ARMA (1, 1) with two auxiliary equations solving the vector estimation using the traditional inverse technique instead of the pseudo-inverse adjoin solution.

Development

The Single-Input Single-Output (SISO) black-box system in the Ricatti differences (1).

$$Y_k = a_k U_k + b_k \Psi_k + c_k \Theta_k \tag{1}$$

Where $Y_k = E\{y_i\} \in \Re$, $U_k = kE\{u(k)\} \in \Re$ $\Psi_k = E\{\psi_i\} \in \Re$, $\Theta_k = E\{\psi_i^2\} \in \Re$ and $u(k)$ as an unitary excitation, $U_k = const$. Therefore the Ricatti form (1) has a complete solution using two auxiliary equations [19].

Theorem 1. The system described in (1) considering two auxiliary equations (2) and (3).

$$\Gamma_{1,k} = a_k \Psi_k + b_k \Theta_k + c_k \Lambda_k \tag{2}$$

$$\Gamma_{2,k} = a_k \Theta_k + b_k \Lambda_k + c_k \Phi_k \tag{3}$$

With its elements defined as: $\Gamma_{1,k} = E\{y_i \psi_i\}$, $\Psi_k = E\{\psi_i\}$, $\Theta_k = E\{\psi_i^2\}$, $\Lambda_k = E\{\psi_i^3\}$ $\Gamma_{2,k} = E\{y_i \psi_i^2\}$, $\Phi_k = E\{\psi_i^4\}$, where the recursive parametres is described from (4) to (6).

$$\hat{a}_k = \beta_k \hat{a}_{k-1} + D_k - E_k - F_k + G_k - H_k + I_k + J_k \tag{4}$$

$$\begin{aligned} \hat{b}_k &= \delta_k \hat{b}_{k-1} + \mathrm{L_K} + \mathrm{M_K} - \mathrm{N_K} - \mathrm{O_K} \\ &+ \mathrm{P'_K} + \mathrm{Q'_K} - \mathrm{R'_K} + \mathrm{S'_K} - \mathrm{T'_K} - \mathrm{U'_K} \end{aligned} \tag{5}$$

$$\begin{aligned} \hat{c}_k &= \varphi_k \hat{c}_{k-1} + \mathrm{D''_K} + \mathrm{E''_K} - \mathrm{F''_K} - \mathrm{G''_K} \\ &+ \mathrm{H''_K} - \mathrm{I''_K} - \mathrm{J''_K} + \mathrm{L''_K} + \mathrm{M''_K} - \mathrm{N''_K} \end{aligned} \tag{6}$$

Proof: In a matrix sense, according to (1), (2) and (3), the matrix solution is described in (7).

$$[M_k] = [A_k]^{-1}[Y_k] \tag{7}$$

In (7) its elements have the form (8).

$$[M_k]^T := [a_k \quad b_k \quad c_k],$$

$$A_k := \begin{vmatrix} k & E\{\psi_i\} & E\{\psi_i^2\} \\ E\{\psi_i\} & E\{\psi_i^2\} & E\{\psi_i^3\} \\ E\{\psi_i^2\} & E\{\psi_i^3\} & E\{\psi_i^4\} \end{vmatrix} \tag{8}$$

$$[Y_k]^T := [E\{y_i\} \quad E\{y_i \psi_i\} \quad E\{y_i \psi_i^2\}]$$

A_k is described as a rotational, symbolically described as $\nabla_k \times (f_k \times \nabla_k)$, converging to zero, instead of being considered $\nabla_k \times (\nabla_k \times f_k)$, practically corresponding to interchange the last two rows of (8), so allowing the inverse an odd description. Here in a stochastic sense $\nabla_k := [E\{\psi_i^2\} \quad E\{\psi_i^3\} \quad E\{\psi_i^4\}]$, and $f_k := [E\{\psi_i\} \quad E\{\psi_i^2\} \quad E\{\psi_i^3\}]$, and the odd rotational $W_k := \nabla_k \times (\nabla_k \times f_k)$ is developed in (9). Without this property, the coefficients matrix has null values.

$$E\{\psi_i^4\}(E\{\psi_i\})^2 - 2E\{\psi_i\}E\{\psi_i^2\}E\{\psi_i^3\} + \left(E\{\psi_i^2\}\right)^3 - kE\{\psi_i^4\}E\{\psi_i^2\} + k\left(E\{\psi_i^3\}\right)^2 \qquad (9)$$

Each parametre is described in (9), (10) and, (11) according to (1), (2) and, (3).

$$\hat{a}_k = \frac{\left((E\{\psi_i^3\})^2 + E\{\psi^2\}E\{\psi^4\}\right)E\{y_i\} + \left(\left(E\{\psi_i\}E\{\psi_i^4\} - E\{\psi_i^2\}E\{\psi_i^3\}\right)\right)E\{y_i\psi_i\} + \left((E\{\psi_i^2\})^2 - E\{\psi_i\}E\{\psi_i^3\}\right)E\{y_i\psi_i^2\}}{W_k} \qquad (10)$$

$$\hat{b}_k = \frac{\left(\left(E\{\psi_i\}E\{\psi_i^4\} - E\{\psi_i^2\}E\{\psi_i^3\}\right)\right)E\{y_i\} + k\left(E\{\psi_i^2\}^2 - E\{\psi_i^4\}\right)E\{y_i\psi_i\} + \left(\left(kE\{\psi_i^3\} - E\{\psi_i\}E\{\psi_i^3\}\right)\right)E\{y_i\psi_i^2\}}{W_k} \qquad (11)$$

$$\hat{c}_k = \frac{\left((E\{\psi_i^2\})^2 - \left(E\{\psi_i\}E\{\psi_i^3\}\right)\right)E\{y_i\} + \left(E\{\psi_i^3\} - E\{\psi_i\}E\{\psi_i^2\}\right)E\{y_i\psi_i\} + k\left(E\{\psi_i\}^2 - E\{\psi_i^2\}\right)E\{y_i\psi_i^2\}}{\mathrm{W_k}} \qquad (12)$$

In symbolic form are (13), (14) and, (15).

$$\hat{a}_k = \frac{\left((T_k)^2 + Q_kU_k\right)P_k + ((S_kU_k - Q_kT_k))R_k + \left((Q_k)^2 - S_kT_k\right)W_k}{U_k(S_k)^2 - 2S_kQ_kT_k + (Q_k)^3 - kU_kQ_k + k(T_k)^2} \qquad (13)$$

$$\hat{b}_k = \frac{((S_kU_k - Q_kT_k))P_k + \left((Q_k)^2 - KU_k\right)R_k + ((KT_k - S_kQ_k))W_{kk}}{U_k(S_k)^2 - 2S_kQ_kT_k + (Q_k)^3 - kU_kQ_k + k(T_k)^2} \qquad (14)$$

$$\hat{c}_k = \frac{\left[(Q_k)^2 - (S_kT_k)\right]P_k + ((KT_k - S_kQ_k))R_k + \left((S_k)^2 - (KQ_k)\right)W_k}{U_k(S_k)^2 - 2S_kQ_kT_k + (Q_k)^3 - kU_kQ_k + k(T_k)^2} \qquad (15)$$

And considering stationary conditions the coefficients are described on (16) to (22).

$$P_k = \frac{1}{k}\left[y_k + (k-1)P_{k-1}\right] \qquad (16)$$

$$Q_k = \frac{1}{k}\left[\psi_k^2 + (k-1)Q_{k-1}\right] \qquad (17)$$

$$R_k = \frac{1}{k}\left[y_k\psi_k + (k-1)R_{k-1}\right] \qquad (18)$$

$$S_k = \frac{1}{k}\left[\psi_k + (k-1)S_{k-1}\right] \qquad (19)$$

$$T_k = \frac{1}{k}\left[\psi_k^3 + (k-1)T_{k-1}\right] \qquad (20)$$

$$U_k = \frac{1}{k}\left[\psi_k^4 + (k-1)U_{k-1}\right] \qquad (21)$$

$$W_k = \frac{1}{k}\left[y_k\psi_k^2 + (k-1)W_{k-1}\right] \qquad (22)$$

These coefficients are applied in (4), (5) and, (6).■

Simulation

The Ricatti observable SISO system in differences has the form (23).

$$y(k) = au(k) + by(k-1) + cy(k-1)^2 \qquad (23)$$

In the first experiment, where $y(k)$ has a periodic function, the identifier is described in (24).

$$\hat{y}(k) = \hat{a}(k)u(k) + \hat{b}(k)y(k-1) + \hat{c}(k)y(k-1)^2 \quad (24)$$

And using (10), (11) and (12), the estimation applied in (24), and the identifier with the reference signal are scheme is shown in figure 1 and the simulation result is shown in figure 2.

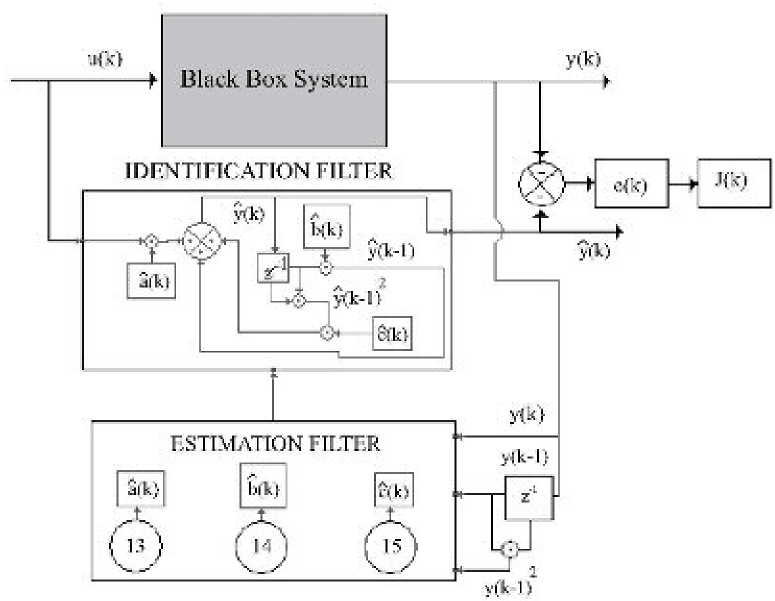

Figure 1 Estimation and identification filter with respect to input-output black box system

Where the error estimation is described by in agreement to figure 1 as $e(k)$ and its second probability moment as $J(k)$

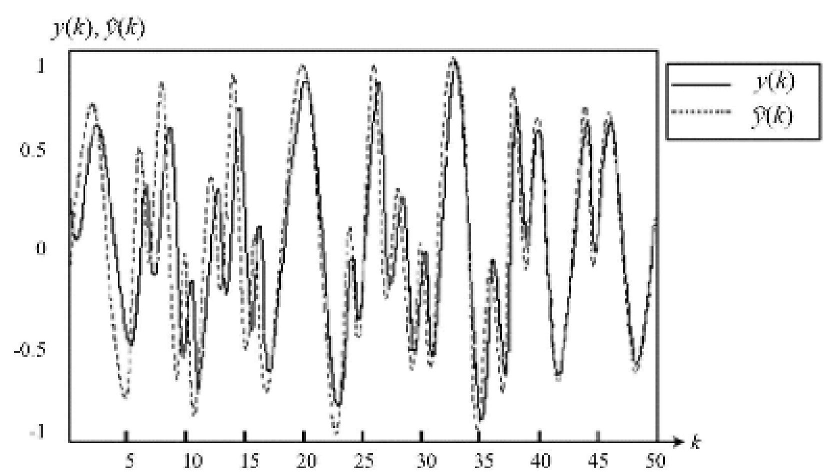

Figure 2 Identification results $\hat{y}(k)$ and reference signal with slow perturbation $y(k)$

In the second experiment described by (24), the parametres are estimated based on (13) to (15), and described in figure 1, and depicted in figure 3, where the three parametres affected the reference signal.

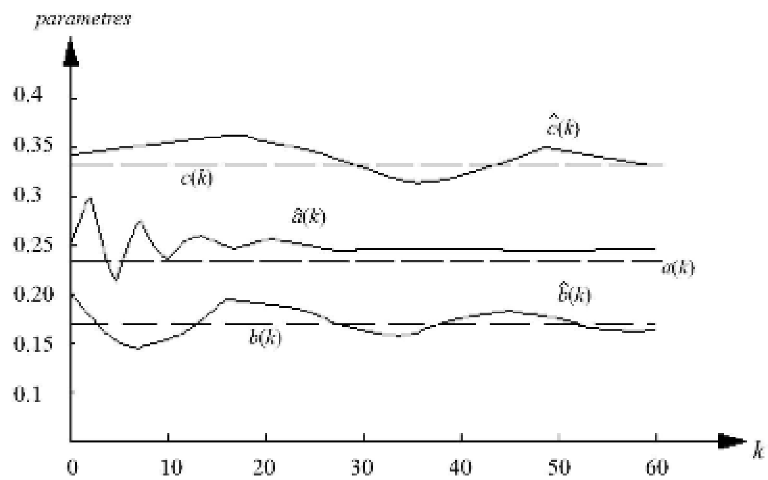

Figure 3 Parametres estimation and its references

The output observable signal with bounded random sequence is shown in figure 4, depicting the output identification evolution.

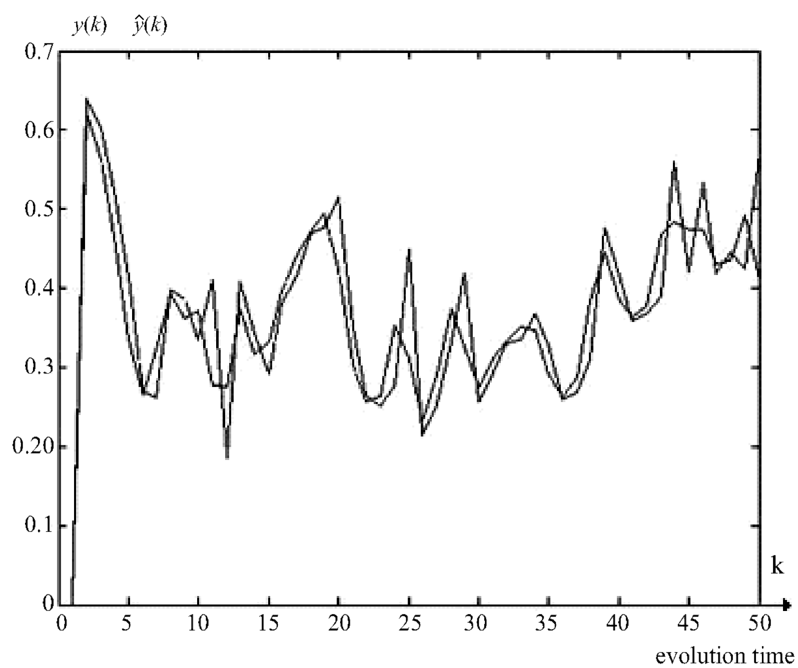

Figure 4 The reference *y(k) and* the identification $\hat{y}(k)$ signals *bounded*

Figure 5 shows LSM and Ricatti functional errors, respectively based on (25)

$$J(k) = \frac{1}{k}\left[e(k)^2 + (k-1)J(k-1)\right] \qquad (25)$$

The identification error is defined as $e(k) := y(k) - \hat{y}(k)$.

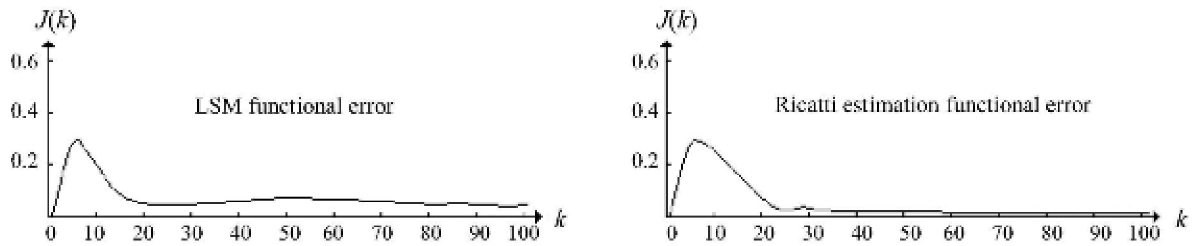

Figure 5 Functional errors

Conclusion

In this paper a stochastic digital filter as a parametre estimation shown by the Ricatti differences equation was proposed. A first difference order model with uncorrelated innovation conditions described in (1) as a SISO model in (23) considered two auxiliary equalities bounding the spatial operation, which allowed an approximation to a real parametres set. Three parametres were estimated, and the recursive forms from (13) to (15) considered stationary conditions listed from (16) to (22). The functional error (25) described illustratively the convergence rate shown in figure 5.

References

1. A. Novales. *Análisis de Regresión Lineal*. Universidad Complutense. Disponible en: http: //www .ucm. es/ info / ecocuan /anc /ectriaqf /Analisis %20de%20 Regresion.pdf. pp. 4-116. Consultado en septiembre de 2010.

2. E. Alameda, H. Ruiz, P, Tienda, M. Carrión. *Un nuevo algoritmo tipo gradiente estocástico para identificación ciega de canales*. Universidad de Coruña. Vol. 6. 2003. pp. 1-4. Disponible en: http:// ursi2003.udc.es/fichas/ficha1284.html.

3. H. Yanga, X. Changa, D. Liua. Improvement of the Liu. "Estimator in Weighted Mixed Regression Communications in Statistics". *Theory and Methods*. Vol. 38. 2009. pp. 285-292.

4. J. Urdánoz. "Medicion de la desproporcionalidad electoral: una critica a los mínimos cuadrados" *Revista Española de Investigaciones Sociológicas*. No. 115. 2006. pp. 271-273.

5. H. López. "Análisis e implementación de un Sistema de Control Adaptativo en Tiempo Real basado en Microcomputador". Tesis de doctorado. Universidad de Oviedo. Departamento de ingeniería eléctrica, electrónica de computadoras y sistemas. Oviedo, Asturias, España. 1989. pp. 14-19.

6. S. Sezginer, P. Bianchi. "Asymptotically Efficient Reduced Complexity Frequency Offset and Channel Estimators for Uplink MIMO-OFDMA". *Systems Signal Processing, IEEE Transactions*. Vol. 56. 2007. pp. 964-979.

7. A. Fernández. *Cancelación de Eco*. Universidad de la Republica Montevideo Uruguay 2003. http://www. fing.edu.uy/iie/ense/asign/tes/materiales/monografias/ Cancelacion_de_Eco_J_Fernandez.pdf. Consultada en enero de 2012.

8. D. Vecchia. *Una Estructura en Cascada FIR para Predicción Lineal Adaptiva*. Universidad de la Republica Montevideo Uruguay (2003). http://www. fing.edu.uy/iie/ense/asign/tes/materiales/monografias/ Cancelacion_de_Eco_J_Fernandez.pdf.. Consultada el 2 enero de 2012.

9. I. Morales, M. González. "Comparación de las técnicas de análisis de variancia y regresión lineal múltiple: aplicación a un experimento de almacenamiento de

mango". *Agronomía Costarricense*. Vol. 27. 2003. pp. 45-53.

10. J. Medel, M. Zagaceta. "Estimación-Identificación como filtro digital integrado: descripción e implementación recursiva". *Revista Mexicana de Física*. Vol. 56. 2010. pp. 1-8.

11. V. Valerii, S. Fedorov, L. Leonov. "Parameter Estimation for Models with Unknown Parameters in Variances, Communications in Statistics". *Theory and Methods*. Vol. 33. 2006 pp. 2627- 2657.

12. F. Kwun, C. Wen. "An M-Estimator for Estimating the Extended Burr Type III Parameters with Outliers Communications in Statistics". *Theory and Methods.* Vol. 3. 2010. pp. 304-322.

13. D. Belie, F. Melkebeek. "Seamless integration of a low-speed position estimator for IPMSM in a current-controlled voltage-source inverter". 1st Symposium on Sensorless Control for Electrical Drives (SLED). Dept. Electr. Energy, Syst. & Autom., G Univ. Ghent, Belgium. 9-10 july 2010. pp.50-55.

14. F. Antreich, J. Nossek, G. Seco, A. Lee. "Time delay estimation in a spatially structured model using decoupled estimators for temporal and spatial parameters". Smart Antennas, WSA International ITG on Digital Object Identifier. Wessling, Germany. 26-27 Feb. 2008. pp. 16-20.

15. C. Breithaupt R. Martin. "Analysis of the Decision-Directed SNR Estimator for Speech Enhancement with respect to Low-SNR and Transient Conditions". *IEEE Transactions on Audio Speech and Language Processing*. Vol. 19. 2010. pp. 277-289.

16. C. Jian, J. Huang, C. Zhi. "Forecasting of target air route based on weighted least squares method". Control Conference 29th Chinese. Huizhou, China. 29-31 July 2010. pp. 3144-3147.

17. L. Ma, C. Cao, N. Hovakimyan, C. Woolsey, H. Guoqiang. "Estimation of an Affine Motion American Control". Proceedings of the American Control Conference. St. Louis, MO, USA. July 2009. pp. 5058-5090.

18. D. Jones, S. Appadwedula, M. Berry, M. Haun, D. Moussa, D. Sachs. *Adaptive Filtering: LMS Algorithm*. Disponible en: http://cnx.org/content/m10481/2.13/. Consultado en febrero de 2012.

19. J. Medel, M. Zagaceta. "Estimador de Parámetros para Sistemas de Orden Superior". *Revista Mexicana de Física*. Vol. 58. 2012. pp. 127-132.