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A heuristic decomposition method for large-scale traffic assignment: Aburra Valley case study

Un método heurístico de descomposición para la asignación de tráfico de gran escala: caso de estudio Valle de Aburrá

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Abstract

Traffic assignment is one of the most important stages in transportation planning; however, its application to real case studies in medium- to large-sized cities makes the solution of the model difficult because of the scale and high computational complexity related to the combinatorial and non-linear nature of the problem. The aim of this paper is to present a decomposition method based on sub-region analysis, and a simple heuristic rule for solving large-scale traffic assignment problems. This reduces the total amount of variables and equations of the model and offers a practical solution in a reasonable computing time. The proposed traffic assignment model is applied to the multimodal main road network of the Aburra Valley, Colombia. Such an application of a great amount of variables and equations converts the model into a large-scale problem. The proposed method considerably reduces the computational complexity of the problem, and it reveals accurate solutions in an execution time which is reasonable for such a large-scale model.

---------- *Keywords:* Traffic assignment problem, large-scale model, decomposition methods, heuristics

Resumen

La asignación de tráfico es una de las etapas más importante de la planificación del transporte, sin embargo su aplicación a casos reales en ciudades de tamaño medio y grande se hace difícil de resolver por la gran escala y complejidad computacional de estos modelos, asociada a su naturaleza combinatoria y no

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lineal. El objetivo de este artículo es presentar un método de descomposición basado en subregiones y una regla heurística sencilla, para resolver modelos de asignación de tráfico de gran escala, que reducen la cantidad de variables y de ecuaciones del modelo, sin comprometer la calidad de la solución. El modelo de asignación de tráfico propuesto es aplicado a la red multimodal de vías principales del Valle de Aburrá, y dicha aplicación resulta en un problema de gran escala, por el alto número de variables y ecuaciones asociadas. El método propuesto reduce significativamente la complejidad computacional del problema y encuentra soluciones adecuadas en un tiempo de ejecución razonable para un modelo de gran escala.

---------- *Palabras clave:* Problema de asignación de tráfico, modelos de gran escala, modelos de descomposición, heurísticas

Introduction

One of the most important stages in urban transportation planning is addressing the traffic assignment problem (TAP). This problem can be seen as a part of a more complete model for urban transportation planning as is the four-step model, along with trip generation, trip distribution, and modal split [1,2], or it can be seen as an independent model, useful for analyzing the different urban traffic schemes, based on origin-destination (OD) matrices. The aim of the TAP is to characterize user behavior in relation to the route chosen for making a trip. The travel demands are assigned to the available paths in the transportation network, minimizing the total travel times, costs, and/or distances.

A solution to the TAP can be as simple as assuming that there are no traffic congestion effects (free flow) and that all demand pairs use the same route; that is, assuming an “all or nothing” type of assignment. On the other hand, a solution can be as complex as assuming congestion effects on the decisions of users, which means including non-linear functions in the objective function of the optimization problem, and considering all the possible paths available for transportation users for minimizing their travel times. The former is an easy way to solve the TAP but is far from realistic, and the latter is more realistic but harder to solve computationally, especially when the problem is a large-scale one, as the cases of medium- and large-sized cities usually are. The latter approach represents significant research challenges, since it demands the formulation of strong solution methodologies that range from heuristics and metaheuristics, to rigorous methods of mathematical programming and decomposition methods.

In most cases, especially in the cities found in developing countries, the information available in order to run a classic TAP model is insufficient, incomplete, or even non-existent. In those cases, it is necessary to develop a specific TAP model that meets the requirements of urban and transportation planners, and to be in agreement with the available information.

The aim of this paper is to propose a novel approach for the TAP model for the case study of the metropolitan area of Medellin, also known as the Aburra Valley, in the Republic of Colombia. With this approach, it can be considered the mathematical thoroughness of the classic TAP and the cost/time efficiency of the decomposition methods.

Background

Back in the 50s, Wardrop [3] established two principles for representing the behavior of users related to the choice of a route. Those principles, known as Wardrop’s principles, are widely used in equilibrium assignment models nowadays. The first principle states that the journey times in all routes actually used are equal and less than those that would be experienced by a single vehicle on any unused route. This is known as the user equilibrium (UE) principle, and it is the principle which is most used by researchers and practitioners because of its assumptions about the individuals as independent beings seeking to minimize their own journey costs. The second principle states that in conditions of equilibrium the average journey time is minimal, which is known as the system optimal (SO) principle. This means that if all users perceive travel times in the same way, under equilibrium conditions, all routes between a travel demand pair have the same minimum time, while routes not being used require at least the same time. This principle ensures the most efficient use of the whole system.

The first basic mathematical formulation of the UE principle was made by Beckman, McGuire, and Winsten [4] and is given by the objective function (1) and is subject to constraints given by the equations (2), (3) and (4), where the arc ij represents the streets connecting the pair of nodes i and j. In it, o and d represent the origin and destination nodes, r is a route of the set of possible routes R, while $t_{ij}(x_{ij})$ is the travel time on arc ij as a function of the flow traversing the arc, x_{ij}; g^{od} is the travel demand matrix from origin o to destination d, and δ_{ijr}^{od} is a matrix whose elements are 1 if the arc ij belongs to the route r that connects the demand pair od, and 0 otherwise. Decision variables are the flow on the arc ij (x_{ij}) and the flow on route r connecting origin o and destination d (f_r^{od}). On the other hand, the mathematical formulation for the SO principle is given by the objective function (5), subject to the same constraints of UE formulation; which are (2–4).

$$\text{Min } z(x) = \sum_r \int_0^{x_{ij}} t_{ij}(x)dx \tag{1}$$

Subject to:

$$\sum_r f_r^{od} = g^{od}, \ \forall o,d \tag{2}$$

$$x_{ij} = \sum_o \sum_d \sum_r f_r^{od} \delta_{ijr}^{od}, \quad \forall i,j \in N \tag{3}$$

$$f_r^{od} \geq 0, \ \forall r,o,d \tag{4}$$

$$\text{Min } z(x) = \sum_{ij} x_{ij} t_{ij}(x_{ij}) \tag{5}$$

This mathematical formulation takes into consideration that there is a known set of routes R between each demand pair od. This kind of formulation is known as the arc-route formulation, according to [5], since the assignments are made on arcs previously defined for a set of routes. This assumption has two main disadvantages: a subset of routes between every demand pair od must be known in advance, and it leaves some alternative routes that could be better than the previous ones defined in the R set, unconsidered.

An alternative formulation for TAP is the arc-node formulation, which is more complete and therefore more computationally complex since it takes into consideration all possible paths between each demand pair od and is not limited by the subset R of routes. The solution for this approach is more difficult for large-scale problems, which is why its application has been limited.

Congestion

According to Thomson and Bull [6], congestion is the condition that prevails if the introduction of a vehicle in a traffic flow increases the travel time of other vehicles. If the effect of congestion is not considered, UE and SO principles are equivalent.

Congestion effects are represented by link performance functions that relate the travel time on a link of the network with the flow going through it. These are strictly increasing functions, non-linear, and asymptotic to the capacity flow [7]. The most highly used link performance is the function proposed by the Bureau of Public Roads (BPR), given by Eq. (6)

$$t_{ij}(x_{ij}) = t_{ij}^0 \left[1 + \alpha \left(\frac{x_{ij}}{K_{ij}}\right)^\beta\right] \tag{6}$$

Where t_{ij} is the travel time of arc ij, x_{ij} is the flow on arc ij, t_{ij}^0 is the free flow time on arc ij, K_{ij} is the capacity of arc ij, and α and β are parameters of the model. (It is assumed that $\alpha = 0.15$ and $\beta = 4.0$, as recommended by the BPR). Other link performance functions are given by [8] and [9]. In the paper by Babonneau and Vial [10], some link performance functions are evaluated.

Solution method for TAP

The TAP is highly sensitive to the problem scale. When the number of nodes and arcs in the network increase, or when the number of demand pairs *od* increase, the computational time for solving the TAP increases in a non-polynomial way due to the combinatory nature of the problem. Besides that, if the model takes congestion effects into consideration, the objective function becomes non-linear and it increases the complexity of the problem. In the large-scale models for TAP, some of the specialized solvers are unable to resolve the mathematical programming problem.

The need to reduce the computational complexity of the problem has led to different alternatives for facing it; some based on mathematical programming principles (decomposition methods), and others based on heuristics and/or metaheuristics.

The strategy of the decomposition methods based on mathematical programming is to break down or decompose the original problem into a set of less complicated problems, separating the constraints that might have a special structure. The method considers the solution of at least two problems: one with the most complicated constraints named the *master problem*, and other(s) with the constraints that have a special structure, these are named *sub-problem(s)*. The method iterates by passing information from the master problem to the sub-problems and vice versa, until the optimal solution for the original problem is reached [11].

The algorithms most used for the solution of TAP are based on linear approximations to the objective function. Some examples are the Frank-Wolfe method [12], the simplicial decomposition method [13], and column generation approaches [14]. The Frank-Wolfe method, although widely used, has not been reported as having good performance for large-scale problems, and it was found that it can generate cycles in the solution [15].

On the other hand, heuristics (simple rules applied to solve a specific problem) and metaheuristics (rules for general problems) have been widely used in the solution of large-scale problems. Although they do not guarantee an optimal solution, they can find a good solution in a reasonable execution time. Some of the heuristics used for solving the TAP are described in Patriksson [16].

Experimental set-up

Proposed model and mathematical formulation

The transportation network can be represented as a graph (with arcs as streets or lines, and nodes as intersections or stations). Our graph is composed of two networks: a road network with information on the main streets, denoted as NetS; and a public transportation network with the lines and stations of the mass transportation systems available in the region (the Metro system, for instance), denoted as NetM. The congestion assumptions affect the flows on NetS and, in the case of NetM, they are not considered.

The transportation modes are grouped into the following categories: public collective (buses and the Metro system), public individual (taxi cabs), private collective (school buses), and private individual (cars and motorcycles). For the purpose of modeling, the travel demand is also grouped into two matrices by the aforementioned transportation modes: one matrix, named ODRS, for travel demands in private modes and the individual public mode, measured in standard vehicle units; and the other matrix for travel demands in the collective public mode, measured in number of trips, which is named ODRX. The interaction between the two networks is allowed by using a set of special nodes in which collective public transportation demands can include a transfer. It is assumed that the only transportation mode that makes transfers is the collective public mode.

The proposed model is based on the UE principle. Some traffic factors assumed in the model include: calculations are made upon the inelastic and known travel demand, congestion is only considered for NetS, and the collective public mode can switch networks (that is, transfers can be made on it) while other transportation modes flow in NetS. Also, the walking times to the nodes are not considered. The model formulation corresponds to the arc-node formulation for TAP, which means that the model considers all possible paths for each demand pair *od*.

Sets

N: nodes. This set is also renamed *i*, *j, o,* and *d* for modeling purposes; *i,j* for denoting nodes in the transportation network; and *o,d* for denoting the origin and destination nodes of travel demand

m: transportation modes

Subsets:

ms_m: transportation modes that use Net S exclusively: individual public and private modes

mx_m: transportation modes that can do transfers from NetS to NetM, and vice versa (only collective public)

AS_{ij}: Arcs *ij,* belonging to NetS

AM_{ij}: Arcs *ij,* belonging to NetM

Parameters:

FFT_{ij}: Free flow time for arc *ij* belonging to NetS [min/st veh]

MT_{ij}: Time for traversing arc *ij* of the NetM [min/trip]

KS_{ij}: Capacity of the arc *ij* of NetS [st veh/hour]

KM_{ij}: Capacity of the arc *ij* of NetM [trips/hour]

$ODRS^{od}$: Travel demand from origin *o* to destination *d* in modes ms_m [st veh/hour]

$ODRX^{od}$: Travel demand from origin *o* to destination *d* in modes mx_m [trips/hour]

FOC_m: Occupancy factor of vehicles by mode [trips/veh]

$FSTV_m$: Conversion to standard vehicle factor [st veh/veh]

FCM_{ij}: Occupancy factor for standard vehicles of Metro system. This is defined only for arcs *ij* of NetM [trips/st veh]

α and *β:* parameters of the BPR performance function link

Decision variables:

Y_{ij}^{od}: standard vehicles traversing arc *ij* of NetS, going from origin *o* to destination *d* in modes ms_m [st veh/hour]

X_{ij}^{od}: total trips traversing arc *ij* belonging to NetS or NetM, going from origin *o* to destination *d* in collective public mode mx_m [trip/hour]

Auxiliary variables:

FSY_{ij}: total flow of vehicles on arc *ij* of NetS [st veh/hour], equation (7)

$$FSY_{ij} = \sum_{od} y_{ij}^{od} + (\sum_{od} x_{ij}^{od}) * \frac{FSTV_m}{FOC_m} \qquad (7)$$
$$\forall ij \in ASij, \forall m$$

So the optimization model can be formulated as minimizing the objective function (8) subject to constraints (9–16). The *objective function* is the total travel time for the different flows in the network necessary for making the trips at rush hour. This function is divided into two terms;

the first term is the minimization of travel times in NetM, where there are no congestion effects, and the second term is the minimization of travel times in NetS where the function $t_{ij}(FSY)$ is the BPR link performance function, and this captures the congestion effects on the flows and travel times in the network, since there is modal competition for the capacity of network arcs.

$$\text{Min } Z = \sum_{ij\in AM}\left(\frac{TM_{ij}}{FCM_{ij}} * \sum_{od} x_{ij}^{od}\right) + \sum_{j\in AS}\int_0^F t_{ij}(FSY)dF \tag{8}$$

$$\sum_{\substack{k\in N\\ ok\in AS}} y_{ok}^{od} = ODRS^{od}, \quad \forall od \tag{9}$$

$$\sum_{\substack{k\in N\\ ok\in(AS \text{ or } AM)}} x_{ok}^{od} = ODRX^{od}, \quad \forall od \tag{10}$$

$$\sum_{\substack{k\in N\\ kd\in AS}} y_{kd}^{od} = ODRS^{od}, \ \forall od \tag{11}$$

$$\sum_{\substack{k\in N\\ kd\in(AS \text{ or } AM)}} x_{kd}^{od} = ODRX^{od}, \quad \forall od \tag{12}$$

$$\sum_{\substack{k\in N\\ ki\in AS}} y_{ki}^{od} = \sum_{\substack{k\in N\\ ik\in AS}} y_{ik}^{od}, \quad \forall od, \forall i \neq o, \forall i \neq d \tag{13}$$

$$\sum_{\substack{k\in N\\ ki\in(AS \text{ or } AM)}} x_{ki}^{od} = \sum_{\substack{k\in N\\ ik\in(AS \text{ or } AM)}} x_{ik}^{od}, \quad \forall od, \forall i \neq o, \forall i \neq d \tag{14}$$

$$y_{ij}^{od} \geq 0, \quad \forall ij, \forall od \tag{15}$$

$$x_{ij}^{od} \geq 0, \quad \forall ij, \forall od \tag{16}$$

Equations (9–14) correspond to demand routing, and it is assumed that there is flow balance on each node of the network. Specifically, equations (9) and (10) state that if node *i* is the origin of travel demand, then the sum of all flows departing from the node in modes ms_m are equal to $ODRS^{od}$ in the former case, and the sum of flows departing from the node must be equal to $ODRX^{od}$, if the mode is mx_m in the latter case. Equations (11) and (12) state that if node *i* is the destination node of the travel demand, then the sum of all flows arriving to the node in modes msm must be equal to $ODRS^{od}$ in the former case, and the sum of flows arriving to the node must be equal to $ODRX^{od}$, if the mode is mx_m in the latter case. Equations (13) and (14) state that if node *i* is not an origin nor a destination node of travel demand, then the flow arriving to the node must be equal to the flow departing from it, which means that the flow is conserved. Equations (15) and (16) are the non-negative constraints on flows.

Proposed heuristic method

The proposed model is highly complex, and since the application to a medium-sized city often results in a high number of decision variables and constraints, it is necessary to find a strategy for reducing the number of variables and constraints, and therefore the complexity of the model, without compromising the integrity of the solution.

In this paper, we propose a zoning decomposition method, which basically consists of solving travel demands into aggregated zones (intra-zonal trips), and then solving a system of travel demands between the defined zones (inter-zonal trips). In order to do this, the trips must be classified into intra-zonal and inter-zonal trips as follows:

- Intra-zonal trips: When both the origin and destination nodes belong to the same zone or sub-region *sr,* the trip is considered to be intra-zonal. It is assumed that congestion in arcs due to the flows generated by these demands only affects the sub-region where the traffic assignment is carried out, therefore every sub-region is independent of other sub-regions.

- Inter-zonal trips: These trips are made between a pair of sub-regions *sr* and *sr'*. It is assumed that they cause congestion in the sub-region *sr* (from the origin node to the exit node of the sub-region *sr*), in sub-region *sr'* (from the entrance node, to sub-region *sr',* to the destination node) and in the sub-regions between *sr* and *sr'*.

Under the assumption that intra-zonal trips are independent of the other sub-regions, it is possible to generate *sr* independent intra-zonal sub-models, where *sr* is the number of zones to model, and where each independent model has fewer variables than the original model for the TAP. The selection of the zones depends on the topology and transportation dynamics of the case study.

According to the proposed formulation, we define two new sets; a set RNSR(*i,sr*) representing the membership of the node *i* to the sub-region *sr*; and a set RSRSR(*i'*, *sr*, *sr'*), where *i'* is the entrance or exit node from sub-region *sr* to another sub-region *sr'*. Therefore, for each independent zone *sr* and for each travel demand pair *od:*

- If ($o \in sr$) and ($d \in sr$), then assign the travel demand pair *od*.
- If ($o \in sr$) and ($d \notin sr$), then assign the travel demand from origin node *o* to the exit node (*i'*) of the sub-region *sr*. That is, assign the travel demand from *o* to *i'*.
- If ($o \notin sr$) and ($d \in sr$), then assign the travel demand from entrance node (*i'*) of the sub-region *sr* to destination node *d*. That is, assign the travel demand from *i'* to *d*.

Once all the intra-zonal trips are assigned, the total flow on the arcs of NetS is added, and the road network for the inter-zonal traffic assignment is pre-loaded with these flows, and then, an inter-zonal traffic assignment is made, but it is done so by departing from the exit nodes of each sub-region and arriving to the entrance nodes of the sub-region destination. Since the entrance and exit nodes of each sub-region are the only ones taken into account, the origin-destination pairs of the inter-zonal traffic assignment are reduced significantly. If SR is the dimension of the set of entrance and exit nodes of the inter-zonal model, then the origin-destination matrices would have a worst-scenario dimension of SR^2–SR, which is smaller than the dimension of the original travel demand matrices.

Case study

This study was carried out for the Aburra Valley, Colombia. The region has around 3.5 million inhabitants living among the 10 municipalities that form the Valley, and Medellin, its main municipality, contains almost 67% of the metropolitan area population. The transportation data is obtained from recent research on mobility in the Aburra Valley [17]. Rush hour for the region is about 6:30 to 7:30 AM, in which almost 17% of the daily trips are carried out. The OD matrix for this study represents the trips made in the AM rush hour. The total travel demand matrix for the Aburra Valley is decomposed into two matrices: the ODRX for pubic collective demands, and the ODRS for other transportation modes. The resulting matrices have 1,994 and 18,994 origin-destination pairs, respectively. The ODRX matrix has a 287,184-trip demand and the ODRS matrix has a 112,485 standard vehicle demand for the rush hour.

The transportation network of the region can also be decomposed into two networks: NetS, having 2,729 directed arcs; and NetM, having only 68 arcs. The complete graph (NetS + NetM) has 1,008 nodes.

The decomposition heuristic proposed for the case study consists primarily of dividing the region into municipalities. According to table 1, since the northern municipalities (Barbosa, Girardota, and Copacabana) have fewer nodes, they are grouped into a single sub-region. The same applies for the southern municipalities (Sabaneta, La Estrella, and Caldas). But on the contrary, more than 70% of the total network nodes are in Medellin, which implies that for a single zone in Medellin, there would be many variables; and since the objective of the decomposition method is to reduce the number of variables, Medellin must be separated into smaller sub-regions. In the case of the current research, the separation is made by neighborhood groups, according to their spatial location, as is shown in figure 1.

Table 1 Proposed sub-regions for the Aburra Valley case study

sub-model	*Description*	*# of nodes*
1	Northern municipalities: Barbosa, Girardota, Copacabana	9
2	Municipality of Bello	89
3	Municipality of Itagüí	60
4	Municipality of Envigado	57
5	Southern municipalities: Sabaneta, La Estrella, Caldas	35
6	East-central Medellin neighborhoods	205
7	West-central Medellin neighborhoods	125
8	Northeastern Medellin neighborhoods	110
9	Northwestern Medellin neighborhoods	150
10	Southeastern Medellin neighborhoods	74
11	Southwestern Medellin neighborhoods	94

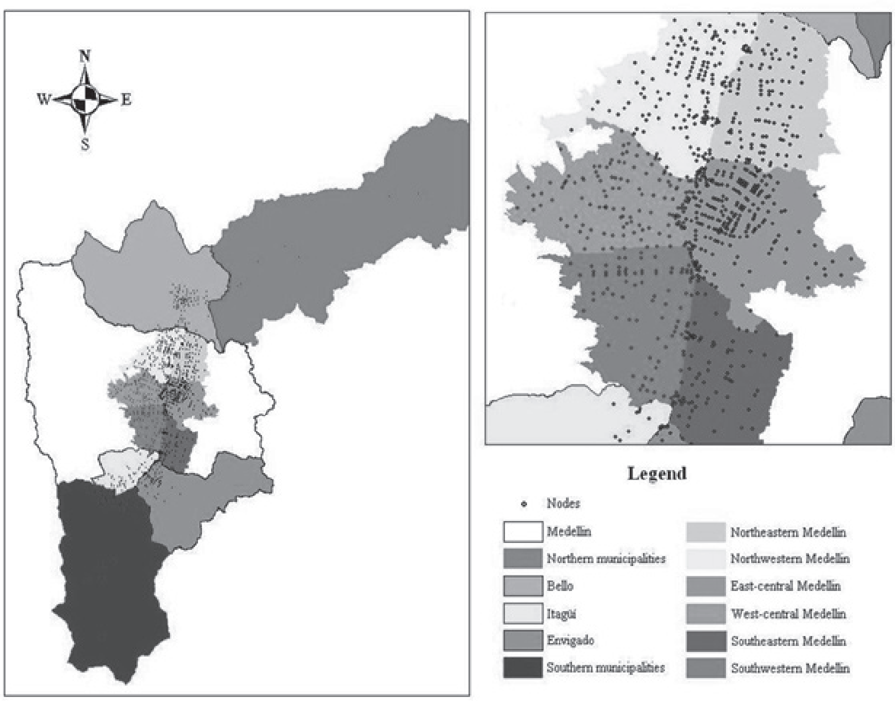

Figure 1 Geographic distribution of sub-regions in the Aburra Valley case study

Results and discussion

The model was implemented in GAMS [18], using the non-linear Minos 5.51 solver [19]. The execution time of the model was about 2.5 hours on a PC with 2 Gb RAM, running under the Windows XP operating system.

Table 2 presents the total number of decision variables and the number of constraints for each independent zone (for every sub-model), comparing the model with and without the heuristic rule which states that each sub-model can only use the arcs of the network of the zone for the traffic assignment. This is due to the congestion assumptions explained previously. In the table, it can be seen that despite the large scale of the problem, the heuristic rule significantly reduces the number of decision variables and constraints, therefore reducing computing complexity without decreasing the realism of the approach.

Table 2 Decision variables and equations for each sub-model with and without the heuristic rule

Sub-model	*# of variables (original)*	*# of variables (heuristic)*	*% of reduction*	*# of equations (original)*	*# of equations (heuristic)*	*% of reduction*
1	239,701	1,741	99.27	85,682	957	98.88
2	4,267,341	429,843	89.93	1,539,984	156,841	89.82
3	1,613,369	103,231	93.60	582,418	45,063	92.26
4	2,150,229	135,175	93.71	775,932	54,484	92.98
5	1,964,929	64,201	96.73	700,916	28,122	95.99
6	2,123,709	399,223	81.20	764,508	176,930	76.86
7	2,181,305	323,493	85.17	785,980	122,474	84.42
8	1,430,519	167,865	88.27	514,944	66,143	87.16
9	2,858,041	485,401	83.02	1,031,288	174,378	83.09
10	1,348,033	114,043	91.54	486,254	47,935	90.14
11	2,320,161	253,342	89.08	837,646	98,017	88.30

Table 3 presents the results of the intra-zonal and inter-zonal traffic assignments. The results are presented in terms of the objective function (total travel time, in minutes, for all travel demands during the rush hour), and they are presented in terms of the computational cost for solving each sub-model. The computational cost of each sub-model is related to the number of variables and equations, presented in Table 3, in a non-polynomial way, since it is an NP-Hard problem.

Table 3 Objective function and computational cost (time) for each sub-model

Sub-model	*Objective function [min/peak hour]*	*CPU time [sec]*
Intra-zonal		
1	6,278.3	0.109
2	131,611.7	491.938
3	41,760.4	138.043

Sub-model	*Objective function [min/peak hour]*	*CPU time [sec]*
4	20,719.6	7.625
5	33,187.7	7.156
6	768,979.6	3,678.215
7	91,628.6	223.375
8	44,610.1	21.250
9	77,558.5	530.715
10	107,255.2	108.605
11	62,831.5	368.516
Inter-zonal	1,677,573.8	2,780.934

Conclusions

The TAP approach presented in this paper is based on the UE principle, and it considers congestion effects. The application to the Aburra Valley case study is a combinatorial, non-linear, and large-scale optimization model. Moreover, since the formulation used is the arc-node instead of the arc-route formulation, the model increases the number of variables and constraints because it has to assess all possible paths for a single travel demand pair *od,* and not only the predefined routes, like in the arc-route formulation. This approach increases the realism of the model, and should give more accurate solutions; therefore, the model has to deal with a massive computational complexity.

The model here presented proposes a series of decompositions: dividing the transportation network into two differentiable networks, each one with different assumptions on congestion and traffic flows, a decomposition of transportation modes into 4 subgroups, taking into account the use of the networks, and finally it is proposed a decomposition of the whole case study region into sub-regions in order to determine the traffic assignment. The modeling approach which considers separated demands (matrices ODRS and ODRX) and separated networks (NetS and NetM) is a novel approach, and it allows for multimodal modeling as well as for the possibility of representing the transfer between networks in the case of the collective-public demand.

In order to solve the resulting large-scale TAP, we proposed a novel approach base on sub-region decomposition. This decomposition method and the heuristic rule which reduces the number of variables and constraints of each sub-problem, make the large-scale problem tractable and give solutions in an execution time which is reasonable for such a large problem. Another advantage of this method is that it allows for a parallel execution of sub-problems, since they are independent; this could considerably reduce the total execution time of the problem. The solution obtained is optimal for each sub-problem, but suboptimal for the original problem, due to a certain lack of interaction between all the flows in separating the problem into intra-zonal and inter-zonal trips. Future research might include the improvement of the solution by allowing more interaction between the intra- and inter-zonal models.

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