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FUZZY E.O.Q MODEL WITH CONSTANT DEMAND AND SHORTAGES: A FUZZY SIGNOMIAL GEOMETRIC PROGRAMMING (FSGP) APPROACH

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ABSTRACT

In this paper, a fuzzy economic order quantity (EOQ) model with shortages under fully backlogging and constant demand is formulated and solved. Here the model is solved by fuzzy signomial geometric programming (FSGP) technique. Fuzzy signomial geometric programming (FSGP) technique provides a powerful technique for solving many non-linear problems. Here we have proposed a new idea that is fuzzy modified signomial geometric programming (FMSGP) and some necessary theorems have been derived. Finally, these are illustrated by some numerical examples and applications.

Keywords: EOQ model, Nearest Interval Approximation (NIA), Fuzzy number, Signomial Geometric Programming.

Mathematics Subject Classification: 90B05, 90C70.

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1. INTRODUCTION

An inventory management deals with the decision that minimizes total averages cost or maximizes total average profit. In an ordinary inventory model are considered all parameters like shortage cost, carrying cost etc. as a fixed, but in a real life situation there some small fluctuations. Therefore, consideration of fuzzy number is more realistic and interesting.

The study of inventory model where demand rates vary with time is the last decades. Datta and Pal investigated an inventory system with power demand pattern and deterioration. Park and Wang studied shortages and partial backlogging of items. Friedman (1978) presented continuous time inventory model with time varying demand.

Ritchie (1984) studied an inventory model with linear increasing demand. Goswami and Chaudhuri (1991) discussed an inventory model with shortages. Gen et. al. (1997) considered classical inventory model with triangular fuzzy number. Yao and Lee (1998) considered an economic production quantity model in fuzzy sense. De, Kundu and Goswami (2003) presented an economic production quantity inventory model involving fuzzy demand rate.

Syde and Aziz (2007) applied sign distance method to fuzzy inventory model without shortage. D.Datta and Pravin Kumar published several paper of fuzzy inventory with or without shortage. Islam, Roy (2006) presented a fuzzy EPQ model with flexibility and reliability consideration and demand depended unit production cost under a space constraint.

A solution method of posynomial geometric programming with interval exponents and coefficients was developed by Liu (2008). Kotba. M. Kotb, Halla. Fergancy (2011) presented Multi-item EOQ model with both demand-depended unit cost and varying lead time via geometric programming.

Jana, Das and Maiti (2014) presented multi-item partial backlogging inventory models over random planning horizon in random fuzzy environment. Samir Dey and Tapan Kumar Roy (2015) presented optimum shape design of structural model with imprecise coefficient by parametric geometric programming.

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A signomial optimization problem often provides much more accurate mathematical representation of real-world nonlinear optimization problems. Initially Passy and Wilde (1967), and Blau and Wilde (1969) generalized some of the prototype concepts and theorems in order to treat signomial programs (SP).

In other work that general type of signomial programming (SP) has been done by Charnes et. al. (1988), who proposed methods for approximating signomial programs with prototype geometric programs. Islam and Roy (2005) proposed EOQ model with shortages under fully backlogging and constant demand is formulated and solved. Here the model is solved by fuzzy signomial geometric programming (FSGP) technique. Fuzzy signomial geometric programming (FSGP) technique provides a powerful technique for solving many non-linear problems.

2. FUZZY NUMBER AND ITS NEAREST INTERVAL APPROXIMATION

2.1. Fuzzy number

A real number $\tilde{A}$ described as fuzzy subset on the real line $\mathcal{R}$ whose membership function $\mu_{\tilde{A}}(x)$ has the following characteristics with $-\infty < a_1 \le a_2 \le a_3 < \infty$

$$\mu_{\tilde{A}}(x) = \begin{cases} \mu_{\tilde{A}}{}^{L}(x) & if a_1 \le x \le a_2, \\ \mu_{\tilde{A}}{}^{R}(x) & if a_2 \le x \le a_3, \\ 0 & otherwise. \end{cases}$$

Where $\mu_{\tilde{A}}{}^{L}(x): [a_1, a_2] \to [0,1]$ is continuous and strictly increasing and $\mu_{\tilde{A}}{}^{R}(x): [a_2, a_3] \to [0,1]$ is continuous and strictly decreasing.

α- level set: The α- level of a fuzzy number is defined as a crisp set where A(α) = [x: μA(x)≥α, xϵX] where αϵ [0,1]. A(α) is a non-empty bounded closed interval contained in X and it can be denoted by Aα = [AL(α), AR(α)]. AL(α) and AR(α) are the lower and the upper bounds of the closed interval, respectively.

2.2. Interval number

An interval number A is defined by an ordered pair of real numbers as follows A = $[a_L, a_R] = \{x: a_L \le x \le a_R\}, x \in \mathcal{R}\}$ where a_L and a_R are the left and the right bounds of interval A, respectively. The interval A, is also defined by center (a_c) and half-width (a_w) as follows

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A = (a_c, a_W) = {x:$a_c - a_w \le x \le a_c + a_w, x \in \mathcal{R}$} where $a_c = \frac{a_R + a_L}{2}$ is the center and $a_w = \frac{a_R - a_L}{2}$ is the half-width of A.

2.3. Nearest interval approximation

Here we want to approximate a fuzzy number by a crisp model. Suppose $\tilde{A}$ and $\tilde{B}$ are two fuzzy numbers with α-cuts are [AL(α), AR(α)] and [BL(α), BR(α)], respectively. Then the distance between $\tilde{A}$ and $\tilde{B}$ is

$$d(\tilde{A}, \tilde{B}) = \sqrt{\int_0^1 (A_L(\alpha) - B_L(\alpha))^2 + \int_0^1 (A_R(\alpha) - B_R(\alpha))^2 d\alpha}.$$

Given a fuzzy number$\hat{A}$, We have to find a closed interval $C_D(\tilde{A})$, which is closest to $\hat{A}$ with respect to some metric. We can do it, since each interval is also a fuzzy number with constant α-cut for all α ∈ [0, 1]. Hence ($C_D(\tilde{A}))\alpha = [C_L, C_R]$. Now we have to minimize

$$d(\tilde{A}, C_D(\tilde{A})) = \sqrt{\int_0^1 (A_L(\alpha) - C_L)^2 + \int_0^1 (A_R(\alpha) - C_R)^2 d\alpha}$$

with respect to $C_L and C_R$.

In order to minimize d($\tilde{A}, C_D(\tilde{A})$), it is sufficient to minimize the function D(C_L,C_R) = ($d^2(\tilde{A}, C_D(\tilde{A})$)). The first partial derivatives are

$\frac{\partial}{\partial C_L} D(C_L, C_R) = -2\int_0^1 A_L(\alpha) d\alpha + 2\, C_L$ and $\frac{\partial}{\partial C_R} D(C_L, C_R) = -2\int_0^1 A_R(\alpha) d\alpha + 2\, C_R$.

Solving $\frac{\partial}{\partial C_L} D(C_L, C_R) = 0$ and $\frac{\partial}{\partial C_R} D(C_L, C_R) = 0,$ we get CL = $\int_0^1 A_L(\alpha) d\alpha$ and CR = $\int_0^1 A_R(\alpha) d\alpha$.

Again, since $\frac{\partial^2}{\partial C_L{}^2}$ (D($C_L{}^*$,$C_R{}^*$)) =2 > 0, $\frac{\partial^2}{\partial C_R{}^2}$ (D($C_L{}^*$,$C_R{}^*$)) =2 > 0 and

H($C_L{}^*$,$C_R{}^*$) = $\frac{\partial^2}{\partial C_L{}^2}$ (D($C_L{}^*$,$C_R{}^*$)).$\frac{\partial^2}{\partial C_R{}^2}$ (D($C_L{}^*$,$C_R{}^*$)) – $\left(\frac{\partial^2}{\partial C_L{}^* C_R{}^*} (D(C_L{}^*, C_R{}^*))\right)^2$ = 4 > 0.

So, D($C_L{}^*$,$C_R{}^*$) i.e. d($\tilde{A}, C_D(\tilde{A})$) is global minimum. Therefore, the interval Cd($\tilde{A}$) = [$\int_0^1 A_L(\alpha) d\alpha, \int_0^1 A_R(\alpha) d\alpha$] is the nearest interval approximation of fuzzy number $\tilde{A}$ with respect to the metric d.

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Let $\tilde{A}$ = (a1, a2, a3) be a triangular fuzzy number. The α-cut interval of $\tilde{A}$ is defined as

Aα = [$A_L(\alpha)$,$A_R(\alpha)$] where $A_L(\alpha)$ = a1+α(a2 - a1) and $A_R(\alpha)$ = a3 - α(a3 – a2). By nearest interval approximation method the lower limit of the interval is

CL = $\int_0^1 A_L(\alpha)d\alpha = \int_0^1 [a_1 + \alpha(a_2 - a_1)]d\alpha = \frac{a_1+a_2}{2}$ and the upper limit of the interval is

CR = $\int_0^1 A_R(\alpha)d\alpha = \int_0^1 [a_3 - \alpha(a_3 - a_2)]d\alpha = \frac{a_3+a_2}{2}$.

Therefore, the interval number corresponding $\tilde{A}$ is $[\frac{a_1+a_2}{2}, \frac{a_3+a_2}{2}] = [m, n]$. In the centre and half –width form the interval number of $\tilde{A}$ is defined as $\langle \frac{1}{4}(a_1 + 2a_2 + a_3), \frac{1}{4}(a_3 - a_1) \rangle$.

2.4. Parametric Interval-valued function

Let [m, n] be an interval, where m > 0, n > 0. From analytical geometry point of view, any real number can be represented on a line. Similarly, we can express an interval by a function. The parametric interval-valued function for the interval [m, n] can be taken as g(s) = $m^{1-s}n^s$ for s ∊ [0, 1], which is a strictly monotone, continuous function and its inverse exits. Let ψ be the inverse of g(s), then

s $= \frac{\log\psi - \log m}{\log n - \log m}$.

3. DETERMINISTIC EOQ MODEL

In many real-life situations shortages occur in an EOQ model. When Shortages occurs, costs are incurred. The purpose of this section is to discuss the deterministic EOQ model in crisp environment. The notations to be used are:

Tac(Q,S): Total average cost of the EOQ model.

Q: Order quantity.

$Q - S$: Maximum shortage that occurs under an ordering policy

c_s: Carrying cost per item per unit time.

c_h: Shortages cost per item per unit time.

c_0: Ordering cost per order.

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D: Demand rate per unit time.

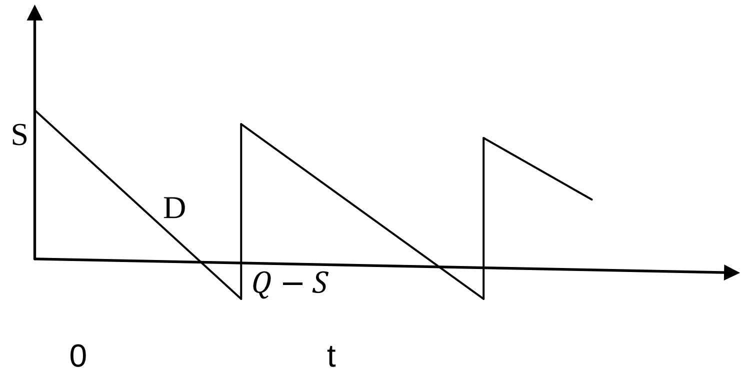

Figure 2: EOQ model

Variables of the EOQ model are Q, S and c_0, c_h, c_s are constant parameters.

Thus,

Total carrying cost = $\frac{c_s S^2}{2D}$,

Total shortages cost = $\frac{c_h (q-s)^2}{2D}$,

So total cost = $c_0 + \frac{c_h (q-s)^2}{2D} + \frac{c_s S^2}{2D}$

And total average cost Tac(Q,S) = $\frac{1}{t}\left[c_0 + \frac{c_h (q-s)^2}{2D} + \frac{c_s S^2}{2D}\right]$

$$= \frac{C_0 D}{Q} + \frac{C_h (Q-S)^2}{2Q} + \frac{C_s (S)^2}{2Q}, \qquad \left[t = \frac{Q}{D}\right].$$

i.e., problem is

$$\text{Minimize Tac(Q,S)} = \frac{C_0 D}{Q} + \frac{C_h (Q-S)^2}{2Q} + \frac{C_s (S)^2}{2Q} \tag{3.1}$$

subject to Q, S $> 0.$

4. FUZZY EOQ MODEL

In the inventory model we take the parameters $\tilde{c_0}$, $\tilde{c_h}$and $\tilde{c_s}$ are fuzzy numbers.

Then from (3.1) we have

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Minimize $\widetilde{Tac}(Q,S) = \frac{\widetilde{c_0}D}{Q} + \frac{\widetilde{c_h}(Q-S)^2}{2Q} + \frac{\widetilde{c_s}(S)^2}{2Q}$ (4.1)

subject to Q, S $> 0.$

5. UNCONSTRAINED FUZZY SIGNOMIAL GP PROBLEM

A problem without any restrictions is called unconstrained problem. I.e., a problem of the form

Minimize $g_0(x_1, x_2, \ldots \ldots \ldots, x_m)$ (5.1)

Subject to $x_j > 0,$ j = 1, 2,……,m,

is called unconstrained problem.

Primal problem:

A primal fuzzy signomial GP programming problem is of the form

Minimize $\tilde{g}_0(x_1, x_2, \ldots \ldots \ldots, x_m)$ (5.2)

Subject to $x_j > 0,$ j = 1, 2,……,m.

Where $\tilde{g}_0(x) = \sum_{i=1}^{k} \sigma_i \tilde{c}_i \prod_{j=1}^{m} x_j^{a_{ij}}.$

Here a_{ij} are real numbers and coefficient $\tilde{c}_i$ are fuzzy triangular, as $\tilde{c}_i = (c_i^1, c_i^2, c_i^3)$.

Using nearest interval approximation method, transformed all triangular fuzzy number into interval number i.e., $[c_i^L, c_i^U]$. Then the fuzzy signomial geometric programming problem is of the following form

Min $\hat{g}_0(x) = \sum_{i=1}^{k} \sigma_i \hat{c}_i \prod_{j=1}^{m} x_j^{a_{ij}}$ (5.3)

Subject to $x_j > 0,$ j = 1, 2,……,m.

Where $\hat{c}_i$ denotes the interval counter parts i.e., $\hat{c}_i \in [c_i^L, c_i^U]. c_i^L > 0, c_i^U > 0,$ for all i. Using parametric interval-valued functional form, the problem () reduces to

Min $g_0(x, s) = \sum_{i=1}^{k} \sigma_i (c_i^L)^{1-s} (c_i^U)^s \prod_{j=1}^{m} x_j^{a_{ij}}$ (5.4)

Subject to $x_j > 0,$ j = 1, 2,……,m.

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This is a parametric geometric programming (PGP) problem.

Dual signomial GP problem:

Dual GP problem of the given primal GP problem is

Maximize $\zeta_0\left[\prod_{i=1}^{n}\left(\frac{(c_i{}^{L})^{1-s}(c_i{}^{U})^{s}}{\delta_i}\right)^{\sigma_i\delta_i}\right]^{\zeta_0}$ (5.5)

Subject to $\sum_{i=1}^{k}\sigma_i\delta_i=\zeta_0$,

$\sum_{i=1}^{k}\sigma_i a_{ij}\delta_i=0$, $j=1,2,\ldots\ldots\ldots..,m$

$\delta_i>0,$

Case I: n>m+1, (i.e. DD >0) so the DP presents a system of linear equations for the dual variables. Here the number of linear equations is less than the number of dual variables. More solutions of dual variable vector exist. In order to find an optimal solution of DP, we need to use some algorithmic methods.

Case II: n< m+1, (i.e. DD <0) so the DP presents a system of linear equations for the dual variables. Here the number of linear equations is greater than the number of dual variables. In this case generally no solution vector exists for the dual variables. However, using Least Square (LS) or Min-Max (MM) method one can get an approximate solution for this system.

Furthermore the primal-dual relation is

$(c_i{}^{L})^{1-s}(c_i{}^{U})^{s}\prod_{j=1}^{m}x_j{}^{*a_{ij}}=\zeta_0\delta^*{}_i\, v(\delta^*,s^*)$. (5.6)

Note: A Weak Duality theorem would say that

$$g_0(x,s)\geq v(\delta,s)$$

For any primal-feasible x and dual-feasible δ but this is not true of the pseudo-dual fuzzy signomial GP problem.

Corollary: When the value of σ_i is 1, then a fuzzy signomial geometric programming (FSGP) problem transform to ordinary geometric programming problem.

Theorem 1: When σ_i is 1, then g_o(x, s) ≥ v(δ, s) (Primal- Dual Inequality).

Proof

The expression for g_0(x, s) can be written as

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g_o(x, s) = $\sum_{i=1}^{n} \delta_k \left(\frac{(c_i^L)^{1-s}(c_i^U)^s \prod_{j=1}^{m} x_j^{\alpha_{kj}}}{\delta_k}\right)$.

Here the weights are $\delta_1, \delta_2, \dots \dots \dots, \delta_n$ and positive terms are $\frac{(c_1^L)^{1-s}(c_1^U)^s \prod_{j=1}^{m} x_j^{\alpha_{1j}}}{\delta_1}$, $\frac{(c_2^L)^{1-s}(c_2^U)^s \prod_{j=1}^{m} x_j^{\alpha_{2j}}}{\delta_2}$, ..., $\frac{(c_n^L)^{1-s}(c_n^U)^s \prod_{j=1}^{m} x_j^{\alpha_{nj}}}{\delta_n}$.

Now applying A.M.-.G.M inequality, we get

$$\left(\frac{(c_1^L)^{1-s}(c_1^U)^s \prod_{j=1}^{m} x_j^{\alpha_{1j}} + (c_2^L)^{1-s}(c_2^U)^s \prod_{j=1}^{m} x_j^{\alpha_{2j}} + \dots + (c_n^L)^{1-s}(c_n^U)^s \prod_{j=1}^{m} x_j^{\alpha_{nj}}}{(\delta_1 + \delta_2 + \dots + \delta_n)}\right)^{(\delta_{01}+\delta_{02}+\dots+\delta_n)}$$

$$\geq \left(\left(\frac{(c_1^L)^{1-s}(c_n^U)^s \prod_{j=1}^{m} x_j^{\alpha_{1j}}}{\delta_1}\right)^{\delta_1}\left(\frac{(c_2^L)^{1-s}(c_2^U)^s \prod_{j=1}^{m} x_j^{\alpha_{2j}}}{\delta_2}\right)^{\delta_2} \dots \left(\frac{(c_n^L)^{1-s}(c_n^U)^s \prod_{j=1}^{m} x_j^{\alpha_{nj}}}{\delta_n}\right)^{\delta_n}\right)$$

Or $$\left(\frac{g_0(x,s)}{\sum_{i=1}^{n} \delta_i}\right)^{\sum_{i=1}^{n} \delta_i} \geq \prod_{i=1}^{n}\left(\frac{(c_i^L)^{1-s}(c_i^U)^s \prod_{j=1}^{m} x_j^{\alpha_{nj}}}{\delta_i}\right)^{\delta_i} \qquad \left[as \sum_{i=1}^{n} \delta_k = 1\right]$$

Or $$g_0(x,s) \geq \left(\frac{(c_i^L)^{1-s}(c_i^U)^s}{\delta_k}\right)^{\sum_{i=1}^{n} \delta_i} \prod_{j=1}^{m} x_j^{\sum_{i=1}^{n} \alpha_{ij}\delta_i}$$

Or $$g_0(x,s) \geq \prod_{i=1}^{n}\left(\frac{(c_i^L)^{1-s}(c_i^U)^s}{\delta_i}\right)^{\delta_i} \prod_{j=1}^{m} x_j^{\sum_{i=1}^{n} \alpha_{ij}\delta_i}$$

$$= \prod_{i=1}^{n}\left(\frac{(c_i^L)^{1-s}(c_i^U)^s}{\delta_i}\right)^{\delta_i} = v(\delta, s) \qquad \left[as \sum_{k=1}^{T_0} \alpha_{0kj}\delta_{ok} = 0\right]$$

i.e., g_0(x, s) ≥ v(δ,s) .

Ex. 1: Minimize $\widetilde{Tac}$(Q,S) = $\frac{\widetilde{c_0}D}{Q} + \frac{\widetilde{c_h}(Q-S)^2}{2Q} + \frac{\widetilde{c_s}(S)^2}{2Q}$

subject to Q, S > 0.

With input values

Table-1 (Input data)

$\widetilde{c_0} = \widetilde{20}$	$\widetilde{c_h} = \widetilde{50}$	$\widetilde{c_s} = \widetilde{50}$	D
(16, 20, 24)	(40, 50, 60)	(40, 50, 60)	10

Using nearest approximation method

$\widetilde{20} = (16, 20, 24) \approx [18, 22] \approx 18^{1-s}22^s \in [18, 22]$;

$\widetilde{50} = (40, 50, 60) \approx [45, 55] \approx 45^{1-s}55^s \in [45, 55]; s \in [0, 1]$.

Then the problem is

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Min. Tac(Q,S,s)= $\frac{18^{1-s}22^{s}.10}{Q}$+ $\frac{45^{1-s}55^{s}(Q-S)^{2}}{2Q}$+ $\frac{45^{1-s}55^{s}(S)^{2}}{2Q}$

Sub. Q, S > 0.

i.e., Min. Tac(Q,S,s)= $\frac{18^{1-s}22^{s}.10}{Q}$+ $\frac{45^{1-s}55^{s}(S)^{2}}{Q}+\frac{45^{1-s}55^{s}Q}{2}-\frac{45^{1-s}55^{s}S}{1}$

Sub. Q, S > 0.

This is primal problem and corresponding dual problem is

$$v(\delta,s)=(\frac{10.16^{1-s}20^{s}}{\delta_1})^{\delta_1}(\frac{45^{1-s}55^{s}}{\delta_2})^{\delta_2}(\frac{45^{1-s}55^{s}}{2\delta_3})^{\delta_3}(\frac{45^{1-s}55^{s}}{\delta_4})^{-\delta_4}$$

Subject to

$$\delta_1+\delta_2+\delta_3-\delta_4=1,$$

$$-\delta_1-\delta_2+\delta_3=0,$$

$$2\delta_2-\delta_4=0,$$

Solving above equations, we have

$$\delta_4=2\delta_2,\ \ \delta_3=1-\delta_1-\delta_2+\delta_4=1-\delta_1-\delta_2+2\delta_2=1-\delta_1+\delta_2,$$

$$\delta_1+\delta_2=1-\delta_1+\delta_2,\ \delta_1=\tfrac{1}{2},\delta_3=\tfrac{1}{2}+\delta_2.$$

i.e., $v(\delta,s)=(\frac{10.16^{1-s}24^{s}}{1/2})^{1/2}(\frac{45^{1-s}55^{s}}{\delta_2})^{\delta_2}(\frac{45^{1-s}55^{s}}{2(0.5+\delta_2)})^{(0.5+\delta_2)}(\frac{45^{1-s}55^{s}}{2\delta_2})^{-2\delta_2}$ (5.7)

Taking log on both side of (5.7) and then partially differentiating with respect toδ_2 and using the conditions of finding optimal solution we get this equation

$$(\log 2.45^{1-s}55^{s}-log2\delta_2)+(log45^{1-s}55^{s}-log2(0.5+\delta_2)-2(log45^{1-s}55^{s}-\ log2\delta_2)=0$$

$$\Rightarrow log(2.45^{1-s}55^{s})/45^{1-s}55^{s}-log2\delta_2(1+2\delta_2)=0$$

$$\Rightarrow \delta_2(1+2\delta_2)=1.$$

From primal-dual relation

$$\frac{10.16^{1-s}24^{s}}{Q}=\delta_1 v(\delta),$$

$$\frac{45^{1-s}55^{s}S^{2}}{2Q}=\delta_2 v(\delta),$$

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$$\frac{45^{1-s}55^{s}Q}{2} = \delta_3 v(\delta),$$

$$45^{1-s}55^{s}S = \delta_4 v(\delta).$$

Solving above relations with difference values of weight, we get the list of values in table-2

Table -2: optimal solution

				Optimal values objectives	
s	1− s	Optimal dual variables	Optimal primal variables	$v(\delta)$	$Tac(Q,S)$
0.1	0.9	$\delta_1^* = 0.5$, $\delta_2^* = 0.5,$ $\delta_3^* = 1, \delta_4^* = 1.$	$S^* = 1.905$ $Q^* = 3.810$	87.464	87.464
0.3	0.7	$\delta_1^* = 0.5$, $\delta_2^* = 0.5,$ $\delta_3^* = 1, \delta_4^* = 1.$	$S^* = 1.944$ $Q^* = 3.889$	92.929	92.929
0.5	0.5	$\delta_1^* = 0.5$, $\delta_2^* = 0.5,$ $\delta_3^* = 1, \delta_4^* = 1.$	$S^* = 1.984$ $Q^* = 3.969$	98.736	98.736
0.7	0.3	$\delta_1^* = 0.5$, $\delta_2^* = 0.5,$ $\delta_3^* = 1, \delta_4^* = 1.$	$S^* = 2.026$ $Q^* = 4.051$	104.906	104.906
0.9	0.1	$\delta_1^* = 0.5$, $\delta_2^* = 0.5,$ $\delta_3^* = 1, \delta_4^* = 1.$	$S^* = 2.068$ $Q^* = 4.135$	111.462	111.462

6. FUZZY MODIFIED SIGNOMIAL GEOMETRIC PROGRAMMING PROBLEM (FMSGP)

6.1. Primal problem:

A primal modified signomial GP programming problem is of the form

Minimize $\tilde{g}_0(x_{lj})$ (6.1)

Subject to $x_{lj} > 0,$ j = 1, 2,……,m.

Where $\tilde{g}_0(x) = \sum_{l=1}^{n} \sum_{i=1}^{k} \sigma_{li} \tilde{c}_{li} \prod_{j=1}^{m} x_{lj}{}^{a_{lij}}.$

Using nearest interval approximation method, transformed all triangular fuzzy number into interval number i.e., $[c_{li}{}^{L}, c_{li}{}^{U}]$. Then the fuzzy signomial geometric programming problem is of the following form

Minimize$\hat{g}_0(x_1, x_2, \dots \dots \dots, x_m)$
(6.2)

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Subject to $x_j > 0$, j = 1, 2,……,m.

Where $\hat{g}_0(x) = \sum_{l=1}^{n} \sum_{i=1}^{k} \sigma_{li} \hat{c}_{li} \prod_{j=1}^{m} x_j^{a_{lij}}$.

Where $\hat{c}_{li}$ denotes the interval counter parts i.e., $\hat{c}_{li} \in [c_{li}^L, c_{li}^U]. c_{li}^L > 0, c_{li}^U > 0,$ for all i. Using parametric interval-valued functional form, the problem () reduces to

Minimize $g_0(x_1, x_2, \dots \dots \dots, x_m, s)$ (6.3)

Subject to $x_j > 0$, j = 1, 2,……,m.

Where $g_0(x,s) = \sum_{l=1}^{n} \sum_{i=1}^{k} \sigma_{li} (c_{li}^L)^{1-s} (c_{li}^U)^s \prod_{j=1}^{m} x_j^{a_{lij}}$.

This is a parametric geometric programming (PGP) problem.

Dual signomial GP problem:

Dual GP problem of the given primal GP problem is

$$\text{Maximize } \zeta_0 \left[\prod_{l=1}^{n} \prod_{i=1}^{k} \left(\frac{(c_{li}^L)^{1-s} (c_{li}^U)^s}{\delta_{li}} \right)^{\sigma_{li} \delta_{li}} \right]^{\zeta_0} \quad (6.4)$$

Subject to $\sum_{l=1}^{n} \sum_{i=1}^{k} \sigma_{li} \delta_{li} = \zeta_0$,

$$\sum_{l=1}^{n} \sum_{i=1}^{k} \sigma_{li} a_{lij} \delta_{li} = 0, \qquad j = 1,2, \dots \dots \dots, m.$$

$$\delta_{li} > 0,$$

Case I: nk≥nm+n, (i.e. DD>0) So the DP presents a system of linear equations for the dual variables. Here the number of linear equations is less than the number of dual variables. More solutions of dual variable vector exist. In order to find an optimal solution of DP, we need to use some algorithmic methods.

Case II: nk<nm+n, (i.e. DD <0) So the DP presents a system of linear equations for the dual variables. Here the number of linear equations is greater than the number of dual variables. In this case generally no solution vector exists for the dual variables. However, using Least Square (LS) or Min-Max (MM) method one can get an approximate solution for this system.

Furthermore the primal-dual relation is

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$(c_{li}{}^{L})^{1-s}(c_{li}{}^{U})^{s}\prod_{j=1}^{m}x_{lj}{}^{\alpha_{lij}} = \zeta_0 \delta^*{}_{li} \sqrt[n]{v(\delta^*)}$., $(l = 1,2, \dots\dots\dots, k; i = 1,2, \dots\dots\dots.., n)$,

$s\in[0,1]$. (6.5)

Note 2: A Weak Duality theorem would say that

$g_0(x_{lj}, s) \geq n\sqrt[n]{v(\delta)}$.

For any primal-feasible x and dual-feasible δ but this is not true of the pseudo-dual fuzzy modified signomial GP problem.

Corollary 2: When the values of σ_{li} is 1, then a fuzzy modified signomial geometric programming (FMSGP) problem transform to ordinary modified geometric programming problem.

Theorem 2: When σ_i is 1, then $g_0(x_{ij}, s) \geq n\sqrt[n]{v(\delta)}$ (Primal- Dual Inequality).

Proof.

The expression for $g_0(x_{ij}, s)$ can be written as

$$g_0(x_{ij}, s) = \sum_{l=1}^{n}\sum_{i=1}^{k}\delta_{ik}\left(\frac{(c_{li}{}^{L})^{1-s}(c_{li}{}^{U})^{s}\prod_{j=1}^{m}x_{ij}{}^{\alpha_{lij}}}{\delta_{ik}}\right).$$

Here the weights are $\delta_{l1}, \delta_{l2}, \dots\dots\dots, \delta_{lk}$ and positive terms are

$\frac{(c_{l1}{}^{L})^{1-s}(c_{l1}{}^{U})^{s}\prod_{j=1}^{m}x_j{}^{\alpha_{l1j}}}{\delta_{l1}}$,

$\frac{(c_{l2}{}^{L})^{1-s}(c_{li}{}^{U})^{s}\prod_{j=1}^{m}x_j{}^{\alpha_{l2j}}}{\delta_{l2}}$, ……… , $\frac{(c_{li}{}^{L})^{1-s}(c_{li}{}^{U})^{s}\prod_{j=1}^{m}x_j{}^{\alpha_{lnj}}}{\delta_{lk}}$.

Now applying A.M.-.G.M inequality, we get

$$\left(\frac{\sum_{l=1}^{n}((c_{l1}{}^{L})^{1-s}(c_{l1}{}^{U})^{s}\prod_{j=1}^{m}x_{ij}{}^{\alpha_{l1j}} + (c_{l2}{}^{L})^{1-s}(c_{l2}{}^{U})^{s}\prod_{j=1}^{m}x_{ij}{}^{\alpha_{l2j}} + \dots + (c_{li}{}^{L})^{1-s}(c_{li}{}^{U})^{s}\prod_{j=1}^{m}x_{ij}{}^{\alpha_{lkj}})}{\sum_{l=1}^{n}(\delta_{l1}+\delta_{l2}+\cdots+\delta_{lk})}\right)^{\sum_{l=1}^{n}(\delta_{l1}+\delta_{l2}+\cdots+\delta_{lk})}$$

$$\geq \sum_{l=1}^{n}\left(\left(\frac{(c_{l1}{}^{L})^{1-s}(c_{l1}{}^{U})^{s}\prod_{j=1}^{m}x_{ij}{}^{\alpha_{l1j}}}{\delta_{l1}}\right)^{\delta_{l1}}\left(\frac{(c_{l2}{}^{L})^{1-s}(c_{l2}{}^{U})^{s}\prod_{j=1}^{m}x_{ij}{}^{\alpha_{l2j}}}{\delta_{l2}}\right)^{\delta_{l2}}\dots\left(\frac{(c_{li}{}^{L})^{1-s}(c_{li}{}^{U})^{s}\prod_{j=1}^{m}x_{ij}{}^{\alpha_{lkj}}}{\delta_{lk}}\right)^{\delta_{lk}}\right)$$

Or $\left(\frac{g_0(x_{ij},s)}{\sum_{l=1}^{n}\sum_{i=1}^{k}\delta_{li}}\right)^{\sum_{l=1}^{n}\sum_{i=1}^{k}\delta_{li}} \geq \prod_{l=1}^{n}\prod_{i=1}^{k}\left(\frac{(c_{li}{}^{L})^{1-s}(c_{li}{}^{U})^{s}\prod_{j=1}^{m}x_{ij}{}^{\alpha_{lij}}}{\delta_{ik}}\right)^{\delta_{ik}}$

Or $\left(\frac{g_0(x_{ij},s)}{n}\right)^{n} \geq \prod_{l=1}^{n}\left(\frac{(c_{li}{}^{L})^{1-s}(c_{li}{}^{U})^{s}}{\delta_{lk}}\right)^{\sum_{i=1}^{k}\delta_{lk}}\prod_{j=1}^{m}x_{ij}{}^{\sum_{i=1}^{k}\alpha_{lij}\delta_{li}}$ $[as\ \sum_{i=1}^{k}\delta_{li} = 1]$

$$= \prod_{l=1}^{n}\prod_{i=1}^{k}\left(\frac{(c_{li}{}^{L})^{1-s}(c_{li}{}^{U})^{s}}{\delta_{li}}\right)^{\delta_{li}}\prod_{j=1}^{m}x_{ij}{}^{\sum_{i=1}^{k}\alpha_{lij}\delta_{li}}$$

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Or

$$\left(\frac{g_0(x_{ij},s)}{n}\right)^n \geq \prod_{l=1}^{n}\prod_{i=1}^{k}\left(\frac{(c_{li}{}^{L})^{1-s}(c_{li}{}^{U})^{s}}{\delta_{li}}\right)^{\delta_{li}} \qquad [as \sum_{i=1}^{k}\alpha_{lij}\delta_{li}=0]$$

$$= v(\delta)$$

i.e., $g_0(x_{ij},s) \geq n\sqrt[n]{v(\delta)}$.

Ex.2: Minimize $\widetilde{Tac}(Q_i,S_i) = \sum_{i=1}^{n}\frac{\widetilde{c_{0i}}D_i}{Q_i} + \frac{\widetilde{c_{hi}}(Q_i-S_i)^2}{2Q_i} + \frac{\widetilde{c_{si}}(S_i)^2}{2Q_i}$

subject to $Q_i, S_i > 0.$

With input values

Table: 3 (Input data)

i	$\widetilde{c_{0i}} = \widetilde{20}$	$\widetilde{c_{hi}} = \widetilde{50}$	$\widetilde{c_{si}} = \widetilde{50}$	D_i
i=1	(16, 20, 24)	(40, 50, 60)	(40, 50, 60)	10
i=2	(6, 10, 14)	(105, 125, 145)	(21, 25, 29)	15

Using nearest approximation method

$\widetilde{10} = (6,10,14) \approx [8,12] \approx 8^{1-s}12^s \in [8,12];$

$\widetilde{20} = (16,20,24) \approx [18,22] \approx 18^{1-s}22^s \in [18,22];$

$\widetilde{25} = (21,25,29) \approx [23,27] \approx 23^{1-s}27^s \in [23,27];$

$\widetilde{50} = (40,50,60) \approx [45,55] \approx 45^{1-s}55^s \in [45,55];$

$\widetilde{125} = (105,125,145) \approx [115,135] \approx 115^{1-s}135^s \in [115,135]; \quad s \in [0,1].$

Then the problem is

$$\text{Min. Tac(Q,S,s)} = \frac{18^{1-s}22^s.10}{Q_1} + \frac{45^{1-s}55^s(Q_1-S_1)^2}{2Q_1} + \frac{45^{1-s}55^s(S_1)^2}{2Q_1} + \frac{18^{1-s}22^s.10}{Q_2} + \frac{45^{1-s}55^s(Q_2-S_2)^2}{2Q_2} + \frac{45^{1-s}55^s(S_2)^2}{2Q_2}$$

Sub. $Q_1, Q_2, S_1, S_2 > 0.$

i.e.,

$$\text{Min .Tac(Q,S,s)} = \frac{18^{1-s}22^s.10}{Q_1} + \frac{45^{1-s}55^s(S_1)^2}{Q_1} + \frac{45^{1-s}55^sQ_1}{2} - \frac{45^{1-s}55^sS_1}{1} + \frac{8^{1-s}12^s.15}{Q_2} + \frac{(115^{1-s}135^s+23^{1-s}27^s)(S_2)^2}{2Q_2} + \frac{115^{1-s}135^sQ_2}{2} - \frac{115^{1-s}135^sS_2}{1}$$

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Sub. $Q_1, Q_2, S_1, S_2 > 0.$

This is primal problem and corresponding dual problem is

$$v(\delta,s) = \left(\frac{10.16^{1-s}20^s}{\delta_{11}}\right)^{\delta_{11}}\left(\frac{45^{1-s}55^s}{\delta_{12}}\right)^{\delta_{12}}\left(\frac{45^{1-s}55^s}{2\delta_{13}}\right)^{\delta_{13}}\left(\frac{45^{1-s}55^s}{\delta_{14}}\right)^{-\delta_{14}}\left(\frac{15.8^{1-s}12^s}{\delta_{21}}\right)^{\delta_{21}}\left(\frac{(115^{1-s}135^s+23^{1-s}27^s)}{2\delta_{22}}\right)^{\delta_{22}}$$

.

Subject to

$$\delta_{11} + \delta_{12} + \delta_{13} - \delta_{14} = 1,$$

$$\delta_{21} + \delta_{22} + \delta_{23} - \delta_{24} = 1,$$

$$-\delta_{11} - \delta_{12} + \delta_{13} = 0,$$

$$-\delta_{21} - \delta_{22} + \delta_{23} = 0,$$

$$2\delta_{12} - \delta_{14} = 0,$$

$$2\delta_{22} - \delta_{24} = 0.$$

Solving above equations, we have

$$\delta_{14} = 2\delta_{12},\ \delta_{13} = 1 - \delta_{11} - \delta_{12} + \delta_{14} = 1 - \delta_{11} - \delta_{12} + 2\delta_{12} = 1 - \delta_{11} + \delta_{12},$$

$$\delta_{11} + \delta_{12} = 1 - \delta_{11} + \delta_{12}$$

$$\delta_{11} = \frac{1}{2}, \delta_{13} = \frac{1}{2} + \delta_{12}.$$

And

$$\delta_{24} = 2\delta_{22},\ \delta_{23} = 1 - \delta_{21} - \delta_{22} + \delta_{24} = 1 - \delta_{21} - \delta_{22} + 2\delta_{22} = 1 - \delta_{21} + \delta_{22},$$

$$\delta_{21} + \delta_{22} = 1 - \delta_{21} + \delta_{22}$$

$$\delta_{21} = \frac{1}{2}, \delta_{23} = \frac{1}{2} + \delta_{22}.$$

i.e., $v(\delta,s) = \left(\frac{10.16^{1-s}24^s}{1/2}\right)^{1/2}\left(\frac{45^{1-s}55^s}{\delta_2}\right)^{\delta_2}\left(\frac{45^{1-s}55^s}{2(0.5+\delta_2)}\right)^{(0.5+\delta_2)}\left(\frac{45^{1-s}55^s}{2\delta_2}\right)^{-2\delta_2}\left(\frac{15.8^{1-s}12^s}{1/2}\right)^{1/2}$

$$\times\left(\frac{115^{1-s}135^s+23^{1-s}27^s}{2\delta_2}\right)^{\delta_2}\left(\frac{115^{1-s}135^s}{2(0.5+\delta_2)}\right)^{(0.5+\delta_2)}\left(\frac{115^{1-s}135^s}{2\delta_2}\right)^{-2\delta_2} \qquad (6.6)$$

Taking log on both side of (6.6) and then partially differentiating with respect to δ_{12} and δ_{22} respectively and using the conditions of finding optimal solution we get;

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$$(\log 2.45^{1-s}55^{s} - log2\delta_{12}) - 0.5 + (log45^{1-s}55^{s} - log2(0.5 + \delta_{12}) - 0.5 - 2(log45^{1-s}55^{s} - \ log2\delta_{12}) + 1 = 0$$

$$\Rightarrow log(2.45^{1-s}55^{s})/45^{1-s}55^{s} - log2\delta_{12}(1 + 2\delta_{12}) = 0$$

$$\Rightarrow \delta_{12}(1 + 2\delta_{12}) = 1.$$

And

$$(\log 2.45^{1-s}55^{s} - log2\delta_{22}) - 0.5 + (log45^{1-s}55^{s} - log2(0.5 + \delta_{22}) - 0.5 - 2(log45^{1-s}55^{s} - log2\delta_{22}) + 1 = 0$$

$$\Rightarrow \log(2.45^{1-s}55^{s})/45^{1-s}55^{s} - log2\delta_{22}(1 + 2\delta_{22}) = 0$$

$$\Rightarrow \delta_{22}(1 + 2\delta_{22}) = 1.$$

From primal-dual relation

$$\frac{10.16^{1-s}24^{s}}{Q_1} = \delta_{11}v(\delta),$$

$$\frac{45^{1-s}55^{s}S_1{}^{2}}{Q_1} = \delta_{12}\sqrt{v(\delta,s)},$$

$$\frac{45^{1-s}55^{s}Q_1}{2} = \delta_{13}\sqrt{v(\delta,s)},$$

$$45^{1-s}55^{s}S_1 = \delta_{14}\sqrt{v(\delta,s)},$$

$$\frac{15.8^{1-s}12^{s}}{Q_2} = \delta_{21}\sqrt{v(\delta,s)},$$

$$\frac{(115^{1-s}135^{s} + 23^{1-s}27^{s})S_2{}^{2}}{2Q_2} = \delta_{22}\sqrt{v(\delta,s)},$$

$$\frac{115^{1-s}135^{s}Q_2}{2} = \delta_{23}\sqrt{v(\delta,s)},$$

$$115^{1-s}135^{s}S_2 = \delta_{24}\sqrt{v(\delta,s)}.$$

Solving above relations with difference values of weight we get the list of values in table-4.

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Table 4: optimal solution

				Optimal values objectives	
s	1− s	Optimal dual variables	Optimal primal variables	$v(\delta, s)$	$Tac(Q, S)$
0.1	0.9	$\delta_{11}{}^* = 0.5$, $\delta_{12}{}^* = 0.5$, $\delta_{13}{}^* = 1, \delta_{14}{}^* = 1$, $\delta_{21}{}^* = 0.5$, $\delta_{22}{}^* = 0.5$, $\delta_{23}{}^* = 1, \delta_{24}{}^* = 1$.	$S_1{}^* = 1.971$, $Q_1{}^* = 3.912$, $S_2{}^* = 0.774$, $Q_2{}^* = 1.549$.	8187.095	195.347
0.3	0.7	$\delta_{11}{}^* = 0.5$, $\delta_{12}{}^* = 0.5$, $\delta_{13}{}^* = 1, \delta_{14}{}^* = 1$, $\delta_{21}{}^* = 0.5$, $\delta_{22}{}^* = 0.5$, $\delta_{23}{}^* = 1, \delta_{24}{}^* = 1$.	$S_1{}^* = 2.008$, $Q_1{}^* = 4.015$, $S_2{}^* = 0.795$, $Q_2{}^* = 1.590$.	9205.062	206.989
0.5	0.5	$\delta_{11}{}^* = 0.5$, $\delta_{12}{}^* = 0.5$, $\delta_{13}{}^* = 1, \delta_{14}{}^* = 1$, $\delta_{21}{}^* = 0.5$, $\delta_{22}{}^* = 0.5$, $\delta_{23}{}^* = 1, \delta_{24}{}^* = 1$.	$S_1{}^* = 2.045$, $Q_1{}^* = 4.090$, $S_2{}^* = 0.816$, $Q_2{}^* = 1.633$.	10349.600	219.326
0.7	0.3	$\delta_{11}{}^* = 0.5$, $\delta_{12}{}^* = 0.5$, $\delta_{13}{}^* = 1, \delta_{14}{}^* = 1$, $\delta_{21}{}^* = 0.5$, $\delta_{22}{}^* = 0.5$, $\delta_{23}{}^* = 1, \delta_{24}{}^* = 1$.	$S_1{}^* = 2.083$, $Q_1{}^* = 4.166$, $S_2{}^* = 0.838$, $Q_2{}^* = 1.677$.	11636.450	232.372
0.9	0.1	$\delta_{11}{}^* = 0.5$, $\delta_{12}{}^* = 0.5$, $\delta_{13}{}^* = 1, \delta_{14}{}^* = 1$, $\delta_{21}{}^* = 0.5$, $\delta_{22}{}^* = 0.5$, $\delta_{23}{}^* = 1, \delta_{24}{}^* = 1$.	$S_1{}^* = 2.122$, $Q_1{}^* = 4.244$, $S_2{}^* = 0.861$, $Q_2{}^* = 1.722$.	13083.300	246.191

7. CONCLUSION

In this paper, a fuzzy EOQ model with shortages under fully backlogging and constant demand is formulated and solved. Here the model is solved by fuzzy signomial geometric programming (FSGP) technique. For fuzzy coefficient we used only triangular fuzzy number (TrFN). In future other types of fuzzy numbers would be used. The methodology proposed in this paper may also be applicable to other EOQ models.

Our approach provide here a simple EOQ model, but in the future it should be used many complex EOQ models. For future research of uncertainty in economic order quantity (EOQ) model, by using different type of fuzzy numbers such as pentagonal, hexagonal fuzzy numbers of generalized fuzzy numbers be analytically more challenging and interesting. Inflation plays an important role in present day-to-day life, but we have neglected it. Therefore, consideration of inflation problem would be more realistic.

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