CO-OCCURRENCE MATRIX AND FRACTAL DIMENSION FOR IMAGE SEGMENTATION

MATRIZ DE CO-OCURRENCIA Y DIMENSIÓN FRACTAL EN SEGMENTACIÓN DE IMÁGENES

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Abstract

One of the most important tasks in image processing problem and machine vision is object recognition, and the success of many proposed methods relies on a suitable choice of algorithm for the segmentation of an image. This paper focuses on how to apply texture operators based on the concept of fractal dimension and cooccurence matrix, to the problem of object recognition and a new method based on fractal dimension is introduced. Several images, in which the result of the segmentation can be shown, are used to illustrate the use of each method and a comparative study of each operator is made.

Keywords: Image segmentation, texture, fractal dimension, co-occurrence matrix.

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Resumen

Una de las tareas más importantes en el procesamiento de imá-
genes es el reconocimiento de objetos, y el éxito de muchos de los
métodos propuestos se basa en la adecuada elección del algoritmo
para la segmentación de la imagen. Este trabajo se enfoca en cómo
aplicar los operadores de textura basados en los conceptos de di-
mensión fractal y de matrices de co-occurencia, al problema de re-
conocimiento de objetos y se introduce un nuevo método basado en
la dimensión fractal. Para ilustrar cada método se utilizan imágenes
en las que se muestra el resultado de la segmentación y se realiza un
estudio comparativo de cada operador.

Palabras clave: Segmentación de Imágenes, texturas, dimensión fractal,
matriz de co-occurencia.

Mathematics Subject Classification: 60J05, 62M40.

1   Introduction

Texture, an intrinsic property of object surface, is used by our visual per-
ception system to understand a scene; therefore texture analysis is an
important component of image processing. But texture is not easy to
quantify since there is neither a consensus on its definition nor a precise
mathematical formulation for it. When the visual examination of an im-
age suggests that the basis for discriminating various structural regions
is a texture rather than color or brightness, it may be possible to use a
simple texture operator. The texture analysis problem includes classifica-
tion/discrimination, description and segmentation of texture regions in a
scene. This paper focuses on the problems of texture segmentation.

Texture analysis methods can be divided into two classes: structural
methods and statistics methods. On the one hand, in structural methods
the texture of the image can be considered as the regular repetition of a
micro-texture with some rule and the image can be considered as a fractal
surface, and it is possible to think that the fractal dimension may be ap-
propriate for the characterization of the image. The fractal dimension has
been used as descriptor for the image segmentation by texture operators
in the last years. It was introduced by Pentland [8] who observed that
fractal dimension was relatively insensitive to image scaling and showed
strong correlation with human judgments of surface roughness. Kuklinski
[2] applied fractal analysis techniques to medical images, for automatic
identification of cancerous growth regions; Zwiggelaar & Bull [12] applied
them to the problem of plant recognition in synthetic aperture radar (SAR)
images. These and other applications can be seen in [7, 9, 10, 11]. These

are just a few applications that clearly demonstrate serious interest in fractal based methods for low-level recognition problems. There are alternative techniques for computing fractal dimension, that use a local operator to measure the local fractal dimension in each pixel of an image. Three different techniques to do it are: the Box-counting Method, the Hurst's Method and the Method of the Range, introduced in this report, which is an easy technique that produces good results.

On the other hand, the purpose of statistical methods is to characterize the stochastic properties in the spatial distribution of the gray level in the image. Haralik [3, 4, 5] developed techniques using first order statistics and introduced second-order statistics, more precisely texture measures based on gray level co-occurrence matrices. Each element of the gray level co-occurrence matrix represents the relative frequency with which two neighboring pixels separated by a distance of Δ_x columns and Δ_y lines occur, one with gray tone g and the other with grey tone g'. Such matrices of gray level spatial dependence frequencies are a function of the angular relationship between the neighboring resolution cells as well as the function of the distance between them. In this paper only five of the texture features extracted from the gray level co-occurrence matrix are presented, they are: entropy, contrast, energy, local homogeneity and correlation.

All the algorithms presented in this paper are unsupervised in the following sense: it does not require a priori knowledge on the texture of the image, no learning steps in particular are necessary; the number of regions or texture classes needs not to be known either; only one parameter is needed in the threshold segmentation procedure. All the techniques discussed in this research are neighborhood operators that access pixels in a small area around each central pixel, perform some calculation with those values, and then derive a new value for the central pixel. In all the cases, the new value is used to produce a new image and it is the original values of pixels which are used in the neighborhood around the next pixel as the operator is repeated throughout the image.

The aim of this report is to compare the effectiveness of some texture features based on fractal dimension and that of based on co-occurrence matrices.

This paper is organized in the following manner: Section 2 (Co-occurrence matrix) presents the gray level co-occurrence matrices, the texture features extracted from them and it also shows the results of these algorithms to some images; Section 3 (Fractal Dimension) presents the concepts of fractal dimension and how to implement these concepts in the images with three different techniques to estimate the fractal dimension, particularly the introduced Method of the Range, describes each algorithm in pseudo code and shows the results of these algorithms, implemented on

C+, to some images. Finally, Section 4 (Conclusions) provides conclusions of comparing these techniques.

2   Co-occurrence matrix

The grey level co-occurrence matrix is defined as a joint distribution of the grey levels of two pixels separated by a given displacement τ. In cartesian coordinates the displacement can be chosen as a vector $\tau = (\Delta_x, \Delta_y)$. Each element in the matrix is $A_\tau(g, g')$ is defined as

$$A_\tau(g, g') = \{\# \operatorname{pair}(g, g') | \operatorname{image}(x, y) = g \wedge \operatorname{image}(x + \Delta_x, y + \Delta_y) = g'\},$$

where g and g' are grey levels in the image. The size of the co-occurrence matrix A_τ is $G \times G$, denoting G the number of grey levels in the image

We illustrate this concept with a binary model and its matrix A_τ for $\tau = (0, 1)$:

1	1	1	1	1
1	1	1	1	0
1	1	1	0	0
1	1	0	0	0
1	0	0	0	0

$$A_\tau = \begin{array}{c|cc} & 0 & 1 \\ \hline 0 & 6 & 0 \\ 1 & 4 & 10 \end{array}$$

Haralick et al, [3] described fourteen not independent features extracted from the grey level co-occurrence matrix, and showed that when they were applied to large rectangular areas in satellite photos, these parameters could distinguish water from grassland, different sandstones and woodland from urban regions. The texture features extracted from the grey level co-occurrence matrix that are applied to this research are presented below and their physical meaning is also given

a) *Energy*

$$\sum_{g,g'} A_\tau(g, g')^2.$$

Energy is also called angular second moment. Energy measures textures uniformity, that is, pixel pairs repetition. When the image under consideration is homogenous (only similar gray level pixels are present), energy reaches its maximum. Thus, high values of energy occur when the grey level distribution over the window has either a constant or a periodic form.

b) *Entropy*

$$-\sum_{gg'} A_\tau(g,g') \log \left(A_\tau(g,g') \right).$$

Entropy measures the disorder of an image. When the image is not texturally uniform, entropy is very large. Conceptually, energy and entropy are inversely correlated.

c) *Contrast*

$$\sum_{g,g'} (g - g')^2 A_\tau(g,g').$$

Spatial frequency is the difference between the highest and the lowest values of a contiguous set of pixels. The gray level co-occurrence matrix contrast is the spatial frequency where the displacement is $\tau = (\Delta_x, \Delta_y)$. Contrast is a measure of the liner variation presented in the image.

d) *Local homogeneity*

$$\sum_{g,g'} \frac{A_\tau(g,g')}{1 + (g - g')^2}.$$

Local homogeneity is also called inverse difference moment and it measures images homogeneity. It assumes larger values for smaller grey tone difference in pair elements. The local homogeneity and the energy are strongly but inversely correlated.

e) *Correlation*

$$\sum_{g,g'} \frac{(g - \mu_1) \left(g' - \mu_2 \right) A_\tau(g,g')}{\sigma_1 \, \sigma_2}.$$

Correlation is a measure of grey level linear dependencies in the image, high correlation values imply a linear relationship between the grey levels of pixel pair. Thus the correlation is uncorrelated to energy and entropy, that is, to pixel pair repetitions

One of the most important decisions when texture features based on co-occurrence matrix are used is how to choose the displacement. Usually the grey levels co-occurrence matrices are computes for different values of τ and that one which maximizes some statistic is chosen. The suggested statistic is

$$\chi^2(\tau) = \sum_{g,g'} \frac{A_\tau^2(g,g')}{A_\tau(g,\cdot) \, A_\tau(\cdot,g')},$$

that can be understood as a normalization of the scene energy to the grey level linear dependence in the image.

(a)

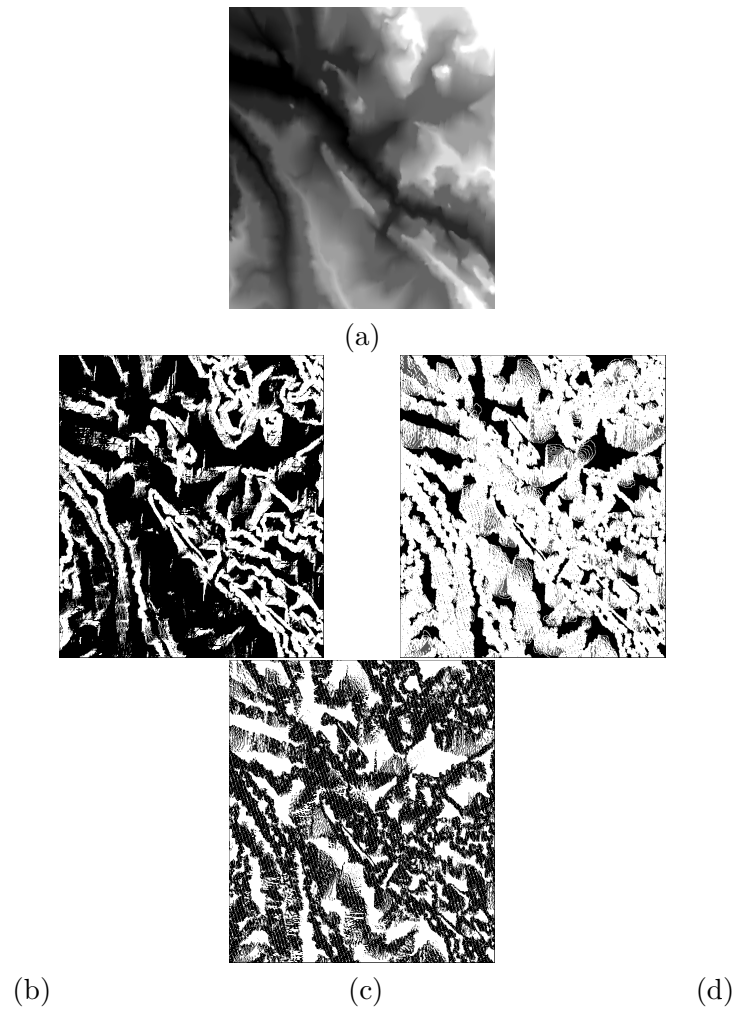(b)   (c)   (d)

Figure 1: *(a) Original Image. (b) Energy Operator (c) Entropy Operator. (d) Contrast Operator.*

Some of these operations are obviously easier than others to calculate for all the pixels in an image. The resulting values are scaled to create a new image that can be segmented by brightness thresholding. Sometimes experimentation with several texture operators is required to find the one that gives the best separation between the object and its surrounding and in consequence the best segmentation.

The results of applying some of these operators is illustrated in figure 1. Figure 1 (a) is the original image, (b) shows the computation of the Energy Operator, (c) shows the computation of the Entropy Operator and (d) shows the computation of the Contrast Operator.

3 Fractal dimension

Let $\mathcal{H}(\Re^n)$ be the space of all nonempty compact subsets of the Euclidean space $\Re^n$. A set $A \subset \mathcal{H}(\Re^n)$ is said to be self similar if A is the union of $N(\epsilon)$ distinct and non overlapping copies of itself each of which is similar to A scaled down by a radio ϵ. The fractal dimension D of A can be defined by

$$D = \lim_{\epsilon \to \infty} \frac{\ln N(\epsilon)}{\ln(1/\epsilon)}, \tag{1}$$

in other words $N(\epsilon)$, the number of rules of length ϵ required to cover the set A, is related to the fractal dimension D by the equation $N(\epsilon).\epsilon^D = 1$.

The following sub-sections focus on three methods for estimating the fractal dimension on an image: the Box-counting method, that is one of the simplest methods to implement and visualize; the Hursts method, that is particularly efficient on medical images, and the Method of the Range which has been introduced in this paper. Some experimental results of each method are shown using one particular image to compare them.

3.1 Estimating fractal dimension

3.1.1 Box-counting Method

There are several approaches for estimating the fractal dimension in an image, one of them is the Box Counting method. The method consists on laying finer grids atop the image and counts the number of grid-squares which contain a part of the image.

There exists several methods to compute this estimate; most of them are applied to images that must be turned into binary images. A method that can be applied to images with different gray levels in the interval $[0, 255]$, where 0 corresponds to black and 255 to white is descibed in [1].

The fractal dimension is based on equation (1). In our method, $N(\epsilon)$ is counted in the following manner. Let us consider that the image of size $M \times M$ pixels has been scaled down to a size $s \times s$ where $M/2 \geq s \geq 1$ and s is an integer. Then we have an estimate of $r = s/M$. Now, let us consider the image as in 3D space with (x, y) denoting 2D position and the third coordinate z denoting the gray level. The (x, y) is partitioned into grids of size $s \times s$. On each grid there is a column of boxes of size $s \times s \times s$, this is, a partition on the gray levels is made. Let k and l be the minimum and maximum gray level of the image in (i, j)-th grid fall in box number, then the contribution of $N(\epsilon)$ in the pixel (s) of the grid is $n_\epsilon(s) = l - k + 1$. Taking contributions from all grids, we have

$$N(\epsilon) = \sum_s n_\epsilon(s).$$

Then $N(\epsilon)$ is computed to different values of ϵ, and the fractal dimension can be estimated from the least square linear fit of $\ln N(\epsilon)$ against $\ln \epsilon$.

Box Dimension Algorithm

- A centered squared window of size $l_1 = 17$ is taken for each pixel.

- The window is divided in $(1/\epsilon)^2$ squares and the range of intensities is divided into $(1/\epsilon)$ levels, for five given values of $\epsilon \in (0, 1]$.

- Denoting l and k the maximum and minimum gray level in the square respectively, $n_\epsilon(s) = l - k + 1$ is computed for each pixel and $N(\epsilon) = \sum_s n_\epsilon(s)$ is computed for each ϵ.

- The line $y = ax + b$ is fitted to the pairs $(\ln N(\epsilon), \ln \epsilon)$ and the fractal dimension is estimated with the slope of the fitted line, $D = a$.

- The histogram with all the values D is constructed and, choosing a threshold h, the image is recolored putting black on each pixel where $D > h$ and white on each pixel where $D \leq h$.

This technique is illustrated in figures 2 and 3. Figure 2 (a) shows a coast, (b) shows the computation of the box dimension in each pixel, and (c) shows the segmented image. This leads us to verify that this technique is particularly efficient in coastal images. Figure 3 is another application of the segmentation method for recognizing certain city blocks in a city.
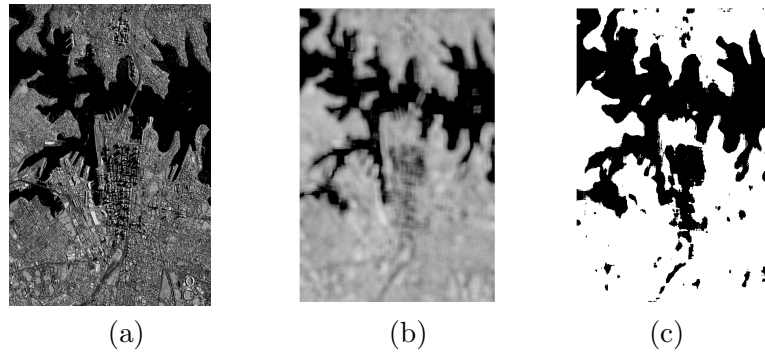
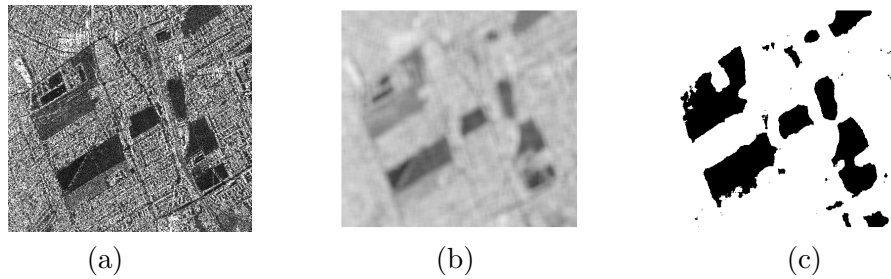Figure 2: *(a) Original Image. (b) Box Dimension. (c) Segmented Image.*

Figure 3: *(a) Original Image. (b) Box Dimension. (c) Segmented Image.*

3.1.2 Hurst's method

A particularly efficient method for computing the fractal dimension of surfaces from elevation images is the Hurst coefficient, or rescaled range analysis. This procedure plots the greatest difference in brightness between points along a linear traverse of the image or surface as a function of the search distance, on log-log axes. When the range is scaled, dividing by the standard deviation of the data, the slope of the resulting line is directly related to the fractal dimension of the profile.

The size of the window used to process the image is a compromise between the desire to include many pixel values, for accurate results, and the need for fast calculation. For each pixel, we consider a neighborhood region consisting of thirty seven pixels, in a seven pixel wide octagonal shape, as shown in Figure 3. Each pixel in the diagram is labeled to indicate its distance from the center of the octagon. The distances, in pixel units, and the number of pixels at each distance are listed in Table 1.

		h	g	h		
	f	e	d	e	f	
h	e	c	b	c	e	h
g	d	b	a	b	d	g
h	e	c	b	c	e	h
	f	e	d	e	f	
		h	g	h		

Figure 4: *Octagonal neighborhood for Hurst's method.*

Pixel class	Number of pixels	Distance from center: d
a	1	0
b	4	1
c	4	$\sqrt{2}$
d	4	2
e	8	$\sqrt{5}$
f	4	$\sqrt{8}$
g	4	3
h	8	$\sqrt{10}$

Table 1: *Classification of the pixels from Figure 4.*

The application of the operator proceeds to examine the pixels in the neighborhood around each pixel in the original image. The brightest and darkest pixel values are found for each of the distance classes and their distance used to construct a Hurst plot. Performing a least squares fit of the slope of the log (brightness difference) vs. log (distance) relationship is simplified because of the distance values, and their logarithms, are unvarying and can be stored beforehand in a short table. It is also unnecessary to divide by the standard deviation of pixel brightness in the image, since this is a constant for each pixel in the image and the slope of the Hurst plot will be arbitrarily scaled to fit the brightness range of the display anyway.

Hurst's method Algorithm

- A centered octagonal window as in figure 1 is taken for each pixel.

- Denoting $M_d = \{$Maximum gray level over the pixels $t : dist(t,s) < d\}$ and $m_d = \{$Minimum gray level over the pixels $t : dist(t,s) <$

d}, the brightness range $r_d = M_d - m_d$ is computed for each distance d.

- The line $y = ax + b$ is fitted to the pairs $(\log(d), \log(r_d))$.

- The fractal dimension is estimated according to the slope of the fitted line, $D = a$.

- The histogram with all the values D is constructed and, choosing a threshold h, the image is recolored putting black on each pixel where $D > h$ and white on each pixel where $D \leq h$.

This technique is illustrated in figures 5 and 6. Figure 5 (a) shows a coast, (b) shows the computation of the Hurst's dimension in each pixel and (c) shows the segmented image. The segmentation is not as good as before. The edges are not well defined. Figure 6 is another application of the method in a medical image, a mammogram, where the technique is more efficient.
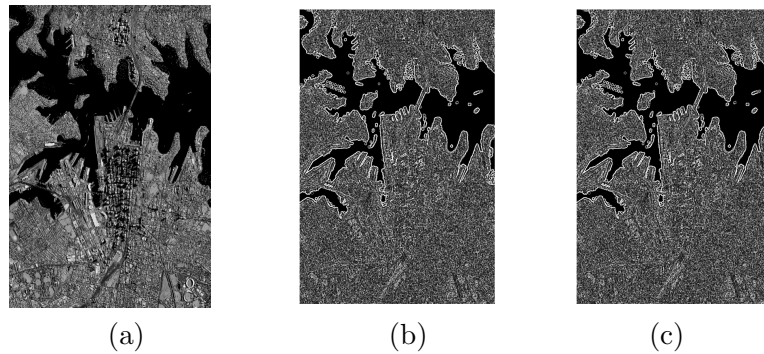

(a)   (b)   (c)

Figure 5: *(a) Original Image. (b) Hurst Dimension. (c) Segmented Image.*

3.1.3   Method of Range

In this subsection a variation of the box counting method is introduced. The method computes the fractal dimension as a function of the brightness range, keeping the idea of refining the grid over the image, while improving the efficiency of the computation, [6].

Method of Range Algorithm

- A centered squared window of size $l_1 = 9$ is taken for each and the brightness range as $r_1 = M_1 - m_1$ is computed, where M_1 is
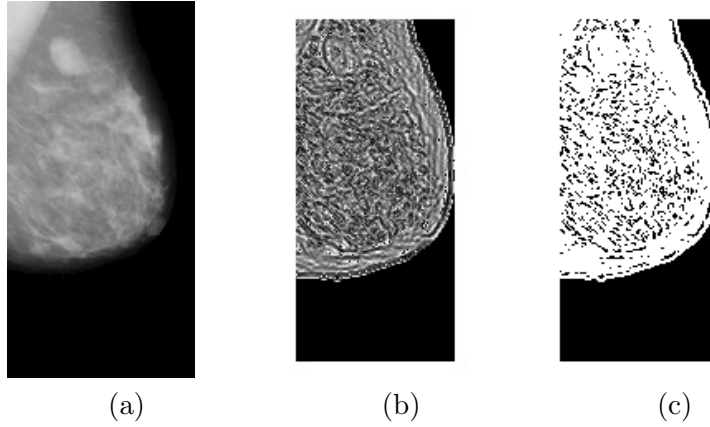
Figure 6: *(a) Original Image. (b) Hurst Dimension. (c) Segmented Image.*

the maximum brightness, and m_1 is the minimum brightness in the window.

- For the same pixel, we take a small squared window of size $l_2 = 5$, centered at that pixel. We compute the brightness range as $r_2 = M_2 - m_2$, where M_2 is the maximum brightness, and m_2 is the minimum brightness in the window.

- We estimate the fractal dimension as

$$D = \frac{r_1 - r_2}{\ln l_1 - \ln l_2}. \tag{2}$$

- The histogram with all the values D is constructed and, choosing a threshold h, the image is recolored putting black on each pixel where $D > h$ and white on each pixel where $D \leq h$.

Equation (2) represents the ratio between the difference of the ranges in each window and the proportion of the length of each window, in log scale. This technique is illustrated in figures 7 and 8. Figure 7 (a) shows a coast, (b) shows the computation of the fractal dimension with the method of Range in each pixel and (c) shows the segmented image, with well defined edges but with no homogeneous regions. Figure 8 is another application where the method is extremely efficient.

4   Conclusions

Now that the algorithms have been described, they could be compared. Perhaps a more important indication of a segmentation algorithm's useful-
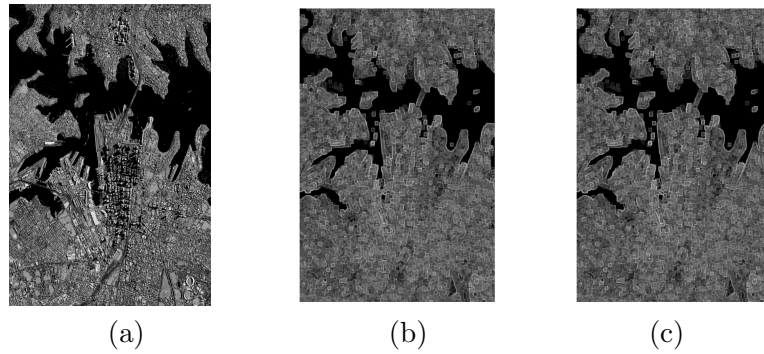
Figure 7: *(a) Original Image. (b) Method of Range. (c) Segmented Image.*

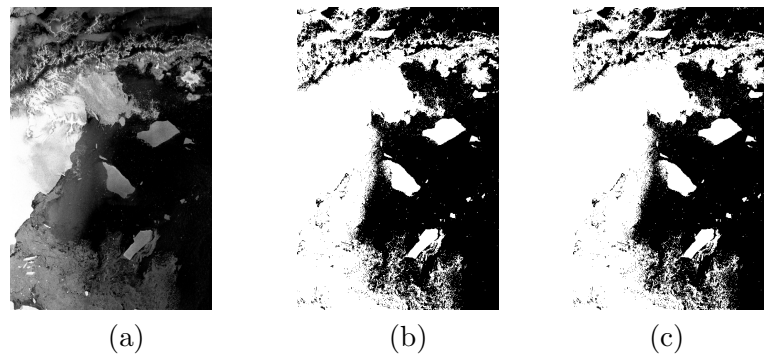Figure 8: *(a) Original Image. (b) Method of Range. (c) Segmented Image.*

ness is its stability. If an algorithm reasonably gives correct segmentation on average, but is unpredictable on any given image, it will be useless as a preprocessing step. Segmentation with low bias and low variance are required, in other words algorithms that have low variability with respect to their parameters. In this paper each method was applied to the segmentation of a series of different images to gauge the strengths and weaknesses of each one.

About the texture features based on co-occurrence matrices, energy or angular second moment, contrast and entropy were chosen. All of them were computed for different values of τ and the chosen was $\tau = (0,1)$. The operator energy has been efficient and the operator entropy has been the best to define edges, but as their computational coast was too expensive, the methods based on fractal dimension are preferred. About the three operators based on local fractal dimension, the Box Dimension, the Hurst's Method, and the introduced Method of the Range, it is well known that

the box dimension algorithm is extremely efficient in coast image, but the Method of the Range has also been effective in those kind of images and it is computationally faster, moreover the Method of the range defined the edges very well, although the segmentation was not so homogeneous in the big regions. The lower computational weight is not the only advantage of the method of the Range, it has surprisingly been effective when the image had dimensions that took values near $D = 2$ and $D = 3$. The Hurst's Method was originally proposed to medical images and was very efficient in them, but it has been heavier, computationally talking, than the method of the Range, which also obtained acceptable results in those kinds of images. The algorithms were compared all using the same group of images some of them natural images, others taken from the Brozart album and few of them SAR images. The introduced method of the Range could be used to build a supervised classifier based on a priori segmented images such as training blocks.

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