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DETECTION OF UNOBSERVED HETEROGENEITY WITH GROWTH MIXTURE MODELS

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Abstract

Latent growth curve models as structural equation models are extensively discussed in various research fields (Duncan et al., 2006). Recent methodological and statistical extension are focused on the consideration of unobserved heterogeneity in empirical data. Muthén extended the classical structural equation approach by mixture components, i. e. categorical latent classes (Muthén 2002, 2004, 2007).

The paper will discuss applications of growth mixture models with data from one of the first panel studies in Germany which explore deviant and delinquent behavior of adolescents (Reinecke, 2006a, 2006b). Observed as well as unobserved heterogeneity will be considered with growth mixture models using the program M*plus* (Muthén & Muthén, 2006). Special attention is given to the distribution of the substantive dependent variables as a count measures (Poisson distribution, zero-inflated Poisson distribution, cf. Nagin, 1999). Different model specifications with respect to substantive questions will also be emphasized.

Keywords: Panel data, growth mixture models, heterogeneity, Poisson distribution.

Resumen

Los modelos latentes de curvas de crecimiento, como modelos de escuaciones estructurales, son ampliamente discutidos en varios campos de investigación (Duncan et al., (2006)). Extensiones metodológicas y estadísticas recientes se enfocan en la consideración de heterogeneidad no observada en datos empíricos. Muthén extendió el enfoque clásico de ecuaciones estructurales por componentes de mezcla, es decir clases latentes categóricas (Muthén 2002, 2004, 2007).

El artículo discute aplicaciones de modelos de crecimiento de mezcla con datos de uno de los primeros estudios de panel en Alemania, que explora comportamiento

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desviado y delinquivo de adolescentes (Reinecke, 2006a, 2006b). La heterogeneidad observada y no observada será considerada con modelos de crecimiento de mezcla usando el programa M*plus* (Muthén & Muthén, 2006). Se dará especial atención a la distribución de las variables sustantivas dependientes como medidas de conteo (distribución de Poisson, distribución cero-inflada de Poisson, cf. Nagin, 1999). Se dará énfasis también a diferentes especificaciones de modelos con respecto a cuestiones importantes.

Palabras clave: Datos de panel, modelos de mezclas de crecimiento, heterogeneidad, distribución de Poisson.

Mathematics Subject Classification: 62P25.

1 Introduction

Longitudinal research studies with repeated measurements are quite often used to examine processes of stability and change in individuals or groups. With panel data it is possible to investigate intraindividual development of substantive variables across time as well as interindividual differences and similarities in change patterns. While the traditional analysis of variance (ANOVA) and the analysis of covariance (ANCOVA) assume homogeneity of the underlying covariance matrix across the levels of the between-subjects factors and the same covariance patterns for the repeated measurements, the structural equation methodology offers an alternative strategy: the *latent growth curve models*. These models describe not only a single individual's developmental trajectory, but also capture individual differences in the intercept and slopes of those trajectories. Based on the formative work of Rao and Tucker's basic model of growth curves (Rao, 1958; Tucker, 1958), Meridith and Tisak (1990) discussed and formalized the model within the structural equation framework. Further developments of the growth curve model were proposed by McArdle and Epstein (1987), McArdle (1988) and Muthén (1991, 1997). Extensive applications of different growth curve models with structural equations using the programs LISREL (Jöreskog, K. G. & Sörbom, 2004), EQS (Bentler, 2001) and M*plus* (Muthén & Muthén, 2006) are discussed by Duncan et al. (2006).

Observed heterogeneity in growth curve models can be captured by covariates explaining part of the variances of the intercept and slope. But the assumption of a single population underlying the growth curves has to be relaxed in the case of unobserved heterogeneity. Instead of considering individual variation around a single growth curve, different classes of individuals should vary around different mean growth curves. A very suitable framework to handle the issue of unobserved heterogeneity is *growth mixture modeling* introduced by Muthén and Shedden (1999). These mixture models differ between continuous and categorical latent variables. The categorical latent variables represent mixtures of subpopulations where the product membership is inferred from the data. Like the conventional growth curve models, intercept and slope variables capture the continuous part of the model. Growth mixture models can also be seen as an extension of the structural modeling approach with techniques of latent class analysis. The inferred membership of each individual to a certain class is produced with the information of the estimated latent class probabilities. Further developments and applications with the program M*plus* (Muthén & Muthén, 2006) are discussed in several papers by Muthén (2001a, 2001b, 2003, 2004). Recently, Muthén (2007) gives an model overview of the so-called *latent variable hybrids* within the continuous and categorical latent variable framework.

The simplest specification of a growth mixture model is *latent class growth analysis* where no variation across individuals are allowed within classes. This model labeled as a "semiparametric group-based approach" was originally discussed by Nagin and Land (1993), Nagin (1999) and Roeder, Lynch and Nagin (1999) with measurements of deviant and delinquent behavior. The authors discuss also the possibility to treat their measurements as counts with the Poisson distribution as the underlying statistical model (see, e. g., Ross, 1993). If the count variables are biased to zero, i. e. the particular behaviors seldom occur, a variant of the Poisson model, the so-called zero-inflated Poisson model (Lambert, 1992), should lead to a better statistical representation of the data than a model without considering the zero inflation.

After the introduction of growth curve and growth mixture models including their special cases (Section 2), applications are shown with longitudinal panel data from representative panel study of adolescents' deviant and delinquent behavior (Section 3). Results of the models are discussed and summarized in Section 4. The article concludes with suggestions for further research with growth mixture models.

2 Growth curve and growth mixture models

The possibility that the individual trajectories of a dependent variable can vary is one of the main advantages of the growth curve model. The formal representation of a growth curve model can be seen either as a multilevel, random-effects model or as a latent variable model, where the random effects are latent variables:

$$y_i = \Lambda \eta_i + \epsilon_i \tag{1}$$

y_i is a $p \times 1$ vector of repeated mesurements for observation i where p is the number of panel waves. η is a $q \times 1$ vector of latent growth factors where q is the number of latent growth factors. ϵ is a $p \times 1$ vector of time-specific measurement errors, and Λ is the $p \times q$ matrix of factor loadings with fixed coefficients representing the functional form of the individual trajectories. Variations of individual trajectories are captured by q-numbers of latent variables η whereas η_1 is the *intercept*, η_2 is the *linear slope* and in case of nonlinear development η_3 represents the *quadratic slope.* Equation 1 assumes that all individuals are drawn from the same population. The means of the latent variables η shows the average development of a particular longitudinal variable within a homogenous population.

Growth mixture models can relaxe the assumption of an homogenous population and can give information about parameter differences across unobserved subpopulations. Instead of considering individual variation of single means of the vector η the growth mixture model allows different classes of individuals to vary around different means. Classes are introduced by a latent categorical variable where the categories represent the unobserved heterogeneity of the data (Muthén & Shedden, 1999):

$$y_{ik} = \Lambda_k \eta_{ik} + \epsilon_{ik} \tag{2}$$

The growth mixture model (abbreviated GMM) in Equation 2 allows the estimation of $k = 1, \ldots K$ latent classes each with its own latent growth model. The probability density function for the GMM is a finite mixture of normal distributions:

$$f(y_i) = \sum_{k=1}^{K} \pi_k \phi_k [y_i; \mu_k(\theta_k) \Sigma(\theta_k)] \tag{3}$$

π_k is the unconditional probability that a measurement belongs to latent class k, ϕ_k is the multivariate probability density function for latent class k. $\mu_k(\theta_k)$ represents the model-implied mean vector given by

$$\mu_k(\theta_k) = \Lambda_k \alpha_k \tag{4}$$

and $\Sigma_k(\theta_k)$ is the model-implied covariance matrix given by

$$\Sigma_k(\theta_k) = \Lambda_k \Psi_k \Lambda_k^{'} + \Theta_k \tag{5}$$

In an unconditional mixture model the latent variables η are only described by their class specific means α_k and variances Ψ_k. A conditional mixture model includes exogenous latent variables ξ_n represent the observed heterogeneity of the data. The relation between ξ_n and the categorical class variable c is given by a multinomial logistic regression equation:

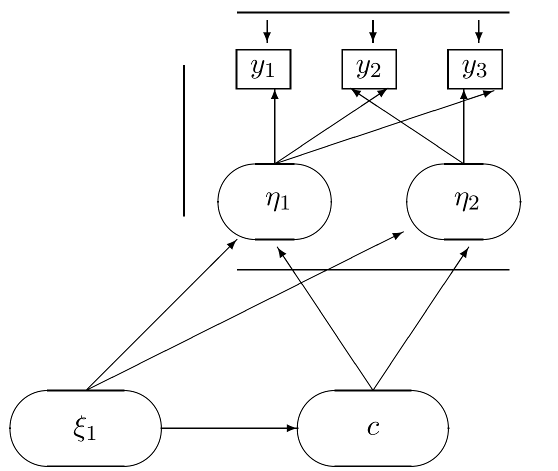
$$logit(\pi_k) = \alpha_k + \Gamma_k \xi_n \tag{6}$$

with $\pi_k = P(c_k = k|\xi_n)$. Γ_c is a $(K-1) \times q$-parameter matrix containing regression coefficients of K classes on ξ_n. Figure 1 gives an example of a growth mixture model with intercept and linear slope variables (η_1 and η_2) and one exogenous latent variable ξ_1.

Growth mixture models are estimated by maximizing the log likelihood function within the admissible range of parameter values given classes and data. M*plus* uses the principle of maximum likelihood estimation and employs the EM-algorithm for maximization (Dempster, Laird & Rubin, 1977; Muthén & Shedden, 1999). For a given solution, each individual's probability of membership in each class is estimated. Individuals can be assigned to the classes by calculating the posterior probability that an individual i belongs to a given class k. Each individual's posterior probability estimate for each class is computed as a function of the parameter estimates and the values of the observed data (Muthén & Muthén, 2001: 367f.). By classifying each individual into his most likely class, a table with rows corresponding to individuals classified into a given class can be constructed. The columns of that table show the average conditional probabilities to be in the particular class. Quality of the classification is summarized by the entropy measure E_K (Muthén & Muthén, 2001: 372), which ranges from zero to one, where values close to one indicate a good classification of the data.

Standard errors of estimates are asymptotically correct if the underlying mixture model is the true model. In general, test statistics require well-defined classes in a mixture

Figure 1: General Growth Mixture Model (GMM).



A measurement model for the exogenous variable ξ_1 is ommitted.

model. In mixture models a k class model is not nested within a $k+1$ group model. Therefore, conventional mixture tests like the Akaike Information Criterion (AIC; Akaike, 1987) and the Bayesian Information Criterion (BIC; Schwartz, 1978) have to be used for model comparisons. Usually, the model with the smallest AIC or BIC is accepted within model comparisons. Furthermore, M*plus* calculates a sample size adjusted BIC with $n = (n + 2)/24$ which was found to give superior performance for model selection (Yang, 1998). But accepting or rejecting a model on the basis of the AIC or BIC is more or less descriptive and does not imply any statistical test.

Lo, Mendell, and Rubin (2001) proposed a likelihood ratio-based method for testing $k-1$ classes against k classes in mixture models. The Lo-Mendell-Rubin likelihood ratio test (LMR-LRT) considers the usual likelihood ratio for testing the $k-1$ model against a k model but with the correct distribution. The p-value from the test represents the probability that H0 is true, i. e., that the model is sufficient with one less class. Therefore, a low p-value indicates that the $k-1$ class model has to be recjected and the k-class model is sufficient to represent the mixture of the data. LMR-LRT has been critized by Jeffries (2003), but importance of the critics in applications is unknown (Muthén, 2004: 356).

McLachlan and Peel (2000) suggested another likelihood-based technique to compare the mixture models: the bootstrapped likelihood ratio test (BLRT). This method uses bootstrap samples to estimate the distribution of the log likelihood difference test statistic. Instead of assuming that the difference distribution follows a known distribution like the chi-square distribution, the BLRT empirically estimates the difference distribution. Similar to the LMR-LRT, the BLRT provides a p-value that can be used to compare the increase in model fit between the $k-1$- and k-class models.

Latent Class Growth Analysis

A special case of the growth mixture model is the latent class growth analysis (LCGA), which has been studied by Nagin and Land (1993), Nagin (1999) and Roeder, Lynch and Nagin (1999). LCGA is a submodel of the GMM and characterized by having zero variances and covariances of the intercept and slope variables (η_1 and η_2 in Figure 1). Individuals within a class are treated as homogeneous with respect to their development.

LCGA can serve as a starting point for growth model analyses although trajectories of the indviduals are fixed within the class. LCGA can give information about the latent classes representing substantially different trajectories. In addition, the zero variance restriction leads to a fast convergence of the estimation (Muthén 2004: 350).

Poisson Models

To study development of deviant and delinquent behavior is one of the main topics in criminal sociology or criminology. Very often the longitudinal data gives information about the incidence rate of that behavior or the number of convictions. From a methodological point of view the distribution of those variables are counts and have to be treated differently compared to continuous data. The so-called "key approach in the modeling of delinquent and criminal careers" (Land, McCall & Nagin, 1996) is the Poisson distribution with the corresponding regression models.

Let $Y = 0, 1, 2 \ldots$ be a random variable for a given time interval and y be the number of observed occurences. The number of events in an interval of a given length is Poisson distributed with the probability density function:

$$Pr(Y = y) = e^{-\nu} \left[\frac{\nu^y}{y!} \right] \tag{7}$$

The expected value or mean of the Poisson distribution is $E(Y) = \nu$ with $Var(Y) = \nu$. Usually, the parameter ν is refered as the mean rate of occurrence of events. Small values of ν yield high probability for zero occurences of the random variable Y. The higher the value of ν, the lower the skewness of the distribution.

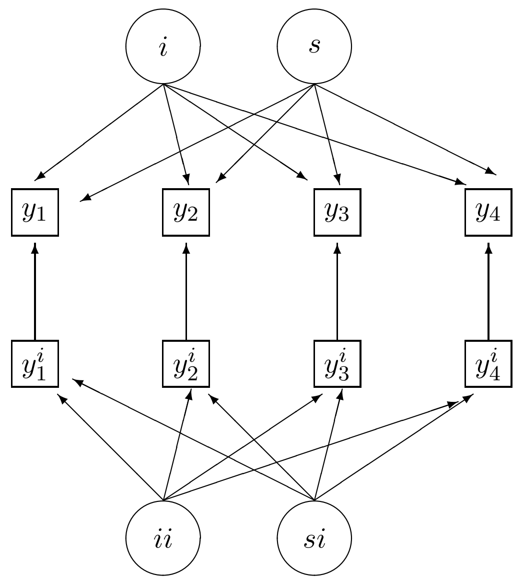
To cover the unobserved heterogeneity, a Poisson-based latent class growth model can be formulated which is also implemented in M*plus* (Muthén & Muthén, 2004: 190):

$$ln(\nu_{itk}) = \lambda_{1tk}\eta_{1k} + \lambda_{2tk}\eta_{2k} \tag{8}$$

ν_{itk} is the expected number of occurences of the measurement y of individual i at time t given the membership in class k. The conditional number of events, $P(y_{itk}|k)$, should follow the Poisson distribution.

If the number of zeros in the count variable are very large, a variant of the Poisson regression model is more appropriate: the so-called *zero-inflated Poisson model* (ZIP) originally proposed by Lambert (1992). The ZIP model combines the Poisson regression model with a logit model to cover the zero inflation in the count variable Y with probability p that Y is zero (Lambert, 1992: 3). Two parallel growth mixture models are estimated simultaneously when zero inflation of the data is assumed: The first model contains the count part of the outcomes with values of zero and above (Variables y_1 to y_4 in Figure 2).

Figure 2: Two-part Growth Mixture Model with Zero-inflated Measurements.



Intercepts of the outcomes are fixed to zero as the default. The means of growth curve variables (Variables i, s) are estimated for each class. The second model refers to the zero-inflation part of the outcome with only values of zero in all measurements (Variables y_1^i to y_4^i). Intercepts of the outcomes are estimated and held equal as the default. The mean of the intercept variable (Variable ii) are fixed to zero for all classes while the mean of the slope (Variable si) is estimated and held equal for all classes (cf. Muthén & Muthén, 2004: 190).

3 Applications with panel data

The empirical basis for the following analysis with mixture models is taken from the longitudinal research project *Juvenile Delinquency in Modern Towns*.[1] The main focus of the study is on the emergence and the development of deviant and delinquent behavior of juveniles, and the social control surrounding it; both formal, meaning the police and the judiciary, and informal, referring to school and family. The panel data contains self-administered interviews with pupils from the town of Duisburg located north of Cologne in West Germany. The initial survey was conducted in the year 2002 with pupils from 7th grade, considering all relevant school types in the community. The same age cohort have been yearly interviewed, and currently the 7th wave is in progress.

[1]This interdisciplinary research project is located at the universities of Münster (Institute of Criminology) and Bielefeld (Faculty of Sociology) and supported by the German National Science Foundation (DFG) under grant numbers Bo1234/6 and Re832/4. More information can be found on the website of the project (www.uni-bielefeld.de/soz/krimstadt).

The analysis of the unobserved heterogeneity in our sample by means of LCGA has been conducted using five panel waves with the subjects' age ranging from about 13 to 17. First of all, some preliminary assumptions regarding the modeling strategies are required. One important aspect concerns the expected development over time of the outcome of interest. The mean values of the prevalence rates over the five panel waves clearly show a curvilinear development. Furthermore, this assumption is confirmed by the comparison of the model fit statistics between the quadratic and the linear growth curve models, which suggests that the linear model has to be rejected in favour of a quadratic development (table not reported).

Another important aspect in the model choice process is the distribution of the outcome variables. Due to the large number of non-offenders, namely those who never reported the commission of deviant behaviors, the data are highly skewed because of the inflation with zero values. Consequently, our analyses assume a zero-inflated Poisson distributed model (LCGA-ZIP) with a curvilinear develpoment. It will form the basis for the subsequent analysis of unobserved heterogeneity.

In order to determine the most adequate number of classes to represent the sample' distribution a comparison of the different model fit measures is needed. The results are reported in Table 1.

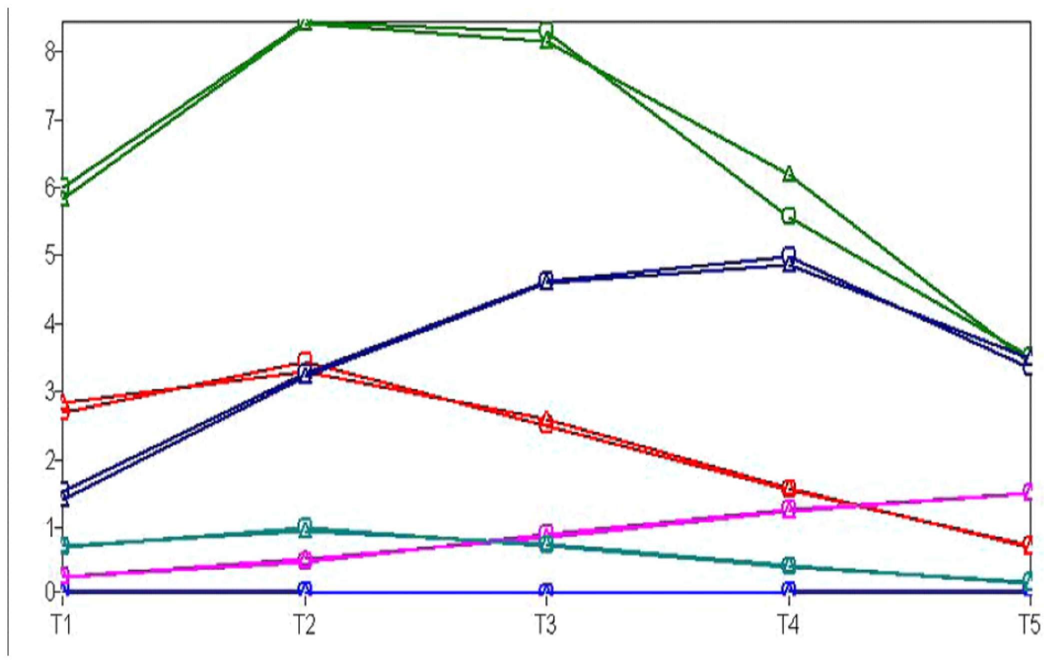
Table 1: Comparison of Unconditional Models with Different Classes (Duisburg).

LCGA-ZIP (Quadratic Model)							
Test	C1	C2	C3	C4	C5	C6	C7
BIC	18880	16557	15890	15699	15606	15535	15513
Adj. BIC	18861	16525	15845	15643	15537	15452	15417
LMR-LRT	–	2274	673	212	118	97	50
p-value	–	0.00	0.00	0.00	0.03	0.02	0.07
p-value							

The BIC and the adjusted BIC favor a solution with seven classes, although their values show a larger decrease between the models with five and six classes (a difference of 85 units compared to 35 units when a seventh class is added). The LMR-LRT points out to a solution with six classes as the best model, whereas its value for a seventh class is not significant. The results are in part confirmed by the BLRT that shows a small value for the model containing six classes. The same value increases when one more class is added, suggesting that a seventh class is of minor importance and redundant. Thus, six classes are chosen to represent the unobserved heterogeneity in the sample.

Figure 3 shows the class trajectories of the zero-inflated model with one class of non-offenders and five classes with different offending patterns. The classification of the subjects is based on the most likely latent class membership, which groups pupils with a high probability to share a common delopmental trajectory over time.

Figure 3: Six Class Quadratic-LCGA-ZIP for Five Panel Waves.



From the bottom to the top of the figure, the first class consists of 826 so called *non-offenders* (53%) who almost never reported offenses over the covered time period, and includes the majority of the participants. This class is well represented by a straight line. A model in which the intercept and slopes of this class are held equal to zero does not show any improvement in model fit, supporting the assumption that sporadic offences might occur even within this group of subjects (the specification of a so called zero-class has been proposed by Kreuter and Muthén, 2008). The second class represents the so called *low increasers*, with 197 subjects (13%). This is the single group which shows a moderate, but still evident increase across the whole time period. The third class, which is the second largest with 279 subjects (18%), is defined as *low stable*. Its developmental pattern shows some deviant activity at the beginning of the survey, which decreases constantly over time heading toward zero at the last time point. The fourth class represents the 3% of the sample (49 individuals), and is named *increasers* after its trajectory shows a clear increment in offenses from a mean value of 1.54 (t_1) to 4.63 (t_3), especially in early adolescence. Afterwards, however, its crime rates tend to stabilize, and in the last wave they show a modest decrement to an average of 3.34 (t_5). Another important group of offenders is represented by the *high-level desisters* which includes 174 pupils (11%). Their criminal activity is quite consistent at the first time point (with a mean rate of 2.69 in t_1), although they constantly desist throughout the entire panel, where in the last panel wave, they report a mean value close to zero (0.74 in t_5). The last class includes 27 (2%) *high rate* offenders. Although they clearly show a tendency to desist, their level of offending starts high. A peak around the age of 14 and 15 with a mean value of 8.44 (t_2) is reached. This group still maintains at a considerable level in the fifth panel wave.

The development of the classes over time is remarkable; all but the non-offenders show a trajectory which tends to decrease by the end of the time period under study. Yet, this is not the case for the *low increasers* which, on the contrary report a slow but constant increase in offending rates. Further waves will be conducted to gain a better understanding of the future development of the different patterns of delinquent behavior.

4 Discussion

The general framework of growth mixture modeling outlined by Muthén (2002, 2004) integrates several approaches to longitudinal growth modeling, e. g., the semiparametric group-based model developed and applied by Nagin and Land (1993) and Nagin (1999). This model is equivalent to the latent class growth model (LCGA), in which the intercept and slope variances are fixed to zero. Due to an easier estimation of the parameters the latent class growth analysis is computationally less demanding and thus useful for a first evaluation of the unobserved heterogeneity in the data. Thereafter, the variability of the class specific intercepts and slopes can be studied with the more general growth mixture models. If count data with largely positive skewed distributions are analyzed, the assumption of continuously distributed variables can be replaced by the Poisson or the zero-inflated Poisson distribution. M*plus* allows the above mentioned tests of growth mixture models assuming different distributions of the manifest variables under study.

Data from a five-wave panel study of adolescents have been used to study unobserved heterogeneity in the development of deviant and delinquent behavior. In a first step the data have been analzyed by means of a general latent growth model. A curvilinear trajectory fits the data well and it is favored to a linear development. A negative quadratic random effect suggests that the level of offending increases across the first three time points and decrease therafter. Due to the large number of zeros in the sample (namely those who never reported offences), the data are highly skewed. This condition justify the assumption of a Poisson distributed model with zero inflation. Following a stepwise procedure - as outlined in Kreuter and Muthén (2008) - the quadratic growth specification is used as the initial model for the analysis of unobserved heterogeneity in the sample. The comparison of several model fit criteria (e.g. BIC, adjusted BIC and LMRT-LRT) for the unconditional latent class growth models (LCGA) suggests that one class of non-offenders and five classes of offenders well represent the mixture of the data. Among the mentioned offender classes, all report a decreasing trajectory, whereas only one class (*low increasers*) shows a constant increase in offending over time. Even the *high rates*, although maintaining a significant involvement in deviant activities, reports signs of desistance (mainly in the last two panel waves). These findings add new informations to the results previously obtained with only four waves with the same sampled cohort. In fact, Reinecke (2006a, 2006b) obtained similar patterns of development with different panel data of the same substantive topic, where only three classes were necessary to represent the mixture of the data. These classes are replicated in the current analysis but with additional patterns of *desisters* and *low increasers*. So, the current panel study shows a greater versatility of delinquent behaviors. All in all, this suggest the importance of expanding the time range of the analysis in order to improve the understanding of the substantive phenomenon, especially in a cohort of adolescents where behavioral instability of different deviant behaviors can be expected.

Great attention has been paid to the process of model specification and model selection. Furthermore, the new results are supported by previous researches on the same data, suggesting a high stability in the mixture distribution. Nonetheless, the present analysis shows some limitations. On the one hand, although the five-wave panel design has given important insight in the subject, much more is to expect when new panel waves will be available. On the other hand, the current discussion on the topic suggests great caution in the interpretation of mixture analyses. As Bauer and Curran (2003) clearly demonstrate, there is the risk that a mixture is found which is only an approximation of a non-normal, but homogeneous, distribution in the data, and do not represent real groups of individuals in the population. To this respect, the introduction of theoretically meaningful covariates, in the form of either antecendents, time varying covariates, or distal outcomes, is expected to enhance the capability of the model to define the unobserved mixture, and eventually spot possible misspecification (Muthén, 2003). Therefore, further tests of growth mixture models should also include substantively relevant time-dependent (Hussong et al., 2004) and time-independent (Muthén, 2003) variables, although those extensions are beyond the scope of the present article.

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