Hydrodynamic Evolution Near QCD Critical Point

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Some consequences of the presence of critical point in the equation of state on the hydrodynamical evolution of the strongly interacting matter are discussed. For this purpose, we apply the low energy effective theory of QCD, the Nambu-Jona-Lasinio model and show some examples.

Keywords: Piston effect; QCD critical point; Hydrodynamics; Spinodal decomposition

I. INTRODUCTION

Hydrodynamics is usually adequate to describe the collective flow phenomena in macroscopic systems where thermodynamical relations are valid locally. Surprisingly enough, sometimes the hydrodynamical description is also useful as a model to describe the collective phenomena in microscopic and semi-microscopic systems. In fact, the collective flow behavior of hot, dense matter produced in relativistic heavy-ion collisions are amazingly well described by the hydrodynamical models [1]. In the most of these models, the hot matter is approximated as an ideal fluid.

In spite of the success, there still remain several open questions for hydrodynamical approach: the setup of the initial conditions, the finite volume effect, causality and dissipation, the freeze out mechanism, and so on [2]. In this work, we focus on one aspect of the equation of state (EOS) used in the hydrodynamical approach. In usual hydrodynamical modelings of relativistic heavy ion collision processes, the bag-model type of EOS's are introduced. For these EOS's, the phase transition between quark and hadronic phases is of the first order. However, it has been suggested that the QCD phase diagram has the critical point and the phase transition is crossover at the low chemical potential region[3].

As a matter of fact, the presence of a critical point gives rise to various interesting effects such as the focusing phenomena[4], the enhancement of susceptibility and critical slowing down[5], the critical opalescence, the piston effect[6] and so on. Accordingly, these effects should have influence on the hydrodynamical evolution of the system, too. In this work, we investigate how the hydrodynamical evolution is affected by the presence of a critical point in the EOS of the QCD matter.

II. CRITICAL POINT AND NJL MODEL

For a systematic study of the effect of critical point, we should construct an appropriate EOS with the critical point usable for the hydrodynamical applications. The most promising way for realistic studies of the dynamics of the QCD matter, will be the one obtained from the lattice QCD calculation, but this is still difficult to discuss the finite chemical potential case. Thus, we have to look for other resources.

Some studies in this direction have already been done. For example, Nonaka and Asakawa discussed the hydrodynamics near the QCD critical point [4]. In this study, they used the so-called mapping relation to derive the EOS. The idea of mapping relation is as follows. First consider the critical behavior of the liquid-gas transition. It is believed that the universality class of the liquid-gas transition belongs to the same as that of the 3D Ising model. If this is the case, then one may extract the critical behavior of the liquid-gas transition from that of the Ising model from the knowledge of the relation between the thermodynamic quantities of the liquid-gas transition and those of the Ising model. For example, the order parameter Ψ of the Ising model is the magnetization density, but that of the liquid-gas transition is defined by the volume. Then, we have the following mapping relation,

$$\Psi = \frac{1}{V}\sum_i s_i \rightarrow \frac{V}{V_c} - 1,$$

where s_i is the spin at the lattice site i, and V_c denotes the critical volume at the critical point of the liquid-gas transition. Substituting the expression to the van der Waals equation of state, we obtain the equation for the order parameter. To match the equation with the Landau free energy of the Ising model, one should choose the following mapping relation,

$$h = -\frac{1}{p_c}(p - p_c) + \frac{1}{p_c}\left(\frac{\partial p}{\partial T}\right)_{cxs}(T - T_c),$$

where h denotes the external magnetic field, P_c and T_c means the pressure and the temperature at the critical point of the liquid-gas transition. The suffix *cxs* indicates the derivative along the coexisting line [6].

Nonaka and Asakawa applied the idea to obtain the QCD equation of state containing the critical point. They assume the universality class of QCD belongs to the same as that of the 3D Ising model, as is the case with the liquid-gas transition. They further assumed the following simple mapping relations between QCD and the Ising maple,

$$(T_{QCD} - T_c)/T_c = h, \quad (1)$$
$$(\mu_{QCD} - \mu_c)/\mu_{QCD} = -(T - T_c)/T_c. \quad (2)$$

By using the above mapping relations, one may construct the EOS corresponding to the QCD matter. However, the EOS obtained in this fashion can only describe the behavior of the matter near the critical point. Thus, they modified the derived EOS so as to reproduce the ideal gas QCD at high temperature and the hadron resonance gas EOS at the low temperature region, respectively.

The derived EOS seems to describe the anomalous behavior near the critical point and contains non-trivial fluctuations. However, they employed many assumptions whose validity is not well established. For instance, the universality class of the QCD phase transition is believed to be the same as that of the Heisenberg model, not the Ising model. In addition, we do not know how we can justify the mapping relation between QCD and the Ising model.

Hama et al. [7] introduced a QCD EOS with the critical point by modifying the usual bag model plus hadronic resonance gas method by a purely empirical interpolation of the two phases and applied to the hydrodynamical study of relativistic heavy ion collisions. However, although the realistic behavior of the hadronic resonance gas and the high temperature QGP state are incorporated correctly, this approach is too empirical to study the detailed behavior of the physical effect of the critical point.

The EOS can be estimated by using effective models of QCD, such as the Nambu-Jona-Lasinio (NJL) model, the sigma model, the QCD-like model, etc. The principal advantage of these models is that the critical point appears naturally even in the mean-field approximation in a simple fashion. Of course, the mean-field approximation cannot describe the effects of fluctuations sufficiently. Furthermore, these the effective models cannot describe the confinement-deconfinement phase transition.

In spite of the above shortcomings, we still expect that an effective theory is able to describe qualitatively the anomalous behaviors of thermodynamical quantities near the QCD critical point to some extent. For example, we can estimate the temperature and chemical potential dependences of the specific heat near the phase transition. At the phase transition, the specific heat shows discontinuity, and it becomes maximum at the critical point[8]. The anomalous behavior of the specific heat affects the behavior of fluid. The Navier-Stokes equation of the non-relativistic fluid is given by

$$\frac{DT}{Dt} - \left(1 - \frac{C_v}{C_p}\right)\left(\frac{\partial T}{\partial P}\right)_\rho \frac{DP}{Dt} = \frac{1}{\rho C_p}\lambda \nabla^2 T,$$

where

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla,$$

and C_v and C_p are the specific heat with constant volume and that with constant pressure, respectively. The r.h.s. of the equation is the thermal conduction term. Near the critical points, the specific heat with constant pressure often diverges. Thus, at first glance, the term seems to disappear near the critical point. This represents the critical slowing down of thermal conduction processes. On the other hand, the isobaric thermal expansion diverges at the critical point. Thus, the competition between them should carefully be investigated.

These singularities can be described in the effective theories. Thus, it is possible that the qualitative effect of the critical point can be discussed by using the EOS of an effective model. In this work, we use the EOS calculated by using the NJL model.

The NJL Lagrangian density in the chiral limit is given by

$$\mathcal{L}(x) = \bar{q}(x)[-i\partial\!\!\!/]q(x) - G[(\bar{q}(x)q(x))^2 + (\bar{q}(x)i\gamma_5\vec{\tau}q(x))^2],$$

where $\vec{\tau}$ is the Pauli matrix. In the mean-field approximation, the thermodynamic potential per unit volume ω is

$$\omega = \frac{m^2}{4G} - 12\int_0^{\Lambda_{\rm UV}} dk \frac{k^2}{2\pi^2}\left\{E_k + \frac{1}{\beta}\ln\left[1+e^{-\beta(E_k+\mu)}\right] + \frac{1}{\beta}\ln\left[1+e^{-\beta(E_k-\mu)}\right]\right\}, \quad (3)$$

where $\beta = 1/T$ and $E_k = \sqrt{k^2+m^2}$ with $m = -2G\langle\bar{q}q\rangle$ being the dynamically generated quark mass. The minimum of ω with respect to m gives the equilibrium value of the thermodynamical potential. Thus, the quark mass m is determined by the self-consistency condition,

$$\frac{\partial\omega(\beta,\mu,m)}{\partial m} = 0.$$

Depending on the possible solutions of the above equation, the phase of chiral transition is determined. The thermodynamical quantities are then calculated from

$$\frac{\partial\omega}{\partial T} = s,\ \frac{\partial\omega}{\partial\mu} = n$$

and

$$\varepsilon = -p + Ts + \mu n$$

where $p = -\omega$.

III. HYDRODYNAMIC EVOLUTION AND SPINODAL DECOMPOSITION

At the QCD critical point, it is considered that the transition is changed from crossover to first order. Thus, for the larger values of the chemical potential, the effect of the first order transition in hydrodynamical evolution becomes important. In a usual hydrodynamical models, where the local thermodynamical equilibrium is assumed, the EOS obtained from the Maxwell construction is applied. However, if the expansion rate of the QGP formed in relativistic heavy ion collisions is very large, there is a possibility that the expanding matter can be supercooled and undergoes the so-called spinodal decomposition. An analogous process is believed to be the case in nuclear fragmentation phenomena [9].

The effect of the critical point to the spinodal decomposition is one of interesting problem. To study this problem, we need to calculate the pressure for the states out of the thermal

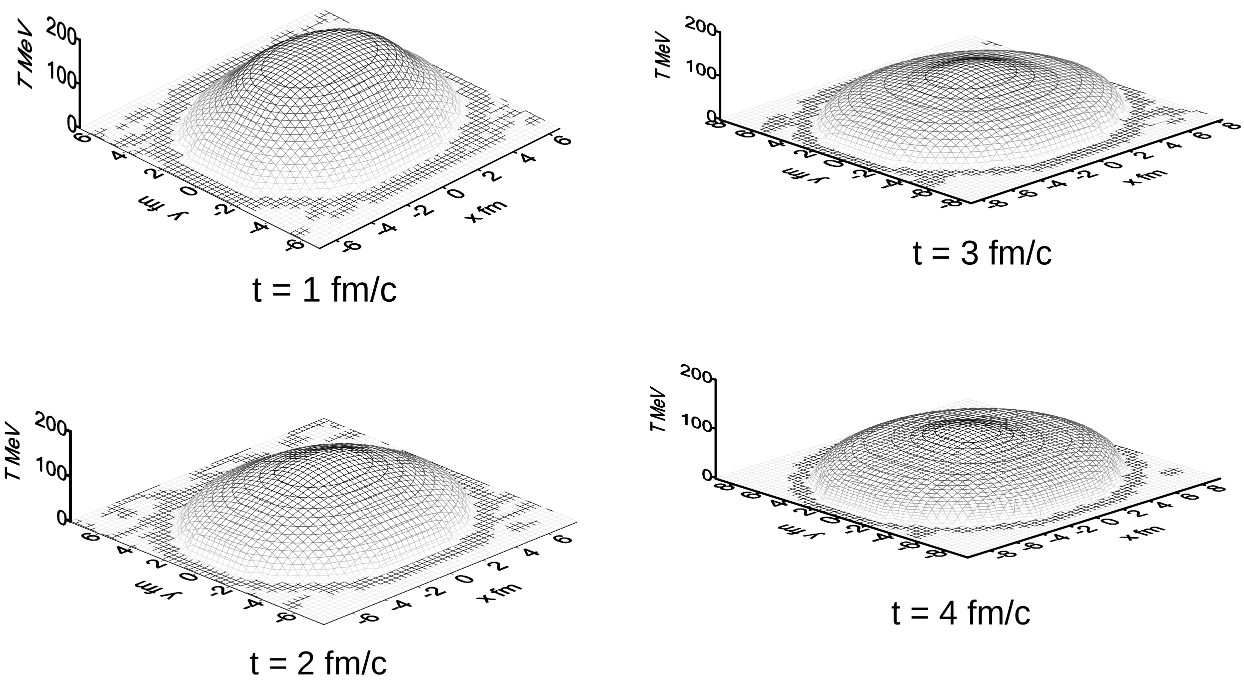

FIG. 1: The time evolution of temperature with the EOS calculated in the NJL model at vanishing chemical potential.

equilibrium. The EOS calculated by the effective field theoretical models can be used to simulate such processes, when the dynamics of the order parameter is known as is the case of the linear sigma model. Aguiar et al. derived the coupled equations of motion between the relativistic hydrodynamical collective motion of the matter and the effective dynamics of the order parameter in terms of variational principle [**?**]. The NJL approach, the quark condensate is the order parameter, but we have to introduce the kinetic energy associated to this parameter.

As the first step of studying the effect of critical point on hydrodynamics, we performed the numerical study of the hydrodynamical motion by using the EOS calculated in the NJL model. We adapted the smoothed particle hydrodynamics (SPH) to solve the hydrodynamic model [1] with our EOS. Here, for simplicity, we discuss the evolution for the transverse direction at vanishing chemical potential.

In Fig. 1, we show the numerical results. One can see that the temperature decreases as the system expands. Because of the second order phase transition at the vanishing chemical potential, the evolution of the temperature shows simple monotonic decrease. This behavior is different from the result with the usual EOS of the first order phase transition, where the evolution of the temperature is decelerated during the coexisting phase because of the disappearance of the gradient of pressure [1]. Thus, the order of the phase transition can affect the physical observables. For example, the elliptic flow with assuming the second order phase transition (or crossover) is expected to be smaller than that with assuming the first order phase transition [7].

IV. SUMMARY AND PERSPECTIVES

We used the NJL model to obtain the EOS with the critical point, and applied the EOS to discuss the hydrodynamic evolution at vanishing chemical potential. To discuss the effect of the critical point, we have to extend our calculation to finite chemical potential case.

Near the critical point, the competition between the second and first order phase transitions will give rise to nontrivial behavior. In particular, there is a possibility of the spinodal decomposition in the first order phase transition, although this has not yet discussed in the relativistic heavy-ion collisions.

To see the effect of the critical point, we have to generalize our numerical code to the finite chemical potential region.

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