# Implications of a Decay Law for the Cosmological Constant in Higher Dimensional Cosmology and Cosmological Wormholes

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*Higher dimensional cosmological implications of a decay law for the cosmological constant term are analyzed. Three independent cosmological models are explored mainly:*

*1) – In the first model, the effective cosmological constant was chosen to decay with times like* $\Lambda_{effective} = Ca^{-2} + D(b/a_I)^2$ *where* $a_I$*is an arbitrary scale factor characterizing the isotropic epoch which proceeds the graceful exit period. Further, the extra-dimensional scale factor decays classically like* $b(t) \approx a^x(t)$*, x is a real negative number.*

*2) – In the second model, we adopt in addition to* $\Lambda_{effective} = Ca^{-2} + D(b/a_I)^2$*the phenomenological law* $b(t) = a(t)\exp(-Qt)$ *as we expect that at the origin of time, there is no distinction between the visible and extra dimensions; Qis a real number.*

*3) – In the third model, we study a* $\Lambda-$*decaying extra-dimensional cosmology with a static traversable wormhole in which the four-dimensional Friedmann-Robertson-Walker spacetime is subject to the conventional perfect fluid while the extra-dimensional part is endowed by an exotic fluid violating strong energy condition and where the cosmological constant in (3+n+1) is assumed to decays like* $\Lambda(a) = 3Ca^{-2}$.
*The three models are discussed and explored in some details where many interesting points are revealed.*

Keywords: Extra-dimensions, Dark energy, Phantom energy, Superacceleration, $\Lambda-$decaying cosmology, Big bounce, wormhole

## I. Introduction

Recent advances in theoretical physics studies and cosmological observations produce a large deposit for modified theories of gravity and a large set of observational data from which an exceptional detailed knowledge of the universe can be extracted and explored. Further, a cosmic concordance has emerged from several different and independent observations of the dynamics of galaxies [1], cluster of galaxies [2] and of Type Ia supernovae (SNeIa) [3] with redshift $z > 0.35$ and the CMB [4] pointing that the Universe is accelerating with time approaching de Sitter (dS) regime. Further, the universe is dominated by a mysterious form of dark energy (DE) characterized by a negative equation of state parameter (EoSP) $w = p/\rho < 0$, which accounts for about 70% of the total energy content and 30% of dark matter. On the other hands recent findings of BOOMERANG experiments [5] strongly suggests that the cosmos is spatially flat. The existence of a considerable amount of DE represents one of the most profound and difficult problem in modern cosmology. The evolution of the EoSP in terms of the redshift leads to the conclusion that the universe had undergone a phase of deceleration before passing to the current accelerated expansion [6]. Two main theoretical problems appear: the reason of the late dominance of dark energy over matter and the fine tuning problem, i.e. the tiny amount of dark energy density during the radiation epoch compared to the radiation and matter density. These pose troubles on the theoretical alternative for the energy fraction which seems to be absent.

In the last several years, numerous theoretical competitive models trying to explain the physical nature of the dark component have been proposed, including the$\Lambda$CDM [7] consisting a mixture of cosmological constant $\Lambda$and cold dark matter (CDM) or WIMPS composed of weakly interacting massive particles which must be relics of a grand unified phase of the Universe, quintessence [8], K-essence [9], viscous fluid [10], Chaplygin gas [11,12], Generalized Chaplygin gas model (GCGM) [13,14], Brans-Dicke (BD) pressureless solutions [15,16,17], decaying Higgs fields [18], decaying dark energy models [19], dual role of Ricci scalar [20], modified gravity and scalar tensor theories [21], higher-order corrections [22], and so on. All these models stimulated a renewed interest in the generalized or extended scalar gravity theories with repulsive gravitational force and time-increasing gravitational constant causing the present accelerated expansion of the Universe. Despite the appealing consequences of these theories, some problems with many difficulties appear. The most embarrassing one is the coincidence problem which concerns the fact that the energy densities of matter and dark energy are of the same order today. Another different question concerns the fact that observations lead to a value for the cosmological constant today 120 order of magnitude smaller than that predicted by quantum field considerations. "Tracker field behavior" quintessence cosmological model [23] with exponential self-interaction and Gaussian potentials has been proposed to solve the CCP and to alleviate the fine tuning problem. In such models, for a wide class of initial conditions, the equation of motion is an attractor like in a sense that for a wide range of initial conditions the equation of motion converges to the same solution. In other words, the equation of state of dark energy tracks the one of the background matter and radiation. As for popular quintessence theory with scalar field $\phi$ acting as fluctuating dark energy, fine tuning parameters and several constraints are required. Another concern is that a universe dominated by dark energy with effective equation of state parameter

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$w = p/\rho < -1$ (superquintessence or phantom energy) may end its existence at a Big Rip singularity at which the scale factor and the energy density and pressure of quintessence diverge at a finite time in the future [24]

On the other hand, theories of extra-dimensions including braneworld models originally developed in the late 1990s have received a lot of interest in recent years because M-theory requires that the space-time of our universe might be an 11-dimensional manifold in which the extra dimensions are compactified by hand *a la* Kaluza-Klein on a Calabi-Yau threefold via a bifold symmetry obtaining a five-dimensional brane effective theory in which only the 4-dimensional is observed experimentally [25]. These large extra-dimensions may provide a possible solution to the hierarchy problem- the unnatural huge discrepancy of sixteen orders of magnitude between the electroweak scale ($10^3 GeV$) and the Planck scale ($10^{19} GeV$). These new concepts have a number of interesting applications in modern cosmology and modified gravity theories. In order to describe the current evolution of the universe, several powerful mathematical tools were developed and many exact solutions of the Einstein's field equations were obtained, but most of them assume some additional matters and need particular settings.

In this paper, we adopt the simplest generalization of standard Friedmann-Robertson-Walker cosmology by including a few extra spatial dimensions, i.e. cosmology in a (1+3+n) dimensional homogeneous anisotropic universe described by the combination of the standard (1+3) FRW metric and n extra-dimensions as [26].

$$ds^2 = -dt^2 + a^2(t)\left[dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right] + +b^2(t)\tilde{\gamma}_{pq}dy^p dy^q, (p,q:1,...,n), \quad (1)$$

where $a(t)$ and $b(t)$ are the scale factors of the expanding universe respectively in three and n-dimensions, $\tilde{\gamma}_{pq}$is the maximally symmetric metric in n-dimensions. The Einstein's field equations [25]

$$R_{AB} - \frac{1}{2}g_{AB}R + \Lambda g_{AB} = \kappa^2 T_{AB}, (A,B:0,1,...,3+n), \quad (2)$$

with$\kappa^2 = 8\pi/M^{2+n}$where $M$is the higher-dimensional Planck's mass and $\Lambda$is the effective cosmological constant. This determines the energy-momentum tensor to be of the form:

$$T_{00} = \rho, \qquad T_{ij} = -p_a\gamma_{ij}, \qquad T_{3+p,3+q} = -p_b\tilde{\gamma}_{pq}, \quad (3)$$

According to the current astrophysical observations dynamical behaviour of the universe is well described by the standard FRW model in the presence of a perfect fluid. Thus it might be worth-while to generalize this approach to the description of the post-inflationary stage in multidimensional cosmological models.

In fact, the paper is divided into two main sections: in **Section II**, we discuss two different cosmological scenarios: 1) – In the first scenario, the effective cosmological constant was chosen to decay with cosmic time like $\Lambda_{effective} = Ca^{-2} + D(b/a_I)^2$ where $a_I$is an arbitrary scale factor characterizing the isotropic epoch which proceeds the graceful exit period. The extra-dimensional scale factor is assumed to decay classically like $b(t) \approx a^x(t), x \in \mathbb{R}^-$.

2) – In the second model, we adopt the phenomenological law $b(t) = a(t)\exp(-Qt)$ as we expect that at the origin of time, there is no distinction between the visible and extra dimensions; $Q$is a real positive number.

Cosmological solutions of both models are explored and compared. In **Section III**, we study a $\Lambda-$decaying extra-dimensional cosmology with a static traversable wormhole and where the cosmological constant in (3+n+1) is assumed to decays like $\Lambda(a) = 3Ca^{-2}$. In fact, our interest to explore wormhole cosmology came from the results obtained recently by Gonzalez-Diaz when exploring the evolution of a classical wormhole embedded in a FRW universe approaching the Big Rip [27]. The author concluded that the wormhole accreting superquintessence expands more rapidly than the surroundings FRW universe and that the radius of the wormhole throat diverges before the occurrence of the Big Rip, consequently the wormhole engulfs the whole universe, which will reappear from the other wormhole throat. It is noteworthy that such geometries connect two regions of the same universe by a traversable throat. The consequential spacetime is not globally hyperbolic and is a causal with closed time-like curves threading the wormhole throat. Such strange scenarios are potentially appealing as constraints. In fact, if it can be proved that phantom energy leads in principle to improper consequences, this may be enough to rule out its existence. Faraoni and Israel argued that if the wormhole is modeled by a thin spherical shell accreting the phantoms field, the wormhole becomes asymptotically comoving with the cosmic fluid and the future evolution of the universe is fully causal [28]. Furthermore, wormholes cosmology have been discussed in higher-dimensional scenarios, in particular brane world where a new class of static and spherically symmetric solutions in vacuum brane and bulk Weyl effects support the wormhole was obtained [29]. Very recently, Lobo has given a wide-ranging formulation for brane wormholes with two possible wormhole configurations with dust and perfect fluid having linear equation of state, as the brane matter [30]. More recently, Gonzalez-Diaz and Martin-Moruno discussed diverse arguments that have been raised against the viability of the Big Rip process [31]. They argued that this process is stable and can in reality occur by accretion of phantom energy onto the wormholes.

In this work, the conservation equation of the stress-energy tensor will be considered conserved in our scenarios which seem to be at odds with current results obtained separately by a number of authors, in particular the renormalization group (RG) approach to cosmology which was proved to be an efficient method to study the promising evolution of the cosmological parameters from the point of view of quantum field theory in curved space-time [32]. Our speculation is dissimilar since the coupling between the cosmological constant and the gravitational coupling constant was chosen so as to outfit the famous equivalence principle. More precisely, the vanishing of the covariant divergence of the Einstein tensor in equation (2) and the usual energy-momentum conservation relation $T^{\mu\nu}_{;\nu} = 0$lead to: $\dot{\Lambda} + \dot{\kappa}^2\rho = 0$and $\dot{\rho} + 3H_a(p_a + \rho) + 3H_b(p_b + \rho) = 0$.

## I. FIRST COSMOLOGICAL MODEL: $\Lambda_{effective} = Ca^{-2} + D(b/a_1)^2$ AND $b(t) \approx a^x(t)$

In reality, it is widely believed that the value of the cosmological constant was large during the early stages of cosmological evolution and strongly influenced its expansion, whereas its present value is too small. Observing that the cosmological constant has dimension of inverse length square, and that extra-dimensions compactify as the visible dimensions expand, the simplest scale factor dependence we adopted thought this work is $\Lambda_{effective} = Ca^{-2} + D(b/a_I)^2$ where $C$ and $D$are real positive constants and $a_I$is an arbitrary scale factor which characterize the isotropic epoch which proceeds the graceful exit period. When$b = a_I$, $\Lambda_{effective} = Ca^{-2} + D$ and there is no distinction between the visible and extra dimensions and consequently $D$plays the role of the false vacuum. One thus expects an effective curvature constant in the theory. In fact, the case where $\Lambda_{effective} = Ca^{-2} + D$ corresponds to the cosmic string matter and has mostly been taken based on dimensional considerations by some authors. [33]. This phenomenological decaying law does not decipher in reality the cosmological constant problem, but it may relate this problem to the age problem (of why the universe is old) and have a radius much larger than the Planck length. One may presuppose that the value of the effective lambda in the early universe have been much superior than its current value and huge enough to compel some spontaneous symmetry breaking which might have occurred in the early epoch. There is also a strong believe that $\Lambda_{effective}$ decay spontaneously in time into a massive and/or massless particles reducing its value to its present tiny value [34]. As we strongly believe that the effective cosmological constant is very close to zero, the term $\Lambda_{effective}$ may be identified to a variable dynamic degree of freedom so that in an expanding it relaxes to zero. The phenomenological law introduced here may explain why the effective cosmological constant is reduced from a large value at early times to a sufficiently small value at late times in consistent with observational upper limit. Within the framework of extra-dimensions, the decaying of the cosmological constant plays a crucial role [35]. It was recently shown that the cosmological constant may be reduced by thermal production of membranes by the cosmological horizon rather than tunneling through it and consequently, black holes are produced out of the vacuum energy associated with the cosmological constant [36]. Further, it was proved that the decaying law plays in (4+D)-dimensional Kaluza-Klein non-singular cosmology with a FRW metric the role of an evolving DE in the universe [37] In fact, the existence of a large long-lived universe demands that the cosmological constant is tiny. Many proponents of the braneworld and extra-dimensions models are dealing with this problem seriously. [38,39,40]. The recent approach of Randall-Sundrum in which the bulk dimensions are extremely warped but not necessarily compactified may explain why the cosmological constant appears to be small [41].

Moreover, we consider naturally time-dependent gravitational coupling in order to retain the energy conservation in the background of the FRW spacetime by assuming conservation of the energy-momentum tensor of matter content, i.e. the variation of lambda is cancelled by the variation of $\kappa^2$. The time variation of the gravitational coupling constant with time in multidimensional theories is widely discussed in literature [37]. It was recently found that the time-variation of the gravitational coupling constant is related to the time variation of the Newton's constant in three-space dimensions and also is related to the time variation of the volume of the extra spatial dimensions [42]. It is worth noticing that early attempts to unify gravity with electromagnetism predicted such kinds of variation. Extra-dimensions theory such as string/M-theory provides a natural and self-consistent framework for such variation [43].

In reality, one would like to describe the late-time behavior of the universe as a transition from a universe filled with dust-like matter pressure to an accelerating one. It was argued that a dust universe can represents a solution to the DE problem and the Cosmic Cosmological Coincidence Problem (CCCP) [44]. The phenomenological approach with a perfect fluid as a matter source is widely used in usual 4-dimensional cosmology. In our framework, we will assume simple equations of state for the anisotropic fluid, namely$p_a = \gamma\rho$and$p_b = \bar{\gamma}\rho$where $\gamma$and $\bar{\gamma}$are real parameters. The continuity equation is obtained easily:

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\gamma+1)\rho + n\frac{\dot{b}}{b}(\bar{\gamma}+1)\rho = 0, \tag{4}$$

The Friedmann equation for this particular case takes the form:

$$3\frac{\dot{a}^2}{a^2} + 3n\frac{\dot{a}}{a}\frac{\dot{b}}{b} + \frac{n(n-1)}{2}\frac{\dot{b}^2}{b^2} = \kappa^2\rho + \frac{C}{a^2} + D\frac{a^{2x}}{a_I^2}. \tag{5}$$

By making the assumption that the extra-dimensions compactify as the visible dimensions expand like $b(t) \approx a^x(t)$ where $x$ is a real negative number [45] and assuming that $\rho = \rho_0 a^{-y}$where$y$is real constant and $\rho_0$is a constant parameter assumed equal to one for mathematical simplicity, equation (4) gives easily:

$$(3(\gamma+1) + nx(\bar{\gamma}+1)) = y. \tag{6}$$

Making use of the relation $\dot{\Lambda} + \dot{\kappa}^2\rho = 0$, one finds easily:

$$\kappa^2 = \frac{2C}{(y-2)}a^{y-2} - \frac{2xD}{a_I^2(2x+y)}a^{2x+y},$$

$$= \frac{2C}{(3\gamma+1+nx(\bar{\gamma}+1))}a^{3\gamma+1+nx(\bar{\gamma}+1)} - \frac{2xD}{a_I^2(2x+3\gamma+3+nx(\bar{\gamma}+1))}a^{2x+3\gamma+3+nx(\bar{\gamma}+1)}, \tag{7}$$

for which a positive gravitational coupling constant corresponds for $x < -1$. The effective cosmological constant decays consequently in its turn like

$$\Lambda_{effective} = Ca^{-2} + D\left(\frac{a^{2x}}{a_I^2}\right), 2x < -2. \quad (8)$$

If for instance $\bar{\gamma} = -1$ and $\gamma = -1/3$, then $y = 2$, i.e. $\rho \propto a^{-2}$and consequently,

$$\kappa^2 = 2C\ln a - \frac{2xD}{2a_I^2(x+1)}a^{2x+2}. \quad (9)$$

If in contrast $\gamma = -1$, then $y = nx(\bar{\gamma}+1)$, i.e. $\rho = \rho_0 a^{-nx(\bar{\gamma}+1)}$and therefore the gravitational constant and the effective cosmological constant vary respectively like:

$$\kappa^2 = \frac{2C}{nx(\bar{\gamma}+1)-2}a^{nx(\bar{\gamma}+1)-2} - \frac{2xD}{a_I^2(2x+nx(\bar{\gamma}+1))}a^{2x+nx(\bar{\gamma}+1)}, \quad (10)$$

$$\Lambda_{effective} = Ca^{-2} + D\left(\frac{a^{2y/n(\bar{\gamma}+1)}}{a_I^2}\right). \quad (11)$$

More generally, the Friedmann equation is:

$$\left(3+3nx+\frac{n(n-1)}{2}x^2\right)\frac{\dot{a}^2}{a^2} = C\frac{y}{y-2}\frac{1}{a^2} + \frac{y}{2x+y}\frac{D}{a_I^2}a^{2x}, \quad (12)$$

and therefore we may discuss the following two special cases: 1-the value $x = -3/2$is interesting as it gives the following modified Friedmann equation:

$$\left(3-\frac{9}{2}n+\frac{9n(n-1)}{8}\right)\frac{\dot{a}^2}{a^2} = C\frac{y}{y-2}\frac{1}{a^2} + \frac{y}{y-3}\frac{D}{a_I^2}\frac{1}{a^3}, \quad (13)$$

while the gravitational coupling constant and the effective cosmological constant vary respectively like:

$$\kappa^2 = \frac{2C}{(y-2)}a^{y-2} - \frac{2xD}{a_I^2(y-3)}a^{y-3}, \quad (14)$$

$$\Lambda_{effective} = Ca^{-2} + D\left(\frac{a^{-3}}{a_I^2}\right). \quad (15)$$

For $y = 5/2 (5 = -3n(\bar{\gamma}+1))$and $\gamma = -1$ (cosmological constant), i.e. $\rho = \rho_0 a^{-5/2}$and $\kappa^2 = 4Ca^{1/2} - 6Da^{-1/2}/a_I^2 \propto 4Ca^{1/2}$ for very large time. Furthermore, $n > 0$is for instance $\bar{\gamma} < -8/3$(phantom energy). In reality an increasing gravitational constant is favored by a lot of experimental limits on the time variation of the gravitational constant [46] including radar ranging data to the Viking landers on Mars, lunar laser ranging experiments, measurements of the masses of young and old neutron stars in binary pulsars.[1] However, from phenomenological point of view the present observed acceleration of the universe may also be attributed to this ever growing gravity. An increasing gravitational coupling constant with cosmic time would cause the Planck length to be an ever-increasing function in time, and the quantum fluctuations on the metric would be too small in the high energy epoch.

The Friedmann equation for this particular case takes the form:

$$\frac{\dot{a}^2}{a^2} = \frac{5C}{\left(3-\frac{9}{2}n+\frac{9n(n-1)}{8}\right)a^2} - \\ + \frac{5D}{\left(3-\frac{9}{2}n+\frac{9n(n-1)}{8}\right)a_I^2}\frac{1}{a^3} \equiv \frac{C_1}{a^2} - \frac{D_1 a_I^{-2}}{a^3}, \quad (16)$$

where

$$C_1 = \frac{5C}{3-\frac{9}{2}n+\frac{9n(n-1)}{8}}, \quad (17)$$

and

$$D_1 = \frac{5D}{3-\frac{9}{2}n+\frac{9n(n-1)}{8}}. \quad (18)$$

The typical solution of equation (16) is obtained making use of the conformal time $dt = ad\tau$, as: [47]

$$a^{1/2} \approx (Const)^{1/2}\cosh\left[\frac{\tau}{2}\right]. \quad (19)$$

provided $n \geq 9$ and the total density is given by:

$$\rho = \rho_m + \rho_{vacuum} = \quad (20)$$
$$-\frac{3(\gamma+1)\rho_1 + nx(\bar{\gamma}+1)\rho_2}{ma^{5/2}} + \frac{C}{a^2} + D\frac{a^{2x}}{a_I^2}, 2x < m < -2.$$

Notice that we have used the freedom in setting the origin of conformal time so that the universe is initially contracting, and then bounces to an expanding phase. We may set $\eta = 0$at the bounce so that the minimal radius is given by $a_{\min}$ =constant. Therefore, the aspect of a bouncing universe would permit one to evade the issue of resolving the big bang or big crunch singularity, which afflicts many cosmological models. The main interesting point here is that classical wormholes in a flat multi-dimensional cosmological universe with spacetime consisting of $n (n \geq 9)$extra-dimensional spaces in the presence of a positively decaying $\Lambda$ and a perfect fluid are solutions of the cosmological model described here.

It is worth-mentioning that the present day variation of the gravitational coupling constant is $(\dot{\kappa}^2/\dot{\kappa}^2)_0 \approx H_0/2$ in agreement with recent astronomical data [46]; $H = \dot{a}/a$is the Hubble parameter.

Two others interesting values are $x = -2$ and $y = 3$ which yield the modified Friedmann equation:

$$\left[2n^2 - 8n + 3\right]\frac{\dot{a}^2}{a^2} = \frac{3C}{a^2} - \frac{3D}{a_I^2}\frac{1}{a^4}, \quad (21)$$

[1] In fact, reference [46] describes a result where effectively the fractional variation of the gravitational coupling is of the same order of the absolute value of the Hubble parameter, but admitting negative values. The dispersion is very high, so it seems difficulty to confirm from actual observations that the gravitational constant increases with cosmic time.

which may be rewritten in the special form

$$\frac{\dot{a}^2}{a^2} = \frac{C_2}{a^2} - \frac{D_2 a_I^{-2}}{a^4}, \quad (22)$$

where

$$C_2 = \frac{3C}{2n^2 - 8n + 3}, \quad (23)$$

and

$$D_2 = \frac{3D}{2n^2 - 8n + 3}, \quad (24)$$

provided $n \geq 4$. Equation (21) accounts for a wormholes corresponding to the conformal scalar field. If in contrast $x = -3$, the same dynamical equation accounts for axion. The solution are given respectively in conformal time by $a(\tau) \approx \cosh(\tau)$for $x = -2$ and by $a^2(\tau) \approx \cosh(2\tau)$for$x = -3$. It is noticed that as long as $x$decreases, the restriction on the number of extra-dimensions decreases. In this special case, the present day variation of the gravitational coupling constant is $(\dot{\kappa}^2/\dot{\kappa}^2)_0 \approx H_0$ in agreement also with recent astronomical data [46]. It is easy to prove that the scalar curvature invariant is non-singular at the early epoch of the universe.
A final point concerns the energy conditions: it is easy to check that in our arguments, in particular the case $y = 5/2$ for which $\bar{\gamma} < -8/3$(phantom field in the higher dimensional space) and $\gamma = -1$(cosmological constant in the reduced four-dimensional space), the null energy condition$p + \rho \geq 0$is obviously violated. This implies a violation of the weak energy condition which includes the statement $\rho \geq 0$besides the null energy condition

## II. SECOND COSMOLOGICAL MODEL: $\Lambda_{effective} = Ca^{-2} + D(b/a_1)^2$ AND $b(t) = a(t)e^{Q_t}$

As we believe that the extra dimensional space is very large at the beginning, and it is much reduced at the present time, we will take a more radical and generic attitude for the extra-dimensions scale factor. We shall assume that $b(t) = a(t)e^{-Qt}$, $Q$ is a real constant. It is evident that for $t$close to zero, $b \approx a$, e.g. after the inflationary epoch and for very large times, $b(t) \to 0$as it is expected. The conservation of energy (4) gives easily:

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\gamma+1)\rho + n(\bar{\gamma}+1)\rho\left(-Q + \frac{\dot{a}}{a}\right) = 0. \quad (25)$$

In the section, we assume also that $\rho = \rho_0 a^{-y}$and therefore:

$$\frac{\dot{a}}{a} = \frac{Qn(\bar{\gamma}+1)}{3(\gamma+1) + n(\bar{\gamma}+1) - y}. \quad (26)$$

The solution is easily deduced and takes the special form:

$$a(t) = a_0 \exp\left(\frac{Qn(\bar{\gamma}+1)}{3(\gamma+1) + n(\bar{\gamma}+1) - y} t\right), \quad (27)$$

where $a_0 = a(t = 0)$and assumed equal to one for mathematical simplicity. This case is identical to the inflationary scenario if for instance one of the following constraints is realized: $Q < 0, \bar{\gamma} < -1$, $3(\gamma+1) + n(\bar{\gamma}+1) > y$, $Q < 0, \bar{\gamma} > -1$, $3(\gamma+1) + n(\bar{\gamma}+1) < y$ , $Q > 0, \bar{\gamma} < -1$, $3(\gamma+1) + n(\bar{\gamma}+1) < y$ or if finally $Q > 0, \bar{\gamma} > -1$, $3(\gamma+1) + n(\bar{\gamma}+1) > y$. Therefore, the extra-dimension varies like:

$$b(t) \propto \exp\left(\frac{Q(y - 3(\gamma+1))}{3(\gamma+1) + n(\bar{\gamma}+1) - y} t\right). \quad (28)$$

Hence, a decaying extra-dimension is realized if for instance $Q < 0$ and $3(\gamma+1) + n(\bar{\gamma}+1) < y < 3(\gamma+1)$ with $\bar{\gamma} > -1$ which corresponds to the more realistic constraint. Making use again of the relation $\dot{\Lambda} + \dot{\kappa}^2\rho = 0$, one finds:

$$\dot{\kappa}^2 = \frac{2C}{\rho_0} a^{y-3}\dot{a} - \frac{2D}{\rho_0 a_I}\left\{a^{y+1}\dot{a} - Qa^{y+2}\right\} e^{-2Qt}, \quad (29)$$

and consequently:

$$\kappa^2 = \frac{2Ca_0^{y-2}}{\rho_0(y-2)} \exp\left(\frac{Qn(\bar{\gamma}+1)(y-2)}{3(\gamma+1) + n(\bar{\gamma}+1) - y} t\right)$$

$$-\frac{2Da_0^{y+2}}{\rho_0 a_I(y+2)} \exp\left(\frac{n(\bar{\gamma}+1)(y+2)}{3(\gamma+1) + n(\bar{\gamma}+1) - y} - 2\right) Qt$$

$$-\frac{Da_0^{y+2}}{\rho_0 a_I} \exp\left(\frac{n(\bar{\gamma}+1)(y+2)}{3(\gamma+1) + n(\bar{\gamma}+1) - y} - 2\right) Qt. \quad (30)$$

while the cosmological constant varies like:

$$\Lambda_{effective} = C\exp\left(\frac{-2Qn(\bar{\gamma}+1)}{3(\gamma+1) + n(\bar{\gamma}+1) - y} t\right) + \frac{D}{a_I^2}\exp\left(\frac{2Qt(n(\bar{\gamma}+1) - 1)}{3(\gamma+1) + n(\bar{\gamma}+1) - y} t\right). \quad (31)$$

If for instance $Q < 0$and $y > 3(\gamma+1) + n(\bar{\gamma}+1)$, then a decaying cosmological constant in time corresponds to $\bar{\gamma} > -1$(dark energy). Notice that the effective lambda is finite at the origin of time. For very large time, $b(t) = a(t)e^{-Qt} \to 0$and therefore, the Friedmann equation takes the form:

$$\frac{\dot{a}^2}{a^2} = \frac{2yC}{3(y-2)a^2}. \quad (32)$$

Therefore for $y > 2$, the solution is given by:

$$a(t) = \sqrt{\frac{2yC}{3(y-2)}t + a_0}, \tag{33}$$

where $a_0 = a(t=0)$. This is interesting as it coincides with a non-singular scale factor evolution dominated by dark energy. The gravitational constant varies in this particular case like:

$$\kappa^2 = \frac{2C}{(y-2)\rho_0}a^{y-2} = \frac{2C}{(y-2)\rho_0}\left(\sqrt{\frac{2yC}{3(y-2)}t + a_0}\right)^{(y-2)/2}, \tag{34}$$

and therefore the present day variation of the gravitational coupling constant is

$$\left(\frac{\dot{\kappa}^2}{\dot{\kappa}^2}\right)_0 = \frac{y-2}{\left(\sqrt{\frac{8yC}{3(y-2)}}\right)^{(y-2)/2}}H_0, \tag{35}$$

in agreement also with recent astronomical data [48], in particular for $y$ close to 2.

## III. THIRD MODEL: NON-SINGULAR ACCELERATED FLAT COSMOLOGY WITH EXTRA-DIMENSIONS AND A WORMHOLE

The purpose of this last paragraph is to explore wormhole dynamics embedded in a (3+n+1) dimensional homogeneous anisotropic universe described by the combination of the standard 4D FRW metric and static wormhole as follows:

$$\begin{aligned} ds^2 &= -e^{\Phi(r)}dt^2 + a^2(t) \times \\ &\times \left[\frac{dr^2}{1-B(r)/r} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right] + \\ &+ b^2(t)\tilde{\gamma}_{pq}dy^p dy^q, (p,q:1,...,n), \end{aligned} \tag{36}$$

where $\Phi(r)$is the lapse function assumed here equal to zero, $a(t)$ and $b(t)$ are the scale factors of the expanding universe respectively in three and n-dimensions, $B(r)$is the wormhole shape function, $\tilde{\gamma}_{pq}$is the maximally symmetric metric in n-dimensions. Further, we will explore the classical wormhole in a $\Lambda-$decaying accelerated cosmology dominated in four-dimension by dust. The metric (36) is the simple realistic alternative for exploring accelerated expansion in extra-dimensions with wormholes. The case of seven dimensional universe with a wormhole is explored in literature where it was incorporated an expanding Gidding-Strominger wormhole at the center of the extra dimensions generating an adiabatic pressure [[48,49]. The model has some extra suitable characteristics. The Einstein's field equations read also

$$R_{AB} - \frac{1}{2}g_{AB}R + \Lambda g_{AB} = \kappa^2 T_{AB}, (A,B:0,1,...,3+n). \tag{37}$$

Here $\Lambda$ is the cosmological constant in (3+n+1) dimensions assumed through this work to decays like $\Lambda(a) = 3C/a^2$where $C$is a real constant. Further, we allow the gravitational coupling to vary with time in order to retain the energy conservation, i.e. the variation of lambda is cancelled by the variation of $\kappa^2$. Of interest for us for instance are the (0,0) components of the Friedman equation

$$3\frac{\dot{a}^2}{a^2} + 3\frac{k_a}{a^2} + 3n\frac{\dot{a}}{a}\frac{\dot{b}}{b} + \frac{n(n-1)}{2}\left[\frac{\dot{b}^2}{b^2} + \frac{k_b}{b^2}\right] + \frac{1}{a^2}\frac{B'(r)}{r^2} = \kappa^2\rho_a(r,t) + 3\frac{C}{a^2}. \tag{38}$$

In our framework, we will assume again simple equations of state for the anisotropic fluid, namely $p_a = \gamma\rho$ and $p_b = \bar{\gamma}\rho$where $\gamma$and $\bar{\gamma}$are real parameters. Here $\rho_a(r,t) = T_{\hat{t}\hat{t}}$is the energy density in the orthonormal frame $\hat{i}\hat{j}$.The energy conservation equation $T^A_{0;A} = 0$is therefore:

$$\dot{\rho} + 3(p_a + \rho)\frac{\dot{a}}{a} + n\frac{\dot{b}}{b}(p_b + \rho) = 0, \tag{39}$$

where $3\rho_a(r,t) = 3\rho + 2P_a - \tau_a$is the effective energy density in the FRW frame, $\tau_a(r,t)$is the surface tension, $P_a(r,t)$and$p_b(r,t)$are the pressures in 3 and n-dimensional spacetime (in the orthonormal frame$\hat{i}\hat{j}$). $k_a = -1, 0, +1$and $k_b = -1, 0, +1$ are the spatial curvatures in ordinary space and universal extra-dimensions respectively. Following [50] we introduce the ansatz:

$$a^2\rho_a(r,t) = a^2\tilde{\rho}_c(t) + \tilde{\rho}_w(r), \tag{40}$$

separating the time-dependent cosmological dynamics from space-dependent wormhole dynamics, one finds easily:

$$a^2\tilde{\rho}_c(t) - \frac{3}{\kappa^2}\left(\dot{a}^2 + [k_a - C] + n\dot{a}a\frac{\dot{b}}{b} + \frac{n(n-1)}{6}a^2\left[\frac{\dot{b}^2}{b^2} + \frac{k_b}{b^2}\right]\right) = \frac{B'(r)}{\kappa^2 r^2} - \tilde{\rho}_w(r) = M, \tag{41}$$

$M$is a parameter independent of $r$and $t$. It is straightforward to see that in order to have a possible solution, we must impose the constraint $B(r) = \kappa^2\int\tilde{\rho}_w(r)r^2dr + c$ where $c$ is an integration constants. If for instance $\rho_w(r) = \rho_{w_0}(r/r_0)^{-\alpha}, \alpha \in \mathbb{R}$ and where$\rho_{w_0} = \rho_w(r = r_0)$ at the throat, then $B(r) = \kappa^2\rho_{w0}r^{3-\alpha}/(3-\alpha) + c$ and therefore the requirement of asymptotic flatness $B(r)/r \to 0$ as $r \to \infty$is verified if $2 < \alpha < 3$. If for instance $\exists r_0/B(r_0) = r_0$, then $c = r_0 - \kappa^2\rho_{w0}r_0^{3-\alpha}/(3-\alpha)$ and hence $B(r) = \kappa^2\rho_{w0}[r^{3-\alpha} - r_0^{3-\alpha}]/(3-\alpha) + r_0$. The flare-out condition at the throat $B'(r_0) < 1$ gives $r_0 < -\kappa^2\rho_{w0}r_0^{2-\alpha}$. Accordingly, equation (38) may be written like:

$$3\frac{\dot{a}^2}{a^2}+3\frac{k_a}{a^2}+3n\frac{\dot{a}}{a}\frac{\dot{b}}{b}+\frac{n(n-1)}{2}\left[\frac{\dot{b}^2}{b^2}+\frac{k_b}{b^2}\right]+\frac{1}{a^2}\kappa^2\rho_{w0}r^{-\alpha} = \kappa^2\rho_a(r,t)+3\frac{C}{a^2}, \tag{42}$$

and obviously, the fifth term on the RHS of equation (42) tends to zero for $r\to\infty$for$2<\alpha<3$. One may also conjecture that the gravitational coupling constant varies like $\kappa^2\propto r^{\alpha}$and accordingly the fifth term on the RHS of equation (2) can be viewed as a cosmological constant term. This may have interesting consequences concerning the presence of dark matter at galactic and cosmological scales similar to the ones proposed by the asymptotically free theories of gravity.

Making use of equation (41), equation (39) and the relation $\tau_a=-P_a$as measured by an observer who always remains at rest at constant $(r,\theta,\phi)$ give:

$$\tilde{\rho}_c=\tilde{\rho}_0 a^{-3}b^{-n(1+\bar{\gamma})}+\frac{m}{1+3\gamma}a^{-2}, \tag{43}$$

where$\tilde{\rho}_0$and $m$ are real positive constants. In reality, one would like to describe the recent behavior of the universe as a transition from a universe filled with dust-like matter pressure $(\gamma=0)$ to an accelerating one. It was argued that a dust universe can represents a solution to the DE problem and the Cosmic Cosmological Coincidence Problem (CCCP). Consequently, equation (38) is written like:

$$\frac{\dot{a}^2}{a^2}+\frac{1}{a^2}\left[k_a-C-\frac{\kappa^2}{3}(m-M)\right]+n\frac{\dot{a}}{a}\frac{\dot{b}}{b}+\frac{n(n-1)}{6}\left[\frac{\dot{b}^2}{b^2}+\frac{k_b}{b^2}\right]=\frac{\kappa^2\tilde{\rho}_0 a^{-3}b^{-n(1+\bar{\gamma})}}{3}. \tag{44}$$

Making use of the relation $\dot{\Lambda}+\dot{\kappa}^2\tilde{\rho}_c=0$, one finds easily:

$$\kappa^2=\frac{6C}{m}\ln\left[\tilde{\rho}_0 b^{-n(1+\bar{\gamma})}+ma\right]. \tag{45}$$

This corresponds to a slowly increasing gravitational constant in time. Notes that for $\bar{\gamma}=-1$,

$$\kappa^2=\frac{6C}{m}\ln\left[\tilde{\rho}_0+ma\right], \tag{46}$$

and thus the gravitational coupling constant does no longer depends on the extra-dimensional scale factor. This procedure leads us to the definition of an effective gravitational constant due to the presence of a wormhole and is given by equation (45). It is interesting to have quintessence generated from the extra-dimensional world. Equation (44) in its turn takes the form:

$$\frac{\dot{a}^2}{a^2}+n\frac{\dot{a}}{a}\frac{\dot{b}}{b}+\frac{n(n-1)}{6}\left[\frac{\dot{b}^2}{b^2}+\frac{k_b}{b^2}\right]=\frac{6C}{m}\ln\left[\tilde{\rho}_0 b^{-n(1+\bar{\gamma})}+ma\right]\left[\frac{\tilde{\rho}_0 b^{-n(1+\bar{\gamma})}}{3a^3}-\frac{(M-m)}{a^2}\right], \tag{47}$$

where we have assumed that $k_a=C=+1$, i.e. positive cosmological constant. Let us finally note that by making the assumption that the extra-dimensions compactify as the visible dimensions expand like $b(t)\approx a^{-x}(t), 0<x<1$, equation (47) can be written for $k_b=0$ as:

$$\left[\frac{n(n-1)}{6}x^2-nx+1\right]\frac{\dot{a}^2}{a^2}=\frac{6}{m}\ln\left[\tilde{\rho}_0 a^{nx(1+\bar{\gamma})}+ma\right]\left[\frac{\tilde{\rho}_0}{3a^{3-nx(1+\bar{\gamma})}}-\frac{(M-m)}{a^2}\right]. \tag{48}$$

For the special case $n=1$, i.e. 5D-cosmology, one obtains easily:

$$\frac{\dot{a}^2}{a^2}=\frac{6}{m(1-x)}\ln\left[\tilde{\rho}_0 a^{x(1+\bar{\gamma})}+ma\right]\left[\frac{\tilde{\rho}_0}{3a^{3-x(1+\bar{\gamma})}}-\frac{(M-m)}{a^2}\right]. \tag{49}$$

Equation (50) tells us that in order to have an asymptotically Euclidean wormhole, $\dot{a}^2$ must remain positive at large scale factor. Thus, we need to have $\bar{\gamma}\geq-1$and$M>m$. Consequently, the equation of state parameter is $w=p/\rho_b=\bar{\gamma}\geq-1$in agreement with the current observations. The model described here is a deviation from the standard scenarios in which the wormholes in FRW models are typically described by a constraint equation of the form $\dot{a}^2 = 1 - Da^{2-n}, n > 2$ [49]. In return to the gravitational coupling constant, there are several models which predict a time dependence of $\kappa^2$. In our framework, the time variation of $\kappa^2$ is related to both the time variation of the FRW scale factor and the internal space scale factor $b \approx a^{-x}$by:

$$\frac{\dot{\kappa}_2^{effective}}{\kappa_2^{effective}} = \frac{\dot{a} - n(1+\bar{\gamma})a^{x(n(1+\bar{\gamma})+1)}}{\left[a^{nx(1+\bar{\gamma})} + a\right] \ln \left[a^{nx(1+\bar{\gamma})} + a\right]}, \tag{50}$$

where we have assumed for simplicity and without loosing generality that$\tilde{\rho}_0 = 1$and $m = 1$. Clearly for $\bar{\gamma} = -1$, $\dot{\kappa}_2^{eff} \big/ \kappa_2^{eff} \approx \dot{a} \big/ (a \ln a)$and the logarithmic term is due to the presence of a wormhole. This variation shows that as long as the universe expands in time, $\dot{\kappa}_2^{eff} \big/ \kappa_2^{eff}$is too small as it is expected. Let us finally investigated about the behavior of the scale factor: one may easily check that at late time-dynamics, the scale factor evolves like $a(t) \approx \exp(Pt^2 + Qt + R)$, $P, Q$and $R$ are real constants. This corresponds to a super-accelerated non-singular universe with a wormhole in which the conventional four-dimensional FRW flat spacetime is dominated by dust-like matter while the extra-dimensional part is endowed by an exotic non-tachyonic fluid or DE violating strong energy condition with EoS parameter $\bar{\gamma} \geq -1$ and slowly increasing gravitational constant. In this context, the cosmological constant which is finite at the origin of time, decays rapidly to zero and this may explain its smallness at the present time. Further, from the fact $b \approx a^{-x}$, this prove that extra-dimensions are compactified to zero rapidly as $t \to \infty$ but there are finite and large at the origin of time $(t = 0)$.

In summary, it is shown that we have a deviation from most of the results elaborated in literature, in particular, the special form of the Friedman equation and the logarithmic behavior of the effective gravitational constant. Unlike the model adopted by Kim in which the curvature is affected by the wormhole even though the universe is flat, we show in the present model that the curvature is not affected the wormhole if the cosmological constant is positive and decays like $\Lambda \propto a^{-2}$. Further, phantom energy are excluded in our framework and the universe is non-singular. Notes that wormholes with phantom energy fields were proposed extensively in literature but from cosmological point of view, they face some strong difficulties. We assumed that the universe started higher-dimensional at high energies limit (Planck's scale) and subsequently the combination of dark energy, dust-like matter and a static wormhole were needed for the dynamical suppression of the extra dimensions. For further research, other models including time-dependent wormhole will be considered and studied.

## IV. CONCLUSIONS

In conclusion, we have argued that the simple physics of extra-dimensions with the special form of the decaying lambda described in this work may actually be a modification to the standard Friedmann equation. Despite the simple phenomenological laws introduced here, many modern features arise naturally, e.g. wormholes, dark energy and phantom fields. Further the universe is free from the initial singularity. It is shown that we have a deviation from most of the results elaborated in literature, in particular, the special form of the Friedmann equation, the behavior of the effective gravitational constant, the effective lambda and the extra-dimensional scale factor. The extra-dimension is coupled to a perfect fluid violating the strong energy condition, but the effect of extra-dimension can be probed through its consequence on the cosmological lambda and the gravitational coupling constant. The three independent models discussed here are free from lot of cosmological problems and can fit well with the present observational data. Instead of looking for the complicated action to obtain wormhole and brane solution, the models introduced here offer possible simple alternative. Our main aim was *to build the theoretical setup*; for future work, we shall extend these ideas for various cosmological solutions augmented by numerical tests.

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