Time evolution of Wigner functions governed by bipartite Hamiltonian system with kinetic coupling

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For the bipartite Hamiltonian system with kinetic coupling, we derive time evolution equation of Wigner functions by virtue of the bipartite entangled state representation and entangled Wigner operator, which just indicates that choosing a good representation indeed provides great convenience for us to deal with the dynamics problem.

Keywords: entangled Wigner operator, Wigner function evolution, entangled state representation

1. INTRODUCTION

As is well known, Wigner function, introduced by Wigner[1] in 1932, is a quasi-probability distribution to fully describes the state of a quantum system in phase space. Its partial negativity is indeed a good indication of the highly nonclassical character of the quantum states[2]. Now Wigner function has become a very popular tool not only to study the nonclassical properties of quantum states but also to monitor the decoherence of some quantum states by discussing its time evolution[3, 4]. For a density operator ρ of a single-mode system, the Wigner function $W(x,p)$ is defined as ($\hbar=1$)[5, 6]

$$\begin{aligned} W(x,p) &= Tr\left[\rho\Delta(x,p)\right] \\ &= \frac{1}{2\pi}\int\left\langle x+\frac{v}{2}\right|\rho\left|x-\frac{v}{2}\right\rangle e^{-ipv}dv, \end{aligned} \tag{1}$$

where $\Delta(x,p)$ is the single-mode Wigner operator, $|x\rangle$ is the eigenvector of the coordinate operator obeying $X|x\rangle = x|x\rangle$,

$$|x\rangle = \pi^{-1/4}\exp\left(-\frac{1}{2}x^2+\sqrt{2}xa^\dagger-\frac{1}{2}a^{\dagger 2}\right)|0\rangle, \tag{2}$$

$a^\dagger$ and a are the Bose creation and annihilation operators, respectively, satisfying $\left[a,a^\dagger\right]=1$; they are related to coordinate operator X and momentum operator P, namely, $X=\left(a+a^\dagger\right)/\sqrt{2}$ and $P=\left(a-a^\dagger\right)/i\sqrt{2}$. Using the technique of integration within an ordered product of operators (IWOP)[7, 8] and the vacuum projector $|0\rangle\langle 0| =: \exp\left(-a^\dagger a\right):$ ($:\ :$ denotes normal ordering), we have obtained the explicitly normal ordering form of $\Delta(x,p)$

$$\begin{aligned} \Delta(x,p) &\equiv \Delta(\alpha,\alpha^*) = \\ &= \frac{1}{\pi}: e^{-(x-X)^2-(p-P)^2}: = \frac{1}{\pi}: e^{-2\left(a^\dagger-\alpha^*\right)(a-\alpha)}:, \end{aligned} \tag{3}$$

where $\alpha = \frac{1}{\sqrt{2}}(x+ip)$. Respectively performing integrations over dx and dp leads to

$$\int dp\Delta(x,p) = \frac{1}{\sqrt{\pi}}: e^{-(x-X)^2}: = |x\rangle\langle x| \tag{4}$$

and

$$\int dx\Delta(x,p) = \frac{1}{\sqrt{\pi}}: e^{-(p-P)^2}: = |p\rangle\langle p|, \tag{5}$$

where

$$|p\rangle = \pi^{-1/4}\exp\left(-\frac{1}{2}p^2+i\sqrt{2}pa^\dagger+\frac{1}{2}a^{\dagger 2}\right)|0\rangle \tag{6}$$

is the eigenvector of the momentum operator satisfying $P|p\rangle = p|p\rangle$. Thus for a state $|\psi\rangle$, its Wigner function's marginal distributions $\langle\psi|\Delta(x,p)|\psi\rangle$ are, respectively, $|\langle x|\ \psi\rangle|^2$ and $|\langle p|\ \psi\rangle|^2$, this is just Wigner's original idea of setting up a function in $x-p$ phase space whose marginal distributions are the probability of finding a particle in coordinate space and momentum space.

On the other hand, the motion equation of Wigner function has attracted an ever increasing attention for a given system[11]. For example, in Ref.[12], for the Hamiltonian $H_1 = P^2/2m+V(X)$ of a single particle system, one has derived the time evolution of the Wigner function governed by H_1,

$$\begin{aligned} &\left(\frac{\partial}{\partial t}+\frac{p}{m}\frac{\partial}{\partial x}-\frac{dV(x)}{dx}\frac{\partial}{\partial p}\right)W(x,p,t) \\ &= \sum_{l=1}^{\infty}\left(\frac{\hbar}{2}\right)^{2l}\frac{(-1)^l}{(2l+1)!}\frac{d^{2l+1}V(X)}{dx^{2l+1}}\left(\frac{\partial}{\partial p}\right)^{2l+1}W(x,p,t). \end{aligned} \tag{7}$$

In the classical limit $\hbar\to 0$, one derives the Liouville equation

$$\left(\frac{\partial}{\partial t}+\frac{p}{m}\frac{\partial}{\partial x}-\frac{dV(x)}{dx}\frac{\partial}{\partial p}\right)W(x,p,t) = 0. \tag{8}$$

In addition, Wigner function with time evolution of a two-body correlated system has been deduced as well, whose Hamiltonian is $H_2 = \frac{1}{2m_1}P_1^2+\frac{1}{2m_2}P_2^2+V(X_1-X_2)$[13]. Thus an interesting question naturally arise: when the Hamiltonian not only is a two-body correlated system, but also contains kinetic coupling, i.e.,

$$H = \frac{1}{2m_1}P_1^2+\frac{1}{2m_2}P_2^2+kP_1P_2+V(X_1-X_2), \tag{9}$$

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where the potential $V(X_1 - X_2)$ just depends on the relative distance of the two particles, and the term kP_1P_2 is usually called kinetic coupling term, what is the time evolution of Wigner function governed by H? As far as we know, this problem has not been derived analytically in the literature before. This problem is the task of the present paper. By mentioning two-body correlated case, enlightened by the paper of Einstein-Podolsky-Rosen (EPR) in 1935[14] who noticed that two particles' relative coordinate $X_1 - X_2$ and total momentum $P_1 + P_2$ are commutative and can be simultaneously measured, we naturally think of the common eigenvector $|\eta\rangle$ of the relative coordinate operator $X_1 - X_2$ and the momentum sum operator $P_1 + P_2$, (see Eq.(11) below)[15]. Correspondingly, because in the entangled case only those states simultaneously describing two entangled particles can be endowed with physical meaning, phase space should be understood with regard to $|\eta\rangle$. In the following discussion, we mainly obtain the time evolution of the entangled Wigner function governed by the bipartite Hamiltonian in Eq.(9) by the aid of the entangled state representation $|\eta\rangle$.

2. WIGNER FUNCTION FOR TWO-BODY CORRELATED SYSTEM

To begin with, we briefly review the aspects of the Wigner function for the two-body correlated system. In Ref.[16], the so-called entangled Wigner operator has been successfully established. Based on it, the corresponding Wigner function can be conveniently derived. This entangled Wigner operator is expressed in the entangled state representation $|\eta\rangle$ as

$$\Delta(\sigma,\gamma) = \int \frac{d^2\eta}{\pi^3} |\sigma-\eta\rangle \langle\sigma+\eta| \exp(\eta\gamma^* - \eta^*\gamma), \quad (10)$$

where $\sigma = \sigma_1 + i\sigma_2$, $\gamma = \gamma_1 + i\gamma_2$ and

$$|\eta\rangle = \exp\left(-\frac{1}{2}|\eta|^2 + \eta a_1^\dagger - \eta^* a_2^\dagger + a_1^\dagger a_2^\dagger\right)|00\rangle, \eta = \eta_1 + i\eta_2, \quad (11)$$

obeys the eigenvector equations

$$(X_1 - X_2)|\eta\rangle = \sqrt{2}\eta_1 |\eta\rangle, (P_1 + P_2)|\eta\rangle = \sqrt{2}\eta_2 |\eta\rangle \quad (12)$$

with $X_j = \left(a_j + a_j^\dagger\right)/\sqrt{2}$ and $P_j = \left(a_j - a_j^\dagger\right)/i\sqrt{2}$, i.e., $|\eta\rangle$ is the common eigenstate of the relative position of two particles $X_1 - X_2$ and their total momentum $P_1 + P_2$, and spans a complete and orthogonal space[15]

$$\int \frac{d^2\eta}{\pi^2} |\eta\rangle\langle\eta| = 1, \qquad \langle\eta|\eta'\rangle = \pi\delta(\eta-\eta')\delta(\eta^*-\eta'^*). \quad (13)$$

Using Eq.(11) and $|00\rangle\langle 00| =: \exp(-a_1^\dagger a_1 - a_2^\dagger a_2):$ and perfoming the integration in Eq.(10) by virtue of the IWOP technique[7, 8], we obtain the normally ordered form of the Wigner operator $\Delta(\sigma,\gamma)$

$$\begin{aligned}
\Delta(\sigma,\gamma) &= \int \frac{d^2\eta}{\pi^3} \exp\left[-\frac{1}{2}|\sigma-\eta|^2 + (\sigma-\eta)a_1^\dagger - (\sigma-\eta)^* a_2^\dagger + a_1^\dagger a_2^\dagger\right]|00\rangle \\
&\times \langle 00| \exp\left[-\frac{1}{2}|\sigma+\eta|^2 + (\sigma+\eta)^* a_1 - (\sigma+\eta)a_2 + a_1 a_2\right] e^{\eta\gamma^* - \eta^*\gamma} \\
&= \int \frac{d^2\eta}{\pi^3} : \exp\left[-|\eta|^2 + \left(\gamma^* - a_1^\dagger - a_2\right)\eta + \left(a_2^\dagger - \gamma + a_1\right)\eta^*\right] \\
&\times \exp\left(-|\sigma|^2 + \sigma a_1^\dagger - \sigma^* a_2^\dagger + \sigma^* a_1 - \sigma a_2 + a_1 a_2 + a_1^\dagger a_2^\dagger - a_1^\dagger a_1 - a_2^\dagger a_2\right): \\
&= \frac{1}{\pi^2} : \exp\left[-|\sigma|^2 - |\gamma|^2 + \gamma(a_1^\dagger + a_2) + \gamma^*(a_2^\dagger + a_1) + \sigma(a_1^\dagger - a_2) + \sigma^*(a_1 - a_2^\dagger) - 2a_1^\dagger a_1 - 2a_2^\dagger a_2\right] \\
&= \frac{1}{\pi^2} : \exp[-(a_1 + a_2^\dagger - \gamma)(a_1^\dagger + a_2 - \gamma^*) - (\sigma - a_1 + a_2^\dagger)(\sigma^* - a_1^\dagger + a_2)]:, \quad (14)
\end{aligned}$$

where we have used the following integral formula[9, 10]

$$\int \frac{d^2z}{\pi} \exp\left(\zeta|z|^2 + \xi z + \eta z^*\right) = -\frac{1}{\zeta}\exp\left[-\frac{\xi\eta}{\zeta}\right], \mathrm{Re}(\zeta) < 0. \quad (15)$$

By setting

$$\gamma = \alpha + \beta^*, \sigma = \alpha - \beta^* \quad (16)$$

with $\alpha = \frac{1}{\sqrt{2}}(x_1 + ip_1)$, $\beta = \frac{1}{\sqrt{2}}(x_2 + ip_2)$ and comparing with Eq.(4), Eq.(14) is rewritten as

$$\begin{aligned}
\Delta(\sigma,\gamma) &= \\
&\frac{1}{\pi^2} : \exp\left[-2(a_1^\dagger - \alpha^*)(a_1 - \alpha) - 2(a_2^\dagger - \beta^*)(a_1 - \beta)\right]: \\
&= \Delta(\alpha,\alpha^*)\Delta(\beta,\beta^*), \quad (17)
\end{aligned}$$

which is just the product of two independent single-mode Wigner operators. We refer to $\Delta(\sigma,\gamma)$ in Eq.(14) as the entangled Wigner operator, since it can lead to the projector operator of the entangled state $|\eta\rangle$ and the marginal distribution in (η_1,η_2) phase space when we perform the integration of $\Delta(\sigma,\gamma)$ over $d^2\gamma$, i.e.

$$\int \Delta(\sigma,\gamma)d^2\gamma = \frac{1}{\pi}|\eta\rangle\langle\eta||_{\eta=\sigma}. \tag{18}$$

For the density matrix ρ of a given two-body correlated system, the corresponding Wigner function is

$$\begin{aligned} W_\rho(\sigma,\gamma) &= \\ &= Tr[\rho\Delta(\sigma,\gamma)] \\ &= \frac{d^2\eta}{\pi^3}\langle\sigma+\eta|\rho|\sigma-\eta\rangle\exp(\eta\gamma^*-\eta^*\gamma). \end{aligned} \tag{19}$$

3. WIGNER FUNCTION EVOLUTION GOVERNED BY H

Based on the above preliminaries, we devote this section to studying the time evolution of the Wigner function for the Hamiltonian in Eq.(9). For the convenient discussion, we first introduce

$$\begin{aligned} M &= m_1+m_2, \quad \mu=(m_1m_2)/M, \quad x_r = X_1-X_2, \\ P &= P_1+P_2, \quad P_r=\mu_2P_1-\mu_1P_2, \quad \mu_i=\frac{m_i}{M}, \end{aligned} \tag{20}$$

where M, μ, P, x_r and P_r are total mass, reduced mass, total momentum, relative coordinate, and mass-weight relative momentum, respectively. It then follows by substituting Eq.(20) into Eq.(9) that

$$\begin{aligned} H &= \left(\frac{1}{2M}+k\mu_1\mu_2\right)P^2 \\ &+ \left(\frac{1}{2\mu}-k\right)P_r^2+k(\mu_2-\mu_1)PP_r+V(x_r). \end{aligned} \tag{21}$$

According to the Heisenberg equation

$$(\partial/\partial t)\rho = -i[H,\rho], \tag{22}$$

we obtain

$$\begin{aligned} &\frac{\partial}{\partial t}\langle\sigma+\eta|\rho|\sigma-\eta\rangle \\ &= -i\langle\sigma+\eta|\left[\left(\frac{1}{2M}+k\mu_1\mu_2\right)P^2+\left(\frac{1}{2\mu}-k\right)P_r^2\right. \\ &\left.+k(\mu_2-\mu_1)PP_r+V(x_r)\right]\rho|\sigma-\eta\rangle \\ &+i\langle\sigma+\eta|\rho\left[\left(\frac{1}{2M}+k\mu_1\mu_2\right)P^2+\left(\frac{1}{2\mu}-k\right)P_r^2\right. \\ &\left.+k(\mu_2-\mu_1)PP_r+V(x_r)\right]|\sigma-\eta\rangle. \end{aligned} \tag{23}$$

In order to simplify Eq.(23), we shall obtain the expression of $\langle\eta|P_r$. By appealing to the Schmidt decomposition of $|\eta\rangle$ in the $|p\rangle$ representation[17]

$$|\eta\rangle = e^{-i\eta_1\eta_2}\int dp\left|p+\sqrt{2}\eta_2\right\rangle_1\otimes|-p\rangle_2 e^{-i\sqrt{2}\eta_1 p}, \tag{24}$$

where $\otimes$ stands for direct product. It follows from Eq.(24) and $\mu_2+\mu_1=1$ that

$$\begin{aligned} P_r|\eta\rangle &= e^{-i\eta_1\eta_2}\int dp\left[\mu_2\left(p+\sqrt{2}\eta_2\right)+\mu_1 p\right]\left|p+\sqrt{2}\eta_2\right\rangle_1 \\ &\otimes|-p\rangle_2 e^{-i\sqrt{2}\eta_1 p} \\ &= \left[i\frac{\partial}{\partial\left(\sqrt{2}\eta_1\right)}-\frac{(\mu_1-\mu_2)\eta_2}{\sqrt{2}}\right]|\eta\rangle. \end{aligned} \tag{25}$$

As a result of Eqs.(12) and (25) we have

$$\begin{aligned} \langle\sigma+\eta|\left(\frac{1}{2M}+k\mu_1\mu_2\right)P^2 &= \left(\frac{1}{M}+2k\mu_1\mu_2\right)(\sigma_2+\eta_2)^2\langle\sigma+\eta|, \\ \langle\sigma+\eta|\left(\frac{1}{2\mu}-k\right)P_r^2 &= \left(\frac{1}{4\mu}-\frac{k}{2}\right)\left[i\frac{\partial}{\partial(\sigma_1+\eta_1)}+(\mu_1-\mu_2)(\sigma_2+\eta_2)\right]^2\langle\sigma+\eta|. \end{aligned} \tag{26}$$

Setting $\sigma+\eta=\tau$, $\sigma-\eta=\lambda$ and using $\partial/\partial(\sigma_1\pm\eta_1)=(\partial/\partial\sigma_1\pm\partial/\partial\eta_1)/2$ and Eqs.(25) and (26) we rewrite Eq.(23) as

$$\begin{aligned} &i\frac{\partial}{\partial t}\langle\sigma+\eta|\rho|\sigma-\eta\rangle \\ &= \left\{\left(\frac{4}{M}+8k\mu_1\mu_2\right)\sigma_2\eta_2-k(\mu_2-\mu_1)\left[i\sigma_2\frac{\partial}{\partial\sigma_1}+i\eta_2\frac{\partial}{\partial\eta_1}+4(\mu_1-\mu_2)\sigma_2\eta_2\right]\right. \\ &+\left(\frac{1}{4\mu}-\frac{k}{2}\right)\left[2i(\mu_1-\mu_2)\left(\sigma_2\frac{\partial}{\partial\sigma_1}+\eta_2\frac{\partial}{\partial\eta_1}\right)-\frac{\partial^2}{\partial\sigma_1\partial\eta_1}+4(\mu_1-\mu_2)^2\sigma_2\eta_2\right] \\ &\left.+V\left[\sqrt{2}(\sigma_1+\eta_1)\right]-V\left[\sqrt{2}(\sigma_1-\eta_1)\right]\right\}\langle\sigma+\eta|\rho|\sigma-\eta\rangle. \end{aligned} \tag{27}$$

Due to $\frac{\mu}{M}=\mu_1\mu_2$, $\mu_1+\mu_2=1$, Eq.(27) can be converted into

$$
\begin{aligned}
& i\frac{\partial}{\partial t}\langle\sigma+\eta|\rho|\sigma-\eta\rangle \\
& =\left\{\left(\frac{1}{\mu}+2k\right)\sigma_2\eta_2-ik(\mu_2-\mu_1)\left(\sigma_2\frac{\partial}{\partial\sigma_1}+\eta_2\frac{\partial}{\partial\eta_1}\right)+A\right. \\
& \left.+\left(\frac{1}{4\mu}-\frac{k}{2}\right)\left[2i(\mu_1-\mu_2)\left(\sigma_2\frac{\partial}{\partial\sigma_1}+\eta_2\frac{\partial}{\partial\eta_1}\right)-\frac{\partial^2}{\partial\sigma_1\partial\eta_1}\right]\right\}\langle\sigma+\eta|\rho|\sigma-\eta\rangle.
\end{aligned}
\tag{28}
$$

where

$$
\begin{aligned}
A &\equiv V\left[\sqrt{2}(\sigma_1+\eta_1)\right]-V\left[\sqrt{2}(\sigma_1-\eta_1)\right] \\
&=2\sum_{s=0}^{\infty}\frac{1}{(2s+1)!}\frac{\partial^{2s+1}V\left(\sqrt{2}\sigma_1\right)}{\partial\left(\sqrt{2}\sigma_1\right)^{2s+1}}\left(\sqrt{2}\eta_1\right)^{2s+1}.
\end{aligned}
\tag{29}
$$

Substituting Eq.(28) into

$$
\frac{\partial}{\partial t}W_\rho(\sigma,\gamma,t)=\frac{\partial}{\partial t}\int\frac{d^2\eta}{\pi^3}\langle\sigma+\eta|\rho|\sigma-\eta\rangle e^{\eta\gamma^*-\eta^*\gamma}, \tag{30}
$$

and performing the integration by parts we derive

$$
\begin{aligned}
i\frac{\partial}{\partial t}W_\rho(\sigma,\gamma,t) &= \int\frac{d^2\eta}{\pi^3}e^{2i(\eta_2\gamma_1-\eta_1\gamma_2)}\left\{\left(\frac{1}{\mu}+2k\right)\sigma_2\eta_2+\left(\frac{1}{4\mu}-\frac{1}{2}k\right)\right. \\
&\times\left[2i(\mu_1-\mu_2)\sigma_2\frac{\partial}{\partial\sigma_1}-\frac{\partial}{\partial\sigma_1}\frac{\partial}{\partial\eta_1}+2i(\mu_1-\mu_2)\eta_2\frac{\partial}{\partial\eta_1}\right] \\
&\left.+k(\mu_2-\mu_1)\left(-i\sigma_2\frac{\partial}{\partial\sigma_1}-i\eta_2\frac{\partial}{\partial\eta_1}\right)+A\right\}\langle\sigma+\eta|\rho|\sigma-\eta\rangle \\
&=\left\{\left(\frac{1}{\mu}+2k\right)\sigma_2\left(-\frac{i}{2}\frac{\partial}{\partial\gamma_1}\right)+\left(\frac{1}{4\mu}-\frac{1}{2}k\right)\right. \\
&\times\left[2i(\mu_1-\mu_2)\sigma_2\frac{\partial}{\partial\sigma_1}-\frac{\partial}{\partial\sigma_1}(2i\gamma_2)+2i(\mu_1-\mu_2)\left(-\frac{i}{2}\frac{\partial}{\partial\gamma_1}\right)(2i\gamma_2)\right] \\
&\left.+k(\mu_2-\mu_1)\left[-i\sigma_2\frac{\partial}{\partial\sigma_1}-i\left(-\frac{i}{2}\frac{\partial}{\partial\gamma_1}\right)(2i\gamma_2)\right]+A\right\} \\
&\times\int\frac{d^2\eta}{\pi^3}e^{2i(\eta_2\gamma_1-\eta_1\gamma_2)}\langle\sigma+\eta|\rho|\sigma-\eta\rangle \\
&=\left\{\left(\frac{1}{\mu}+2k\right)\sigma_2\left(-\frac{i}{2}\frac{\partial}{\partial\gamma_1}\right)+\left(\frac{1}{4\mu}-\frac{1}{2}k\right)\right. \\
&\times\left[2i(\mu_1-\mu_2)\sigma_2\frac{\partial}{\partial\sigma_1}-\frac{\partial}{\partial\sigma_1}(2i\gamma_2)+2i(\mu_1-\mu_2)\left(-\frac{i}{2}\frac{\partial}{\partial\gamma_1}\right)(2i\gamma_2)\right] \\
&\left.+k(\mu_2-\mu_1)\left[-i\sigma_2\frac{\partial}{\partial\sigma_1}-i\left(-\frac{i}{2}\frac{\partial}{\partial\gamma_1}\right)(2i\gamma_2)\right]+A\right\}W_\rho(\sigma,\gamma,t).
\end{aligned}
\tag{31}
$$

Therefore, the equation of motion for the Wigner function is

$$
\begin{aligned}
& i\frac{\partial}{\partial t}W_\rho(\sigma,\gamma,t) \\
& =\left\{\left(\frac{1}{\mu}+2k\right)\sigma_2\left(-\frac{i}{2}\frac{\partial}{\partial\gamma_1}\right)\right. \\
& +\left(\frac{1}{4\mu}-\frac{k}{2}\right)\left[2i(\mu_1-\mu_2)\left(\sigma_2\frac{\partial}{\partial\sigma_1}+\gamma_2\frac{\partial}{\partial\gamma_1}\right)-2i\gamma_2\frac{\partial}{\partial\sigma_1}\right] \\
& \left.-ik(\mu_2-\mu_1)\left(\sigma_2\frac{\partial}{\partial\sigma_1}+\gamma_2\frac{\partial}{\partial\gamma_1}\right)+A\right\}W_\rho(\sigma,\gamma,t).
\end{aligned}
\tag{32}
$$

Further, inserting Eq.(29) into Eq.(32) yields

$$\left\{\frac{\partial}{\partial t}+\left\{\frac{1}{2\mu}\left[\sigma_2-(\mu_1-\mu_2)\gamma_2\right]+k\sigma_2\right\}\frac{\partial}{\partial\gamma_1}+\left\{\frac{1}{2\mu}\left[\gamma_2-(\mu_1-\mu_2)\sigma_2\right]+k\gamma_2\right\}\frac{\partial}{\partial\sigma_1}-2\frac{\partial V\left(\sqrt{2}\sigma_1\right)}{\partial\left(\sqrt{2}\sigma_1\right)}\frac{\partial}{\partial\left(\sqrt{2}\gamma_2\right)}\right\}W_\rho(\sigma,\gamma,t)$$
$$=\sum_{s=1}^{\infty}\frac{2(-1)^s}{(2s+1)!}\frac{\partial^{2s+1}V\left(\sqrt{2}\sigma_1\right)}{\partial\left(\sqrt{2}\sigma_1\right)^{2s+1}}\frac{\partial^{2s+1}}{\partial\left(\sqrt{2}\gamma_2\right)^{2s+1}}W_\rho(\sigma,\gamma,t). \tag{33}$$

To see the physical meaning of Eq.(33) more clearly, from Eq.(16) we find that

$$\gamma_1=\frac{1}{\sqrt{2}}(x_1+x_2),\gamma_2=\frac{1}{\sqrt{2}}(p_1-p_2),$$
$$\sigma_1=\frac{1}{\sqrt{2}}(x_1-x_2),\sigma_2=\frac{1}{\sqrt{2}}(p_1+p_2), \tag{34}$$

which lead to

$$\frac{1}{2\mu}\left[\sigma_2-(\mu_1-\mu_2)\gamma_2\right]=\frac{1}{\sqrt{2}}\left(\frac{p_1}{m_1}+\frac{p_2}{m_2}\right),$$
$$\frac{1}{2\mu}\left[\gamma_2-(\mu_1-\mu_2)\sigma_2\right]=\frac{1}{\sqrt{2}}\left(\frac{p_1}{m_1}-\frac{p_2}{m_2}\right). \tag{35}$$

Eq.(33) is equal to

$$\left\{\frac{\partial}{\partial t}+\left[\frac{1}{\sqrt{2}}\left(\frac{p_1}{m_1}+\frac{p_2}{m_2}\right)+\frac{(p_1+p_2)k}{\sqrt{2}}\right]\frac{\partial}{\partial\gamma_1}+\left[\frac{1}{\sqrt{2}}\left(\frac{p_1}{m_1}-\frac{p_2}{m_2}\right)+\frac{(p_1-p_2)k}{\sqrt{2}}\right]\frac{\partial}{\partial\sigma_1}-2\frac{\partial V\left(\sqrt{2}\sigma_1\right)}{\partial\left(\sqrt{2}\sigma_1\right)}\frac{\partial}{\partial\left(\sqrt{2}\gamma_2\right)}\right\}W_\rho(\sigma,\gamma,t)$$
$$=\sum_{s=1}^{\infty}\frac{2(-1)^s}{(2s+1)!}\frac{\partial^{2s+1}V\left(\sqrt{2}\sigma_1\right)}{\partial\left(\sqrt{2}\sigma_1\right)^{2s+1}}\frac{\partial^{2s+1}}{\partial\left(\sqrt{2}\gamma_2\right)^{2s+1}}W_\rho(\sigma,\gamma,t). \tag{36}$$

It is seen from Eq.(36) that this equation is expressed by center-of-mass coordinate, relative coordinate and relative momentum. Especially, when $k=0$, Eq.(36) becomes

$$\left\{\frac{\partial}{\partial t}+\frac{1}{\sqrt{2}}\left(\frac{p_1}{m_1}+\frac{p_2}{m_2}\right)\frac{\partial}{\partial\gamma_1}+\frac{1}{\sqrt{2}}\left(\frac{p_1}{m_1}-\frac{p_2}{m_2}\right)\frac{\partial}{\partial\sigma_1}-2\frac{\partial V\left(\sqrt{2}\sigma_1\right)}{\partial\left(\sqrt{2}\sigma_1\right)}\frac{\partial}{\partial\left(\sqrt{2}\gamma_2\right)}\right\}\times W_\rho(\sigma,\gamma,t)$$
$$=\sum_{s=1}^{\infty}\frac{2(-1)^s}{(2s+1)!}\frac{\partial^{2s+1}V\left(\sqrt{2}\sigma_1\right)}{\partial\left(\sqrt{2}\sigma_1\right)^{2s+1}}\frac{\partial^{2s+1}}{\partial\left(\sqrt{2}\gamma_2\right)^{2s+1}}W_\rho(\sigma,\gamma,t), \tag{37}$$

which agrees with that of Ref.[13]. Eq.(37) is formally comparable with Eq.(7) as expected.

It is instructive to consider the classical limit of this equation. For this purpose, we need to reexpress Eq.(29) as

$$A=2\sum_{s=0}^{\infty}\frac{\hbar^{2s+1}}{(2s+1)!}\frac{\partial^{2s+1}V\left(\sqrt{2}\sigma_1\right)}{\partial\left(\sqrt{2}\sigma_1\right)^{2s+1}}\left(\frac{\sqrt{2}\eta_1}{\hbar}\right)^{2s+1}, \tag{38}$$

and at the same time change $e^{2i(\eta_2\gamma_1-\eta_1\gamma_2)}$ in Eq.(31) into $e^{2i(\eta_2\gamma_1-\eta_1\gamma_2)/\hbar}$, then formally set $\hbar=0$. Provided the derivatives of the Wigner function on the right hand side of this equation do not become singular, the right hand side vanishes. So we have

$$\left\{\frac{\partial}{\partial t}+\left[\frac{1}{\sqrt{2}}\left(\frac{p_1}{m_1}+\frac{p_2}{m_2}\right)+\frac{(p_1+p_2)k}{\sqrt{2}}\right]\frac{\partial}{\partial\gamma_1}+\left[\frac{1}{\sqrt{2}}\left(\frac{p_1}{m_1}-\frac{p_2}{m_2}\right)+\frac{(p_1-p_2)k}{\sqrt{2}}\right]\frac{\partial}{\partial\sigma_1}-2\frac{\partial V\left(\sqrt{2}\sigma_1\right)}{\partial\left(\sqrt{2}\sigma_1\right)}\frac{\partial}{\partial\left(\sqrt{2}\gamma_2\right)}\right\}W_\rho(\sigma,\gamma,t)=0. \tag{39}$$

In this sense the terms on the right hand side of Eq.(37) constitute the quantum mechanical corrections to the classical Liouville equation. Comparing with Eq.(8), Eq.(39) is called the Liouville-like equation for the two-body correlated system.

4. CONCLUSIONS

At present, Wigner function is also an important tool to transform the operator equation of motion for the density operator into a c-number equation. However, this equation is rather complicated and brings out the nonlocal nature of the Wigner function. In this present work, we have considered a two-body correlated system with kinetic coupling term in Eq.(9) and successfully derived the time evolution equation of the Wigner function (see Eq.(36)) by virtue of the bipartite entangled state representation, which just indicates that choosing a good representation indeed provides great convenience for us to deal with the dynamics problem.

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