Pairing gap energy correction in Shell model for the neutron-rich tin isotopes

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The region in the vicinity of ^{132}Sn, gathering numerous isotopes of important masses, shows lots of interests for explaining the most interesting phenomena. It offers among others the possibility to extract the $N-N$ empirical interactions so as to test theoretical shell model description of nuclear structure in this region. The researches on the neutron-rich tin isotopes present the anomalies in a systematic variation on various spectroscopic data such as the first 2+ excited state and reduced probabilities $B(E2; 2^+ \rightarrow 0^+)$. In order to improve the OXBASH interactions which are based on the shell model calculations, we have modified some neutron-neutron and proton-proton tow body matrix elements of CW5082 interaction of ^{132}Sn region by adding to them the pairing gap energy. The calculations turn out a new interaction so called CWD5082, allowing to determine the excited energies and the reduced probabilities $B(E2)$ of Sn and Te isotopes for $A = 134$ and 136 masses. The obtained results display a remarkable enhancement in the predictive power of experimental values.

1. INTRODUCTION

As experience has progressed toward the nuclear drip line, it becomes possible to examine the systematic properties of nuclear isotopic chains composed of nuclei with great N and Z numbers. That offers new opportunities to evaluate the developed theoretical models for the grandest exotic phenomena, and therefore imply the development of the applicable effective interaction for these channels. One of the regions in which recent experimental progress has been made is the neutron-rich tin isotopes ^{132}Sn. This region shows a closure shells ($Z = 50$) for protons and ($N = 82$) for neutrons with the nuclei that can provide direct information on the nucleon-nucleon effective interaction, the effective charges values, and particles or holes energies [1]. In addition, the experimental and/or theoretical study of neutron-rich nuclei in the ^{132}Sn region is very important for modeling the r-process astrophysics nucleosynthesis.

The structure of the pairing correlations and the strength of the pairing interaction as we move towards the neutron drip-line in the neutron-rich ^{132}Sn region, are a great interest subject in experimental and theoretical nuclear physics [2][3]. Since the pairing plays an important role in exotic nuclei, weakly bound nuclei with a magnitude of chemical potential close to that of the pairing gap [4].

In this article we are interested studying and understanding the role of the pairing effect on the shell model calculations for the neutron-rich tin isotopes, by modifying the CW5082 interaction of Chou and Warburton [5] in the OXBASH code [6]. In these modifications, we have considered the pairing gap energy. In addition, the obtained results are confronted to the experimental values, the values obtained by the CW5082 original interaction and those of S. Saha et al. [7].

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2. PAIRING GAP ENERGY AND SHELL MODEL

The shell model calculations of the neutron-rich tin isotopes have been developed using the OXBASH code [6]. The space model is composed of an inert core ^{132}Sn and all the orbitals between ^{132}Sn and ^{208}Pb: $1g_{9/2}$, $2d_{5/2}$, $2d_{3/2}$, $3s_{1/2}$ and $1h_{11/2}$ for protons with the energies: -9.6629, -8.7005, -7.2233, -6.9657 and -6.8714 [8] respectively, expressed in MeV, and $1h_{9/2}$, $2f_{7/2}$, $2f_{5/2}$, $3p_{3/2}$, $3p_{1/2}$ and $1i_{13/2}$ for neutrons with the energies: -0.8944, -2.4553, -0.4507, -1.6016, -0.7996 and $+0.2397$ [8] respectively, expressed in MeV. These single energies are taken from experimental values, except that of the single state $3s_{1/2}$ which was derived from a local system [9]. While the energy of the state $1i_{13/2}$ was taken from the reference [10].

The CW5082 interaction is construct from the Kuo-Herling interaction so-called KH5082 [5], by replacing the tow body matrix elements (TBME) of protons in the sub-shell $N = 4$ using the effective interaction Kruse and Wildenthal [5] from the derivation of a least squares fit of binding energies with the surface delta interaction (SDI) as a starting point.

The binding energies of neutron orbitals have been reduced by 100 keV and the proton-neutron TBME are modified to reproduce the levels of spin $J = 0$ and 1 of the ^{134}Sb nucleus.

All two body matrix elements of KH5082 interaction (Kuo-Herling bare + partcule-hole "or" bare G matrix + core polarization") of the ^{208}Pb region have been multiplied by the mass factor $(132/208)^{1/3}$. Moreover, the six neutron-neutron diagonal elements of the spin $Jp = 0^+$ which are more attractive and produce a deficit in the binding energies of the states of the Kuo-Herling interaction, were then reduced by a factor of 0.6 [11].

To obtain the binding energy of -6.365 MeV the ground state of ^{134}Sn nucleus, we have multiplied the six neutron-neutron diagonal elements of the spin. $J^\pi = 0^+$ by the factor 0.48 [5]. Then, calculate the quantity:

$$2\Delta_n - E_{(CW5082)}(2^+) \qquad (1)$$

where $2\Delta_n$ corresponds to the energy needed to break a pair of neutrons, thus allows the jump of a neutron toward the

superior orbital.

$$\Delta_n = \frac{(-1)^{A-Z+1}}{4}[-M(Z,N+1)+3M(Z,N)$$
$$- M(Z,N-1)+M(Z,N-2)] \qquad (2)$$

After that, we subtract the latter part of the neutron-neutron matrix elements, corresponding to three excited states 2^+, 4^+ and 6^+ of ^{134}Sn of the multiple configurations $(\nu\,2f_{7/2})^2$ and a matrix element of the configuration $(\nu\,1h_{9/2}\nu\,2f_{7/2})^2$ of the excited state 8+ of the original interaction CW5082. Finally, we have multiplied these obtained elements by a factor of 0.6 that is proposed by Terasaki [13] in order to renormalize for the reduction of the neutrons pairing gap above $N = 82$.

According to the modification steps outlined above, we have also changed the proton-proton matrix elements for ^{134}Te nucleus of the multiple configurations from the multiplet $(\pi 2g_{7/2})^2$ of the spin $J = 0^+$, 2^+, 4^+ and 6^+. We determine also the quantity:

$$2\Delta_p - E_{(CW5082)}(2^+) \qquad (3)$$

as to calculate the elements:

$$\left[\langle V_{12}\rangle_{CW5082} - \left(2\Delta_p - E_{CW5082}(2^+)\right)\right] \qquad (4)$$

With Δ_p is expressed by the following formula:

$$\Delta_p = \frac{(-1)^{A-Z+1}}{4}[-M(Z+1,N)+3M(Z,N)$$
$$- M(Z-1,N)+M(Z-2,N)] \qquad (5)$$

$2\Delta_p$ corresponds to the energy needed to break a pair of protons, allowing the jump of a proton to the superior orbital. These elements are multiplied by a factor of renormalization proton value 0.9 [13]. The same reduction was made for these matrix elements. Finally, by the modification of 16 matrix elements of CW5082 interaction, we have reproduced a new interaction named by CWΔ5082.

3. RESULTS, COMPARISON, AND DISCUSSION

This work is divided into two parts: the first is devoted to calculate the low spin spectra energy of the even-even $^{134,136}Sn$, $^{134,136}Te$ and ^{136}Xe isotopes. While the second part concerns the calculation of the electric transition reduced probabilities B $(E2 : 0^+ \rightarrow 2^+)$ for some of these nuclei. These calculations are performed by using the effective interaction CWΔ5082 in the framework the OXBASH code.

The calculations with the original CW5082 interaction have also been included.

The two figures (1) and (2) illustrate the results of the obtained energy spectra. These are compared with experimental data and spectra calculated by S. Saha et al., [5] [14].

On the figures (1.a) and (2.a) showing the interaction between the valence neutrons in the isotopes $^{134,136}Sn$, we see clearly that the obtained results by CWΔ5082 for ^{134}Sn are very close to those of experience and of SMN, although the agreement is better for the first three excited states 2^+, 4^+ and 6^+. On the figure (2.a) there is no experimental data of
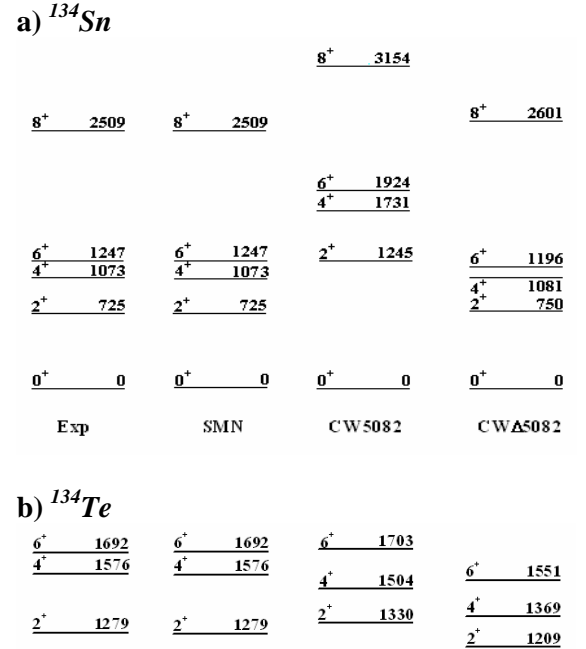

FIG. 1: Comparison of calculated spectra with experimental ones for isobars: (a) ^{134}Sn and (b) ^{134}Te.

^{136}Sn nucleus that we allow comparing with the experience, but we put in this figure our results with those calculated by CW5082 interaction and those of SMN. We note that the different between the CWΔ5082 and SMN for the excited 2^+, 4^+ and 6^+ levels is less than 80keV.

The (1b) and (2.c) figures exhibit the energy spectra of ^{134}Te and ^{136}Xe isotones, respectively, where there is a significant gap between the excited states energies obtained by CWΔ5082 and those of the experiment. Moreover, this gap grows with increasing number of valence protons. One has ΔE (^{134}Te)=100 keV from the state 4^+ and ΔE (^{136}Xe)= 300 keV. While this difference is not observed between the experimental spectra energy and those of the CWΔ5082 interaction for ^{136}Te nucleus as is revealed in the figure (2.b). These results reflect the interaction between the valence neutrons of $N = 82 - 126$ shells and valence protons of $Z = 50 - 82$ shells.

In general, we find that both SMN interactions and SMPN of S. Saha give the closest results to the experimental ones in the both figures (1) and (2)

On the basis of these results, we can show two very important points:

● The first point exposes the effect of the pairing energy gap on the neutron-neutron and proton-proton interaction in the shell model calculations. You can also see that the $2\Delta_n$ effect is more interesting than the $2\Delta_p$ one. The original CW5082 interaction represents the proton-proton interaction better than the modified CWΔ5082 interaction, as the case of ^{134}Te and ^{136}Xe nuclei.

● The second one, shows the influence of inert core ^{132}Sn ($Z = 50$ and $N = 82$) on the calculations of energy spectra
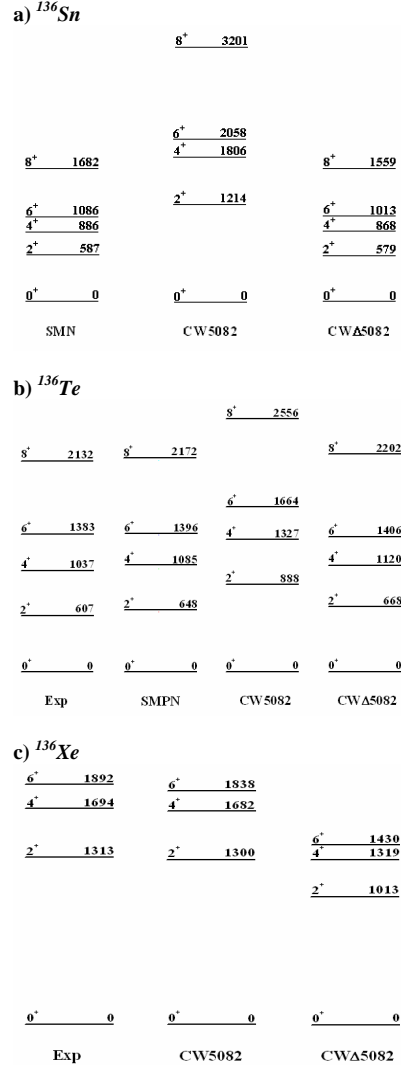
FIG. 2: Comparison of calculated spectra with experimental ones for isobars: (a) ^{136}Sn, (b) ^{136}Te and (c) ^{136}Xe.

of the nuclei with valence protons i.e. the effect of the interaction of neutrons located in the $N = 50 - 82$ shells on the valence protons in the $Z = 50 - 82$ shells.

In the second part of this work, we have calculated the reduced probabilities $B(E2)$ of the electric transition of the ground state to first excited state $0^+ \rightarrow 2^+$ (absorption) for $^{134,136}Sn$ and $^{134,136}Te$ isotopes. The table below shows the results. The actual charges used correspond to a proton effective charge of 1.47e and a neutron effective charge of 0.72e.

The comparison between the available experimental values of $B(E2)$ with those of CWΔ5082 and SMN, demonstrates a very good agreement between the calculated results and those of experience for the ^{134}Te nucleus. By against, we can not find this agreement for ^{136}Te nucleus.

The study of the behavior of the transition probability $B(E2 : 0^+ \rightarrow 2^+) \uparrow$ contributes in the classification of the energy of the first excited 2^+ level ($E2^+$) according to the relation of Grodzins [13] where we find that B(E2)$\uparrow$ increases with decreasing of E2$^+$ (table (1)).

TABLE 1: Comparison between the values of $B(E2)$ calculated with the experimental (in $e^2 fm^4$) using the of $e_p = 1.47e$ and $e_n = 0.72e$ effective charges

Isotopes	$I_i^\pi \rightarrow I_f^\pi$	Exp	SMN	CW5082	CWΔ5082
^{134}Sn	$0^+ \rightarrow 2^+$	290.0	366.0	413.9	369.5
^{136}Sn	$0^+ \rightarrow 2^+$	-	-	858.4	561.5
^{134}Te	$0^+ \rightarrow 2^+$	960.0	869.8	857.9	824.9
^{136}Te	$0^+ \rightarrow 2^+$	1030.0	2165.0	2989.0	1794.0

4. CONCLUSION

The adding of the pairing gap Δ_n and Δ_p in the modifications of the CW5082 interaction produced a new interaction called CWΔ5082. The application of the latter interaction in the shell model calculations produced the good results for the $^{134,136}Sn$, $^{134,136}Te$ and ^{136}Xe even-even nuclei compared to those of the original interaction CW5082. Despite this, the CWΔ5082 interaction does not describe the effect of the modifications in proton-proton matrix elements for the energy spectra calculations of ^{134}Te and ^{136}Xe isotones, because of the addition of the pairing gap in the proton-proton matrix elements of the original CW5082 interaction. The calculated values of reduced probabilities of electric dipole transition $B(E2)$ using the effective charges 1.47e and 0.72e of proton and neutron, respectively, are in good agreement with the experiment for ^{134}Sn and ^{134}Te nuclei.

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