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GENERAL AND APPLIED PHYSICS

The Dielectric Tensor for Magnetized Dusty Plasmas with Superthermal Plasma Populations and Dust Particles of Different Sizes

Renato Andrade Galvão · Luiz Fernando Ziebell · Rudi Gaelzer · Marcelo Camargo de Juli

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Abstract We present general expressions for the components of the dielectric tensor of magnetized dusty plasmas, valid for arbitrary direction of propagation and for situations in which populations of dust particles of different sizes are present in the plasma. These expressions are derived using a kinetic approach which takes into account the variation of the charge of the dust particles due to inelastic collisions with electrons and ions, and features the components of the dielectric tensor in terms of a finite and an infinite series, containing all effects of harmonics and Larmor radius, and is valid for the whole range of frequencies above the plasma frequency of the dust particles, which are assumed to be motionless. The integrals in velocity space which appear in the dielectric tensor are solved assuming that the electron and ion populations are described by anisotropic non-thermal distributions characterized by parameters $\kappa_\parallel$ and $\kappa_\perp$, featuring the Maxwellian as a limiting case. These integrals can be written in terms of generalized dispersion functions, which can be expressed in terms of hypergeometric functions. The formulation therefore becomes specially suitable for numerical analysis.

Keywords Dusty plasmas · Dielectric tensor · Kappa distributions · Waves and instabilities

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1 Introduction

The plasma environment inside the heliosphere is dominated by the solar wind, a continuous outflow of completely ionized plasma from the solar corona. The solar wind is mostly composed by electrons and protons, with a small population of alpha particles and heavier ions, and a much smaller concentration of charged dust particles [1, 2]. A characteristic which is important as motivation to the present work is that the velocity distribution functions of the main components of the solar wind feature markedly non-thermal characteristics, which can be associated with a variety of instabilities. For instance, the anisotropic proton distributions can be associated with firehose instabilities, mirror instabilities, ion-cyclotron instabilities, and Weibel instabilities [3–5].

The observations of non-thermal characteristics of solar wind proton distributions are extensive. For instance, the proton distribution was measured by the satellites IMP 6, HELIOS, Wind, and Ulysses, within the radial interval from 0.3 to 2.5 AU. It displays a significant temperature anisotropy, with the temperature in the direction perpendicular to the local interplanetary magnetic field which is larger than the parallel temperature, at distances $\simeq 0.3$ AU, but with parallel temperature larger than perpendicular temperature for radial distances greater than 1 AU. Another characteristic that is repeatedly observed is the presence of a second proton component, streaming away from the Sun along the direction parallel to the magnetic field, with a relative velocity drift of the order the Alfvén speed. The number density of this population is between 10% and 20% of the total proton density, giving the complete distribution a distinct *double-humped* profile [1, 6–8]. These anisotropies in the proton distribution have been observed both in the slow solar wind and in the fast solar wind [9, 10].

As it is known, these two denominations designate two basic states of the solar wind. The slow wind is characterized by flow speeds in the range 250–400 km/s and is well characterized as an adiabatic flow coming out of active regions in the photosphere of the Sun. The fast wind features flow speeds in the range 400–800 km/s, and its origin is usually associated with magnetic loops above the photosphere [1, 2, 11].

The electron distribution function in the solar wind is also known to display non-thermal characteristics. It is typically observed to be constituted by a dense thermal core and two tenuous but hot superthermal populations. These are the *halo*, which is present at all pitch angles, and the *Strahl*, which is a highly anisotropic, field-aligned population propagating in the anti-sunward direction [12–21]. These non-thermal distributions have been modeled in recent years by Lorentzian or kappa distributions [22–24] and the same distributions have been proposed for modeling the proton VDF as well [25].

The kappa distribution can be characterized by the κ index, which is a measure of the departure of the distribution from the Maxwellian, with the latter being the asymptotic limit of the former when $\kappa \to \infty$. Using combinations of kappa distributions, [23] and [24] have been able to measure the κ index of the electron VDF by fitting the data acquired by spacecraft in the range 0.3–4 AU. The results show that the κ index drops monotonically with radial distance for both the halo and Strahl components and both in the so-called slow and fast solar winds, with $\kappa \simeq 2$ for distances of order or 4 AU, and $10 \lesssim \kappa \lesssim 16$ for a distance around 0.3 AU, implying that the physical processes operating in the solar wind as it streams away from the Sun contribute for an increasing departure of its thermo-statistical properties from the traditional Boltzmann–Gibbs thermodynamics. Even at relatively close distances to the Sun the distribution functions are noticeably different from the Maxwellian, with the fast wind exhibiting two clearly different regimes ($\kappa \simeq 7$ for the halo and $\kappa \simeq 14$ for the Strahl), while both components are in the same state ($\kappa \simeq 9.5$) for the slow wind at 0.3 AU.

Another feature of the solar wind which is relevant as motivation for the present work is the observed presence of dust particles. In fact, observations report the presence of dust particles at distances from the Sun ranging from a fraction of astronomical unit, in the region of the so-called inner solar system, up to a few astronomical units [26–28]. In the more distant regions of the solar system the dust is usually thought as originated from the interstellar environment, while in the inner solar system it is thought to originate mostly from cometary tails and asteroids. Typically, the ratio between dust number density and plasma number density is very small. However, since dust particles are much more massive than electrons and protons, the total mass of dust in the solar wind is significant, being at least of the same order of magnitude as the total solar wind mass [29].

These observations can be used as arguments supporting investigations on waves and instabilities in dusty plasmas. Upon consulting the literature, one notices that a major fraction of the theoretical analyses already printed about the subject has been based on fluid theory, and only a few of these analyses properly take into account the collisional charging of the dust particles [30, 31], despite the well-known importance of this effect to the propagation and damping of waves [32, 33]. However, the fluid formulation has an important limitation since it cannot describe purely kinetic effects such as the Landau damping. Moreover, it has been shown that the dust-charging process must be included in a kinetic approach, for proper derivation of the wave damping [34]. In fact, it is not possible to separate the conventional Landau damping and the damping due to the interaction of ions and electrons with the dust particles, at least for ion-acoustic waves [35].

These considerations offer motivation for the study of dusty plasmas using a kinetic formulation. Some examples are an early paper by Rosenberg [36], which contains a study of instabilities of ion-acoustic waves produced by current in a fully ionized collisionless dusty plasma, without taking into account the effect of variations of the dust charge, a more recent investigation of the effect of dust-charge variation on high-frequency electrostatic plasma waves [37], and several studies on low-frequency electromagnetic waves [38–43] and on electrostatic waves in dusty plasmas [44–46] in which some of us have been engaged, which also take into account the effect of dust-charge variation, incorporating the dependence on the frequency of inelastic collisions of plasma electrons and ions with the dust particles into the components of the dielectric tensor. It may be added that the kinetic formulation employed in Refs.

[38, 46] can be connected to another formulation which can be found in the literature [34, 47, 48], under some restrictive assumptions [44].

The kinetic formulation that has been employed to the study of dusty plasmas in some of our recent publications features expressions for the components of the dielectric tensor which are written in terms of an infinite and a finite summations, formally incorporating effects of all harmonics and all orders of Larmor radius, and keeping effects due to the charging of the dust particles due to inelastic collisions with electrons and ions. Details of the derivation can be found, for instance, in [45]. The formulation is quite general in terms of frequency range and direction of propagation, but was up to now limited to the case in which only one population of dust particles occurs in the plasma. In the present paper, we generalize this basic kinetic approach so that it can be applied to situations in which different populations of dust particles are present, with different sizes. On the other hand, we particularize the formulation to the case in which the populations of plasma particles are described by anisotropic kappa distributions, featuring the Maxwellian case as a limiting case. The resulting formulation can be therefore very useful for the investigation of waves and instabilities in dusty plasmas featuring a variety of non-thermal distributions. In fact, a preliminary investigation on non-thermal features coupled with the presence of dust particles, considering a single population of dust particles, was recently conducted for the case of isotropic kappa distribution for electrons and Maxwellian distribution for ions [43].

The structure of the paper is the following. In Section 2, we briefly outline the model used to describe the dusty plasma and the essential features of the kinetic formulation which leads to the dielectric tensor for dusty plasmas, and present the new formulation which leads to the components of the dielectric tensor expressed in terms of double summations and a small number of basic integrals. Section 3 shows details of the evaluation of the basic integrals appearing in these expressions, for the case of anisotropic kappa distributions. The equilibrium condition is briefly discussed in Section 4. Final remarks are presented in Section 5.

2 The Dusty Plasma Model and the Dielectric Tensor for a Magnetized Dusty Plasma

The formulation to be developed here relies on previously published developments, particularly on the formulation whose detailed derivation appears in [45], to which we introduce some modifications in order to allow the study of plasmas containing more than a single population of dust particles. We consider a dusty plasma in a homogeneous external magnetic field along z direction, $\mathbf{B_0} = B_0\,\mathbf{e_z}$ and take into account the presence of n populations of spherical dust grains with constant radius a_j and variable charge q_j, $j = 1, \ldots, n$. The charge in the dust grains is assumed to be originated from the capture of plasma electrons and ions during inelastic collisions between these particles and the dust particles. Since the electron thermal speed is much larger than the ion thermal speed, the dust charge becomes preferentially negative. As a cross section for the charging process of the dust particles, we use expressions derived from the orbital motion limited theory [49, 50].

For simplicity, the ions are assumed to be simply charged.

We assume that the electrostatic energy of the dust particles is much smaller than their kinetic energy, therefore restricting the analysis to the so-called weakly coupled dusty magneto-plasmas, which comprise a large variety of natural and laboratory dusty plasmas [51].

We assume that the dust particles are immobile, therefore restraining the validity of the analysis to waves with frequencies much higher than the characteristic dust frequencies, which excludes the analysis of modes which can arise from the dust dynamics, as the so-called dust-acoustic wave.

As in previous studies, we assume that the distribution function of particles of species β, f_β, satisfies Vlasov's equation appended with a term describing binary inelastic collisions with dust particles,

$$\frac{\partial f_\beta}{\partial t} + \frac{\mathbf{p}}{m_\beta}\cdot\nabla f_\beta + q_\beta\left[\mathbf{E} + \frac{\mathbf{p}}{m_\beta c}\times\mathbf{B}\right]\cdot\nabla_{\mathbf{p}} f_\beta = -\int \mathrm{d}q\,\frac{p}{m_\beta}\sum_j \sigma_\beta^j\left(f_d^j f_\beta - f_{d0}^j f_{\beta 0}\right), \qquad (1)$$

where f_{d0}^j and $f_{\beta 0}$ represent respectively the equilibrium distribution functions of dust particles of species j and of plasma particles of species β. The formalism includes the interaction between the plasma particles and dust particles of all populations.

The distribution function for the dust particles of population j, $f_d^j \equiv f_d^j(\mathbf{r}, q, t)$, satisfies the following equation,

$$\frac{\partial f_d^j}{\partial t} + \frac{\partial}{\partial q}\left[I^j(\mathbf{r}, q, t)\, f_d^j\right] = 0\,, \qquad (2)$$

where

$$I^j(\mathbf{r}, q, t) = \sum_\beta \int d^3p\, q_\beta\, \sigma_\beta^j(p, q) \frac{p}{m_\beta} f_\beta(\mathbf{r}, \mathbf{p}, t)\,,$$

is the current of electrons and ions which charge the dust particles [34].

Considering small amplitude oscillations, the perturbed distribution function satisfies the following equation,

$$\frac{\partial f_{\beta 1}}{\partial t} + \frac{\mathbf{p}}{m_\beta} \cdot \nabla f_{\beta 1} + q_\beta \left(\frac{\mathbf{p}}{m_\beta c} \times \mathbf{B}_0 \right) \cdot \nabla_{\mathbf{p}} f_{\beta 1} + \left(\sum_j \nu_{\beta d}^{j0}(p) \right) f_{\beta 1} = - \left(\sum_j \nu_{\beta d}^{j1}(\mathbf{r}, p, t) \right) f_{\beta 0} - q_\beta \left[\mathbf{E}_1 + \frac{\mathbf{p}}{m_\beta c} \times \mathbf{B}_1 \right] \cdot \nabla_{\mathbf{p}} f_{\beta 0}\,, \tag{3}$$

where

$$\nu_{\beta d}^{j0}(p) \equiv \int_{-\infty}^{0} \sigma_\beta^j(p, q) \frac{p}{m_\beta} f_{d0}^j(q) dq\,,$$

$$\nu_{\beta d}^{j1}(\mathbf{r}, p, t) \equiv \int_{-\infty}^{0} \sigma_\beta^j(p, q) \frac{p}{m_\beta} f_{d1}^j(\mathbf{r}, q, t) dq\,,$$

and σ_β^j is the charging cross-section of dust particles of radius j by particles of species β, given by [52]

$$\sigma_\beta^j(p, q) = \pi a_j^2 \left(1 - \frac{2qq_\beta m_\beta}{a_j p^2} \right) H \left(1 - \frac{2qq_\beta m_\beta}{a_j p^2} \right). \tag{4}$$

Using Fourier–Laplace transform in the system of equations, it is readily shown that the perturbed distribution function for species β can be written as

$$\hat{f}_\beta(\mathbf{p}) = \hat{f}_\beta^C + \hat{f}_\beta^N, \tag{5}$$

where

$$\hat{f}_\beta^C = -q_\beta \int_{-\infty}^{0} d\tau\, e^{i\left\{\mathbf{k}\cdot\mathbf{R} - \left[\omega + i\left(\sum_j \nu_{\beta d}^{j0}(p)\right)\right]\tau\right\}} \times \left(\hat{\mathbf{E}} + \frac{\mathbf{p}'}{m_\beta c} \times \hat{\mathbf{B}} \right) \cdot \nabla_{\mathbf{p}'} f_{\beta 0}(p_\perp, p_\parallel)\,,$$

$$\hat{f}_\beta^N = - \int_{-\infty}^{0} d\tau\, e^{i\left\{\mathbf{k}\cdot\mathbf{R} - \left[\omega + i\left(\sum_j \nu_{\beta d}^{j0}(p)\right)\right]\tau\right\}} \left(\sum_j \hat{\nu}_{\beta d}^j(\mathbf{k}, p, \omega) \right) f_{\beta 0}.$$

These expressions are similar to those appearing in Refs. [53], except for the fact that in the present paper there is a summation over the inelastic equilibrium collision frequencies $\nu_{\beta d}^{j0}(p)$ in the argument of the exponential functions, instead of a single collision frequency, as well as a summation over the Fourier–Laplace transforms of the perturbed collision frequency $\nu_{\beta d}^{j1}$ in the integrand of $\hat{f}_\beta^N$, instead of a single value.

It is noticed that $\hat{f}_\beta^C$ has the same formal structure as the perturbed distribution obtained in the evaluation of the dielectric tensor of a conventional homogeneous magnetized plasma, with $\omega + i \sum_j \nu_{\beta d}^{j0}(p)$ instead of ω in the argument of the exponential function, and originates a contribution to the dielectric tensor which will be called the "conventional" contribution, denoted as ε_{ij}^C [45, 53]. The quantity identified as $\hat{f}_\beta^N$, in addition to a similar contribution to the argument of the exponential function, features an integrand which is proportional to $\sum_j \hat{\nu}_{\beta d}^j$ and which vanishes in the case of dustless plasma. It will have a contribution to the dielectric tensor which is exclusive to dusty plasmas, and which will therefore be called the "new" contribution, denoted as ε_{ij}^N [45, 53]. The components of the dielectric tensor therefore will be written as a summation of these two contributions [45, 53, 54]

$$\varepsilon_{ij} = \varepsilon_{ij}^C + \varepsilon_{ij}^N\,. \tag{6}$$

The development of the expression proceeds as in previous derivations, until that the components of the "conventional" contribution are written as follows [45]

$$\varepsilon_{ij}^C = \delta_{ij} + \delta_{iz}\delta_{jz} e_{zz} + N_\perp^{\delta_{iz}+\delta_{jz}} \chi_{ij}^C\,, \tag{7}$$

where

$$\chi_{xx} = \frac{1}{z^2} \sum_\beta \frac{\omega_{p\beta}^2}{\Omega_*^2} \frac{1}{n_{\beta 0}} \sum_{m=1}^{\infty} \left(\frac{q_\perp}{r_\beta} \right)^{2(m-1)} \times \sum_{n=-m}^{m} n^2 a(|n|, m - |n|) J(n, m, 0;\, f_{\beta 0})\,,$$

$$\chi_{xy} = i \frac{1}{z^2} \sum_\beta \frac{\omega_{p\beta}^2}{\Omega_*^2} \frac{1}{n_{\beta 0}} \sum_{m=1}^{\infty} \left(\frac{q_\perp}{r_\beta} \right)^{2(m-1)} \times \sum_{n=-m}^{m} nma(|n|, m - |n|) J(n, m, 0;\, f_{\beta 0})\,,$$

$$\chi_{yx} = -i \frac{1}{z^2} \sum_\beta \frac{\omega_{p\beta}^2}{\Omega_*^2} \frac{1}{n_{\beta 0}} \sum_{m=1}^{\infty} \left(\frac{q_\perp}{r_\beta} \right)^{2(m-1)} \times \sum_{n=-m}^{m} nma(|n|, m - |n|) J(n, m, 0;\, f_{\beta 0})\,,$$

$$\chi_{xz} = \frac{1}{z} \frac{v_*}{c} \sum_\beta \frac{1}{r_\beta} \frac{\omega_{p\beta}^2}{\Omega_*^2} \frac{1}{n_{\beta 0}} \sum_{m=1}^{\infty} \left(\frac{q_\perp}{r_\beta} \right)^{2(m-1)} \times \sum_{n=-m}^{m} na(|n|, m - |n|) \times \left[J(n, m, 1;\, f_{\beta 0}) + i J_\nu(n, m, 0;\, f_{\beta 0}) \right],$$

$$\chi_{zx} = \frac{1}{z}\frac{v_*}{c}\sum_\beta \frac{1}{r_\beta}\frac{\omega_{p\beta}^2}{\Omega_*^2}\frac{1}{n_{\beta 0}}\sum_{m=1}^{\infty}\left(\frac{q_\perp}{r_\beta}\right)^{2(m-1)} \times \sum_{n=-m}^{m} na(|n|, m-|n|)J(n,m,1;\ f_{\beta 0}),$$

$$\chi_{yy} = \frac{1}{z^2}\sum_\beta \frac{\omega_{p\beta}^2}{\Omega_*^2}\frac{1}{n_{\beta 0}}\sum_{m=1}^{\infty}\left(\frac{q_\perp}{r_\beta}\right)^{2(m-1)} \times \sum_{n=-m}^{m} b(|n|, m-|n|)J(n,m,0;\ f_{\beta 0}),$$

$$\chi_{yz} = -i\frac{1}{z}\frac{v_*}{c}\sum_\beta \frac{1}{r_\beta}\frac{\omega_{p\beta}^2}{\Omega_*^2}\frac{1}{n_{\beta 0}}\sum_{m=1}^{\infty}\left(\frac{q_\perp}{r_\beta}\right)^{2(m-1)} \times \sum_{n=-m}^{m} a(|n|, m-|n|)(m) \times\left[J(n,m,1;\ f_{\beta 0}) + iJ_\nu(n,m,0;\ f_{\beta 0})\right]$$

$$\chi_{zy} = i\frac{1}{z}\frac{v_*}{c}\sum_\beta \frac{1}{r_\beta}\frac{\omega_{p\beta}^2}{\Omega_*^2}\frac{1}{n_{\beta 0}}\sum_{m=1}^{\infty}\left(\frac{q_\perp}{r_\beta}\right)^{2(m-1)} \times \sum_{n=-m}^{m} a(|n|, m-|n|)(m)J(n,m,1;\ f_{\beta 0}),$$

$$\chi_{zz} = \frac{v_*^2}{c^2}\sum_\beta \frac{1}{r_\beta^2}\frac{\omega_{p\beta}^2}{\Omega_*^2}\frac{1}{n_{\beta 0}}\sum_{m=1}^{\infty}\left(\frac{q_\perp}{r_\beta}\right)^{2(m-1)} \times \sum_{n=-m}^{m} a(|n|, m-|n|) \times\left[J(n,m,2;\ f_{\beta 0}) + iJ_\nu(n,m,1;\ f_{\beta 0})\right],$$

$$e_{zz} = -\frac{1}{z^2}\sum_\beta \frac{\omega_{p\beta}^2}{\Omega_*^2}\frac{1}{n_{\beta 0}}\int \mathrm{d}^3u\,\frac{u_\parallel}{u_\perp}\mathcal{L}(f_{\beta 0}) + \frac{1}{z^2}\sum_\beta \frac{\omega_{p\beta}^2}{\Omega_*^2}\frac{1}{n_{\beta 0}}a(0,0) \times\left[J(0,0,2;\ f_{\beta 0}) + i\,J_\nu(0,0,1;\ f_{\beta 0})\right],$$

where we have defined the following integral expressions,

$$J(n,m,h;\ f_{\beta 0}) \equiv z\int \mathrm{d}^3u\,\frac{u_\parallel^h u_\perp^{2(m-1)} u_\perp L(f_{\beta 0})}{z - nr_\beta - q_\parallel u_\parallel + i\sum_j \tilde{\nu}_{\beta d}^{j0}}, \tag{8}$$

$$J_\nu(n,m,h;\ f_{\beta 0}) = \int \mathrm{d}^3u\,\frac{\left(\sum_j \tilde{\nu}_{\beta d}^{j0}\right) u_\parallel^h u_\perp^{2(m-1)} u_\perp \mathcal{L}(f_{\beta 0})}{z - nr_\beta - q_\parallel u_\parallel + i\sum_j \tilde{\nu}_{\beta d}^{j0}}, \tag{9}$$

and where we have utilized the following dimensionless variables

$$z = \frac{\omega}{\Omega_*},\quad q_\parallel = \frac{k_\parallel v_*}{\Omega_*},\quad q_\perp = \frac{k_\perp v_*}{\Omega_*},$$

$$r_\beta = \frac{\Omega_\beta}{\Omega_*},\quad \tilde{\nu}_{\beta d}^{j0}(u) = \frac{\nu_{\beta d}^{j0}(u)}{\Omega_*}.$$

The differential operators appearing in (8) and (9) are defined as follows,

$$L = \left[\left(1 - N_\parallel^* u_\parallel\right)\frac{\partial}{\partial u_\perp} + N_\parallel^* u_\perp \frac{\partial}{\partial u_\parallel}\right],$$

$$\mathcal{L} = u_\parallel \frac{\partial}{\partial u_\perp} - u_\perp \frac{\partial}{\partial u_\parallel},$$

Moreover, we have the following quantities,

$$\omega_{p\beta}^2 = \frac{4\pi n_{\beta 0} q_\beta^2}{m_\beta},\qquad \Omega_\beta = \frac{q_\beta B_0}{m_\beta c},$$

$$a(n,m) = \left(\frac{1}{2}\right)^{2(|n|+m)} \frac{(-1)^m[2(|n|+m)]!}{[(|n|+m)!]^2(2|n|+m)!m!},$$

$$b(n,m) = \begin{cases} a(1, m-2), & \text{for } n = 0, \\ \frac{1}{4}\left[a(n-1,m) + a(n+1,m-2) - 2\frac{|n|+m-1}{|n|+m}a(n,m-1)\right], & \text{for } n > 0, \end{cases}$$

with

$$\frac{1}{(-m)!} = 0, \qquad \text{for } m \geq 1,$$

and the explicit form for the equilibrium value of the inelastic collision frequency,

$$\nu_{\beta d}^{j0}(u) = \frac{\pi a_j^2 n_{d0}^j v_*}{u}\left(u^2 + \frac{2Z_{d0}^j e q_\beta}{a_j m_\beta v_*^2}\right) H\left(u^2 + \frac{2Z_{d0}^j e q_\beta}{a_j m_\beta v_*^2}\right). \tag{10}$$

The subscript $\beta = e, i$ identifies electrons and ions, respectively, $q_{d0} = -eZ_{d0}$ is the equilibrium charge of the dust particles, assumed to be negative due to the greater collisional rate of the electrons with the dust particles, compared with the ions, and H denotes the Heaviside function.

The quantities Ω_* and v_* are some characteristic frequency and velocity, respectively. For instance, v_* may be the light speed c, or the Alfvén speed v_A, or the ion sound speed c_s, depending on the application for which the formulation is utilized. The integral forms given by (8) and (9) depend on the quantity $\mathbf{u}$, which is the normalized momentum, defined as $\mathbf{u} = \mathbf{p}/(m_\alpha v_*)$.

The form of the "conventional" components of the dielectric tensor, as given by (7), is obtained after expansion of the Bessel functions which appear in components of the dielectric tensor, for magnetized plasmas. The expansion introduces the coefficients $a(n, m)$ and $b(n, m)$, and allows the ε_{ij}^C to be written in such a way that they depend on a double series, on harmonic and Larmor radius contributions. It is seen that the ε_{ij}^C also depend on a small number of integrals, which have to be evaluated depending on the equilibrium distribution function.

The evaluation of the "new" contribution also proceeds along steps similar to those of previous versions, modified whenever necessary. We arrive to the following form of the "new" contribution, in the form of a product of two terms, as follows,

$$\varepsilon_{ij}^N = \sum_k \mathcal{U}_i^k \mathcal{S}_j^k , \tag{11}$$

with

$$\mathcal{U}_x^k = \frac{1}{z}\frac{1}{z + i\left(\tilde{\nu}_{ch}^k + \tilde{\nu}_1^{kk}\right)} \sum_\beta \frac{\omega_{p\beta}^2}{\Omega_*^2}\frac{1}{n_{\beta 0}} \sum_{m=1}^{\infty}\sum_{n=-m}^{+m} \times \left(\frac{q_\perp}{r_\beta}\right)^{2m-1} n a(|n|, m-|n|) J_U(n, m, 0, 0;\ f_{\beta 0}, k) ,$$

$$\mathcal{U}_y^k = -i\frac{1}{z}\frac{1}{z + i\left(\tilde{\nu}_{ch}^k + \tilde{\nu}_1^{kk}\right)} \sum_\beta \frac{\omega_{p\beta}^2}{\Omega_*^2}\frac{1}{n_{\beta 0}} \sum_{m=1}^{\infty}\sum_{n=-m}^{+m} \times \left(\frac{q_\perp}{r_\beta}\right)^{2m-1} m a(|n|, m-|n|) J_U(n, m, 0, 0;\ f_{\beta 0}, k) ,$$

$$\mathcal{U}_z^k = \frac{1}{z}\frac{1}{z + i\left(\tilde{\nu}_{ch}^k + \tilde{\nu}_1^{kk}\right)} \sum_\beta \frac{\omega_{p\beta}^2}{\Omega_*^2}\frac{1}{n_{\beta 0}} \sum_{m=0}^{\infty}\sum_{n=-m}^{+m} \left(\frac{q_\perp}{r_\beta}\right)^{2m} \times a(|n|, m-|n|) J_U(n, m, 1, 0;\ f_{\beta 0}, k) ,$$

$$\mathcal{S}_x^k = -\frac{a_k\Omega_*}{2v_*}\frac{1}{z}\sum_\ell B_{k\ell}\frac{n_{d0}^k}{n_{d0}^\ell}\sum_\beta \frac{\omega_{p\beta}^2}{\Omega_*^2}\frac{1}{n_{\beta 0}} \sum_{m=1}^{\infty}\sum_{n=-m}^{+m} \times \left(\frac{q_\perp}{r_\beta}\right)^{2m-1} n a(|n|, m-|n|) J_{\nu L}(n, m, 0;\ f_{\beta 0}, \ell) ,$$

$$\mathcal{S}_y^k = -i\frac{a_k\Omega_*}{2v_*}\frac{1}{z}\sum_\ell B_{k\ell}\frac{n_{d0}^k}{n_{d0}^\ell}\sum_\beta \frac{\omega_{p\beta}^2}{\Omega_*^2}\frac{1}{n_{\beta 0}} \sum_{m=1}^{\infty}\sum_{n=-m}^{+m} \times \left(\frac{q_\perp}{r_\beta}\right)^{2m-1} m a(|n|, m-|n|) J_{\nu L}(n, m, 0;\ f_{\beta 0}, \ell) ,$$

$$\mathcal{S}_z^k = -\frac{a_k\Omega_*}{2v_*}\frac{1}{z}\sum_\ell B_{k\ell}\frac{n_{d0}^k}{n_{d0}^\ell} \times \left[\sum_\beta \frac{\omega_{p\beta}^2}{\Omega_*^2}\frac{1}{n_{\beta 0}} \sum_{m=0}^{\infty}\sum_{n=-m}^{+m} \left(\frac{q_\perp}{r_\beta}\right)^{2m} a(|n|, m-|n|) \times \left[J_{\nu L}(n, m, 1;\ f_{\beta 0}, \ell) + i J_{\nu\nu}(n, m;\ f_{\beta 0}, \ell)\right] - \sum_\beta \frac{\omega_{p\beta}^2}{\Omega_*^2}\frac{1}{n_{\beta 0}} J_{\nu 0}(f_{\beta 0}, \ell)\right] ,$$

$$\tilde{\nu}_{ch}^k = \frac{a_k\Omega_*}{2v_*}\sum_\beta \frac{\omega_{p\beta}^2}{\Omega_*^2}\frac{1}{n_{\beta 0}} J_{ch}(f_{\beta 0}, k) ,$$

where

$$J_U(n, m, h, l;\ f_{\beta 0}, k) = z\int \mathrm{d}^3u \left(\frac{\tilde{\nu}_{\beta d}^{k0}}{z}\right)^l \times \frac{f_{\beta 0}}{z - nr_\beta - q_\parallel u_\parallel + i\sum_j \tilde{\nu}_{\beta d}^{j0}} \frac{u_\parallel^h u_\perp^{2m}}{u} H\left(u^2 + \frac{2Z_{d0}^k e q_\beta}{a m_\beta v_*^2}\right) , \tag{12}$$

$$J_{\nu L}(n, m, h;\ f_{\beta 0}, k) = z\int \mathrm{d}^3u \left(\frac{\tilde{\nu}_{\beta d}^{k0}}{z}\right) \times \frac{u_\parallel^h u_\perp^{2m-1} L(f_{\beta 0})}{z - nr_\beta - q_\parallel u_\parallel + i\sum_j \tilde{\nu}_{\beta d}^{j0}} , \tag{13}$$

$$J_{\nu\nu}(n, m;\ f_{\beta 0}, k) = z\int \mathrm{d}^3u \left(\frac{\tilde{\nu}_{\beta d}^{k0}}{z}\right)\left(\sum_q \frac{\tilde{\nu}_{\beta d}^{q0}}{z}\right) \times \frac{u_\perp^{2m-1} \mathcal{L}(f_{\beta 0})}{z - nr_\beta - q_\parallel u_\parallel + i\sum_j \tilde{\nu}_{\beta d}^{j0}} , \tag{14}$$

$$J_{\nu 0}(f_{\beta 0}, k) = \int \mathrm{d}^3u \left(\frac{\tilde{\nu}_{\beta d}^{k0}}{z}\right) \frac{\mathcal{L}(f_{\beta 0})}{u_\perp} , \tag{15}$$

$$J_{ch}(f_{\beta 0}, k) = \int \mathrm{d}^3u\ f_{\beta 0}\frac{1}{u} H\left(u^2 + \frac{2Z_{d0}^k e q_\beta}{a_k m_\beta v_*^2}\right) , \tag{16}$$

and where

$$\tilde{\nu}_1^{k\ell} = -i\,\frac{a_\ell \Omega_*}{2v_*}\sum_\beta \frac{n_{d0}^\ell}{n_{d0}^k}\frac{\omega_{p\beta}^2}{\Omega_*^2}\frac{1}{n_{\beta 0}}\sum_{m=0}^{\infty}\sum_{n=-m}^{+m}\left(\frac{q_\perp}{r_\beta}\right)^{2m}$$
$$\times\, a(|n|, m-|n|) J_U(n, m, 0, 1;\; f_{\beta 0}, \ell)\,.$$

In the expressions for the components S_i^k, we find the following quantity which is associated to the derivation in the case of several dust populations,

$$B_{j\ell} = \Bigg[\delta_{j\ell} + C_{j\ell} + \sum_{\ell_2} C_{j\ell_2} C_{\ell_2 \ell}$$
$$+ \sum_{\ell_2}\sum_{\ell_3} C_{j\ell_3} C_{\ell_3\ell_2} C_{\ell_2\ell} + \ldots\Bigg],$$

with $C_{\ell_1\ell_2} = -i(1-\delta_{\ell_1\ell_2})\nu_1^{\ell_1\ell_2}/(\omega + i\nu_{ch}^{\ell_2} + i\nu_1^{\ell_2\ell_2})$. In the case of a single dust population, all $C_{j\ell}$ vanish, and the S_i will correspond to those appearing in [45].

The formulation presented here, which is up to now valid for general forms of the distribution functions of ions and electrons, provided that they feature azimuthal symmetry, and which can be applied to situations in which dust particles of different sizes are present in the plasma, is a generalization of previous formulations developed for the case of a single population of dust particles. We have therefore only briefly sketched the steps of the derivation, for the sake of economy of space. More details about the derivation can be found in [45].

3 Evaluation of the Integrals Which Appear in the Expressions for the Components of the Dielectric Tensor

For further development, we assume that ions and electrons are represented by anisotropic kappa distributions [25, 55, 56],

$$f_{\beta,\kappa}(\mathbf{v}) = n_{\beta 0} A \left(1 + \frac{v_\parallel^2}{\kappa_\parallel v_{\beta\parallel}^2}\right)^{-\kappa_\parallel}\left(1 + \frac{v_\perp^2}{\kappa_\perp v_{\beta\perp}^2}\right)^{-\kappa_\perp}, \tag{17}$$

where

$$v_{\beta\parallel}^2 = \frac{2T_{\beta\parallel}}{m_\beta}\,, \qquad v_{\beta\perp}^2 = \frac{2T_{\beta\perp}}{m_\beta}\,.$$

Assuming that the distribution is normalized to the unity, and using the integral

$$\int_0^\infty \mathrm{d}t\, \frac{t^{z-1}}{(1+t)^{w+z}} = \frac{\Gamma(z)\Gamma(w)}{\Gamma(w+z)}\,, \quad (\Re z > 0\,,\ \Re w > 0)\,, \tag{18}$$

the normalization constant A can be easily determined, and the distribution function can be written as follows, using dimensionless variables,

$$f_{\beta,\kappa}(\mathbf{u}) = \frac{n_{\beta 0}}{\pi^{3/2}\kappa_\perp \kappa_\parallel^{1/2} u_{\beta\perp}^2 u_{\beta\parallel}}\frac{\Gamma(\kappa_\perp)\Gamma(\kappa_\parallel)}{\Gamma(\kappa_\perp - 1)\Gamma(\kappa_\parallel - 1/2)}$$
$$\times\left(1 + \frac{u_\parallel^2}{\kappa_\parallel u_{\beta\parallel}^2}\right)^{-\kappa_\parallel}\left(1 + \frac{u_\perp^2}{\kappa_\perp u_{\beta\perp}^2}\right)^{-\kappa_\perp}. \tag{19}$$

The differential operators which appear in the components of the dielectric tensor can therefore be easily evaluated,

$$L(f_{\beta,\kappa}) = -2u_\perp\Bigg[\frac{\kappa_\perp}{\kappa_\perp u_{\beta\perp}^2 + u_\perp^2} - \frac{q_\parallel}{z}u_\parallel$$
$$\times\left(\frac{\kappa_\perp}{\kappa_\perp u_{\beta\perp}^2 + u_\perp^2} - \frac{\kappa_\parallel}{\kappa_\parallel u_{\beta\parallel}^2 + u_\parallel^2}\right)\Bigg] f_{\beta,\kappa}\,, \tag{20}$$

$$\mathcal{L}(f_{\beta,\kappa}) = -2u_\parallel u_\perp\left(\frac{\kappa_\perp}{\kappa_\perp u_{\beta\perp}^2 + u_\perp^2} - \frac{\kappa_\parallel}{\kappa_\parallel u_{\beta\parallel}^2 + u_\parallel^2}\right) f_{\beta,\kappa}\,. \tag{21}$$

For the evaluation of the relevant integrals, we assume for simplicity that each velocity-dependent collision frequency can be replaced by its average value. This approximation is adopted in order to arrive at a relatively simple estimate of the effect of the charging of dust particles due to collisions with electrons and ions, effect frequently neglected in analysis of the dispersion relation for waves in dusty plasmas, and which, to our knowledge, has never been considered in the case of anisotropic kappa distributions.

$$\nu_\beta^k = \frac{1}{n_{\beta 0}}\int \mathrm{d}^3u\, \nu_{\beta d}^{k0}(u)\, f_{\beta 0}(u)\,. \tag{22}$$

Using the expression for the collision frequency, from (10), we proceed as follows,

$$\nu_\beta^k = \frac{1}{n_{\beta 0}} \int \mathrm{d}^3 u \, \frac{\pi a_k^2 n_{d0}^k v_*}{u} \left(u^2 + \frac{2 Z_{d0}^k e q_\beta}{a_k m_\beta v_*^2} \right) \times H\left(u^2 + \frac{2 Z_{d0}^k e q_\beta}{a_k m_\beta v_*^2} \right) \times \frac{n_{\beta 0}}{\pi^{3/2} \kappa_\perp \kappa_\parallel^{1/2} u_{\beta\perp}^2 u_{\beta\parallel}} \frac{\Gamma(\kappa_\perp)\Gamma(\kappa_\parallel)}{\Gamma(\kappa_\perp - 1)\Gamma(\kappa_\parallel - 1/2)} \times \left(1 + \frac{u_\parallel^2}{\kappa_\parallel u_{\beta\parallel}^2}\right)^{-\kappa_\parallel} \left(1 + \frac{u_\perp^2}{\kappa_\perp u_{\beta\perp}^2}\right)^{-\kappa_\perp} .$$

$$\nu_\beta^k = \frac{\left(a_k^2 n_{d0}^k v_*\right)(2\pi)}{\pi^{1/2} \kappa_\perp \kappa_\parallel^{1/2} u_{\beta\perp}^2 u_{\beta\parallel}} \frac{\Gamma(\kappa_\perp)\Gamma(\kappa_\parallel)}{\Gamma(\kappa_\perp - 1)\Gamma(\kappa_\parallel - 1/2)} \times \int_0^\pi \mathrm{d}\theta \, \sin\theta \int_{u_{lim}^\beta}^\infty \mathrm{d}u \, u \left(u^2 + \frac{2 Z_{d0}^k e q_\beta}{a_k m_\beta v_*^2} \right) \times \left(1 + \frac{u_\parallel^2}{\kappa_\parallel u_{\beta\parallel}^2}\right)^{-\kappa_\parallel} \left(1 + \frac{u_\perp^2}{\kappa_\perp u_{\beta\perp}^2}\right)^{-\kappa_\perp} ,$$

where

$$u_{lim}^e = \left(\frac{2 Z_{d0}^k \left(e^2/a_k\right)}{m_e v_*^2} \right)^{1/2} , \qquad u_{lim}^i = 0 .$$

Integrating over $\mu = \cos\theta$, and dividing by Ω_*, we obtain the following form for the normalized average collision frequencies,

$$\tilde{\nu}_\beta^k = \frac{\nu_\beta^k}{\Omega_*} = \frac{c^3}{\Omega_*^3} \frac{a_k^2 \Omega_*^2}{c^2} \frac{v_*}{c} (\epsilon_k n_{i0}) \times \frac{2(2\pi)}{\pi^{1/2} \kappa_\perp \kappa_\parallel^{1/2} u_{\beta\perp}^2 u_{\beta\parallel}} \frac{\Gamma(\kappa_\perp)\Gamma(\kappa_\parallel)}{\Gamma(\kappa_\perp - 1)\Gamma(\kappa_\parallel - 1/2)} \times \int_{u_{lim}^\beta}^\infty \mathrm{d}u \, u \left(u^2 + \frac{2 Z_{d0}^k e q_\beta}{a_k m_\beta v_*^2} \right) \left(\frac{\kappa_\perp u_{\beta\perp}^2}{u^2 + \kappa_\perp u_{\beta\perp}^2} \right)^{\kappa_\perp} \times F_1\left(\frac{1}{2}; \kappa_\parallel, \kappa_\perp; \frac{3}{2}; -\frac{u^2}{\kappa_\parallel u_{\beta\parallel}^2}, \frac{u^2}{u^2 + \kappa_\perp u_{\beta\perp}^2} \right), \quad (23)$$

where $F_1(\alpha; \beta, \beta'; \gamma; x, y)$ indicates the hypergeometric function of two variables.

3.1 Evaluation of the Integrals $J(n, m, h;\ f_{\beta 0})$, for $f_{\beta 0} = f_{\beta,\kappa}$

We assume, for simplicity, that the collision frequency is replaced by the average value,

$$J(n, m, h;\ f_{\beta 0}) = z(2\pi) \int_0^\infty \mathrm{d}u_\perp \, u_\perp^{2m} \int_{-\infty}^\infty \mathrm{d}u_\parallel \times \frac{u_\parallel^h L(f_{\beta 0})}{z - n r_\beta - q_\parallel u_\parallel + i \sum_j \tilde{\nu}_\beta^j} .$$

Using (20),

$$J(n, m, h;\ f_{\beta 0}) = -2z(2\pi) \int_0^\infty \mathrm{d}u_\perp \, u_\perp u_\perp^{2m} \int_{-\infty}^\infty \mathrm{d}u_\parallel \times \frac{u_\parallel^h}{z - n r_\beta - q_\parallel u_\parallel + i \sum_j \tilde{\nu}_\beta^j} \times \left[\frac{\kappa_\perp}{\kappa_\perp u_{\beta\perp}^2 + u_\perp^2} - \frac{q_\parallel}{z} u_\parallel \times \left(\frac{\kappa_\perp}{\kappa_\perp u_{\beta\perp}^2 + u_\perp^2} - \frac{\kappa_\parallel}{\kappa_\parallel u_{\beta\parallel}^2 + u_\parallel^2} \right) \right] f_{\beta,\kappa}$$

$$= \frac{2z}{q_\parallel} \frac{(2\pi) n_{\beta 0}}{\pi^{3/2} \kappa_\perp \kappa_\parallel^{1/2} u_{\beta\perp}^2 u_{\beta\parallel}} \times \frac{\Gamma(\kappa_\perp)\Gamma(\kappa_\parallel)}{\Gamma(\kappa_\perp - 1)\Gamma(\kappa_\parallel - 1/2)} \times \int_{-\infty}^\infty \mathrm{d}u_\parallel \, \frac{u_\parallel^h}{u_\parallel - u_{\parallel,res}} \left(1 + \frac{u_\parallel^2}{\kappa_\parallel u_{\beta\parallel}^2}\right)^{-\kappa_\parallel} \times \int_0^\infty \mathrm{d}u_\perp \, u_\perp^{2m+1} \left(1 + \frac{u_\perp^2}{\kappa_\perp u_{\beta\perp}^2}\right)^{-\kappa_\perp} \times \left[\left(\frac{1}{u_{\beta\perp}^2} - \frac{q_\parallel}{z} \frac{u_\parallel}{u_{\beta\perp}^2} \right) \left(1 + \frac{u_\perp^2}{\kappa_\perp u_{\beta\perp}^2}\right)^{-1} + \frac{q_\parallel}{z} \frac{u_\parallel}{u_{\beta\parallel}^2} \left(1 + \frac{u_\parallel^2}{\kappa_\parallel u_{\beta\parallel}^2}\right)^{-1} \right],$$

where

$$u_{\parallel,res} = \frac{z - nr_\beta + i\sum_j \tilde{\nu}_\beta^j}{q_\parallel}\,.$$

Changing variable in the integrals over $u_\perp$,

$$\begin{aligned}
J(n,m,h;\ f_{\beta 0}) = {} & \frac{2\omega}{v_* k_\parallel}\frac{(2\pi)n_{\beta 0}}{\pi^{3/2}\kappa_\perp\kappa_\parallel^{1/2}u_{\beta\perp}^2 u_{\beta\parallel}} \\
& \times\frac{\Gamma(\kappa_\perp)\Gamma(\kappa_\parallel)}{\Gamma(\kappa_\perp-1)\Gamma(\kappa_\parallel-1/2)} \\
& \times\int_{-\infty}^{\infty}\mathrm{d}u_\parallel\,\frac{u_\parallel^h}{u_\parallel-u_{\parallel,res}}\left(1+\frac{u_\parallel^2}{\kappa_\parallel u_{\beta\parallel}^2}\right)^{-\kappa_\parallel} \\
& \times\frac{\left(\kappa_\perp u_{\beta\perp}^2\right)^{m+1}}{2}\int_0^\infty \mathrm{d}t\, t^m\,(1+t)^{-\kappa_\perp} \\
& \times\left[\left(\frac{1}{u_{\beta\perp}^2}-\frac{q_\parallel}{z}\frac{u_\parallel}{u_{\beta\perp}^2}\right)(1+t)^{-1}\right. \\
& \left.+\frac{q_\parallel}{z}\frac{u_\parallel}{u_{\beta\parallel}^2}\left(1+\frac{u_\parallel^2}{\kappa_\parallel u_{\beta\parallel}^2}\right)^{-1}\right].
\end{aligned}$$

Using (18) to solve the t integrals, and introducing variable $s = u_\parallel/u_{\beta\parallel}$,

$$\begin{aligned}
J(n,m,h;\ f_{\beta 0}) = {} & \frac{z}{q_\parallel}\frac{(2\pi)n_{\beta 0}}{\pi^{3/2}\kappa_\parallel^{1/2}}(\kappa_\perp u_{\beta\perp}^2)^m (u_{\beta\parallel})^{h-1}(m!) \\
& \times\frac{\Gamma(\kappa_\perp-m-1)\Gamma(\kappa_\parallel)}{\Gamma(\kappa_\perp-1)\Gamma(\kappa_\parallel-1/2)} \\
& \times\int_{-\infty}^{\infty}\mathrm{d}s\,\frac{s^h}{s-\hat{\zeta}_\beta^n}\left(1+\frac{s^2}{\kappa_\parallel}\right)^{-\kappa_\parallel} \\
& \times\left[\left(\frac{1}{u_{\beta\perp}^2}-\frac{q_\parallel}{z}\frac{u_{\beta\parallel}}{u_{\beta\perp}^2}s\right)\frac{\kappa_\perp-m-1}{\kappa_\perp}\right. \\
& \left.+\frac{q_\parallel}{z}\frac{u_{\beta\parallel}}{u_{\beta\parallel}^2}s\left(1+\frac{s^2}{\kappa_\parallel}\right)^{-1}\right],
\end{aligned}$$

where

$$\hat{\zeta}_\beta^n = \frac{z - nr_\beta + i\sum_j \tilde{\nu}_\beta^j}{q_\parallel u_{\beta\parallel}}\,.$$

At this point, we introduce the plasma dispersion function of order m, for κ distributions,

$$Z_\kappa^{(m)}(\xi) = \frac{1}{\pi^{1/2}}\frac{\Gamma(\kappa)}{\kappa^{1/2}\Gamma(\kappa-1/2)}\int_{-\infty}^{\infty}\frac{\mathrm{d}s}{(s-\xi)(1+s^2/\kappa)^{\kappa+m}}\,, \tag{24}$$

which reduces to the distribution defined by Summers and Thorne [57, 58] in the case $m = 1$, and which can be written in terms of the Gauss hypergeometric function ${}_2F_1(a, b, c; z)$, as follows,

$$\begin{aligned}
Z_\kappa^{(m)}(\xi) = {} & \frac{i\Gamma(\kappa)\Gamma(\kappa+m+1/2)}{\kappa^{1/2}\Gamma(\kappa-1/2)\Gamma(\kappa+m+1)} \\
& \times {}_2F_1\left[1, 2\kappa+2m, \kappa+m+1; \frac{1}{2}\left(1+\frac{i\xi}{\kappa^{1/2}}\right)\right],
\end{aligned} \tag{25}$$

for $\kappa > -m - 1/2$. This function will be useful in the subsequent calculations.

We proceed by considering particular values of h, those which are useful for the evaluation of the components of the dielectric tensor. We start with the case of $h = 0$. Considering the integral $J(n, m, h;\ f_{\beta 0})$ in the case of $h = 0$, and by addition and subtraction of $\hat{\zeta}_\beta^n$ to the places where the variable s appears in the numerator, we obtain

$$\begin{aligned}
J(n,m,0;\ f_{\beta 0}) = {} & \frac{z}{q_\parallel}\frac{(2\pi)n_{\beta 0}}{\pi^{3/2}\kappa_\parallel^{1/2}}\left(\kappa_\perp u_{\beta\perp}^2\right)^m (u_{\beta\parallel})^{-1}(m!) \\
& \times\frac{\Gamma(\kappa_\perp-m-1)\Gamma(\kappa_\parallel)}{\Gamma(\kappa_\perp-1)\Gamma(\kappa_\parallel-1/2)} \\
& \times\int_{-\infty}^{\infty}\mathrm{d}s\,\frac{1}{s-\hat{\zeta}_\beta^n}\left(1+\frac{s^2}{\kappa_\parallel}\right)^{-\kappa_\parallel} \\
& \times\left[\left(\frac{1}{u_{\beta\perp}^2}-\frac{q_\parallel}{z}\frac{u_{\beta\parallel}}{u_{\beta\perp}^2}\left(s-\hat{\zeta}_\beta^n+\hat{\zeta}_\beta^n\right)\right)\right. \\
& \times\frac{\kappa_\perp-m-1}{\kappa_\perp}+\frac{q_\parallel}{z}\frac{u_{\beta\parallel}}{u_{\beta\parallel}^2}\left(s-\hat{\zeta}_\beta^n+\hat{\zeta}_\beta^n\right) \\
& \left.\times\left(1+\frac{s^2}{\kappa_\parallel}\right)^{-1}\right],
\end{aligned}$$

which leads to the following form,

$$
\begin{aligned}
&J(n,m,0;\ f_{\beta 0})\\
&=\frac{z}{q_\parallel}\frac{(2\pi)n_{\beta 0}}{\pi^{3/2}\kappa_\parallel^{1/2}}(\kappa_\perp u_{\beta\perp}^2)^m(u_{\beta\parallel})^{-1}(m!)\\
&\times\frac{\Gamma(\kappa_\perp-m-1)\Gamma(\kappa_\parallel)}{\Gamma(\kappa_\perp-1)\Gamma(\kappa_\parallel-1/2)}\\
&\times\left\{2\int_0^\infty \mathrm{d}s\left(1+\frac{s^2}{\kappa_\parallel}\right)^{-\kappa_\parallel}\right.\\
&\quad\times\left[-\frac{q_\parallel}{z}\frac{u_{\beta\parallel}}{u_{\beta\perp}^2}\frac{\kappa_\perp-m-1}{\kappa_\perp}+\frac{q_\parallel}{z}\frac{u_{\beta\parallel}}{u_{\beta\parallel}^2}\left(1+\frac{s^2}{\kappa_\parallel}\right)^{-1}\right]\\
&\quad+\int_{-\infty}^{\infty}\mathrm{d}s\,\frac{1}{s-\hat{\zeta}_\beta^n}\left(1+\frac{s^2}{\kappa_\parallel}\right)^{-\kappa_\parallel}\\
&\quad\times\left[\left(\frac{1}{u_{\beta\perp}^2}-\frac{q_\parallel}{z}\frac{u_{\beta\parallel}}{u_{\beta\perp}^2}(\hat{\zeta}_\beta^n)\right)\frac{\kappa_\perp-m-1}{\kappa_\perp}\right.\\
&\quad\quad\left.\left.+\frac{q_\parallel}{z}\frac{u_{\beta\parallel}}{u_{\beta\parallel}^2}(\hat{\zeta}_\beta^n)\left(1+\frac{s^2}{\kappa_\parallel}\right)^{-1}\right]\right\},
\end{aligned}
$$

The integrals without the denominator $s-\hat{\zeta}_\beta^n$ can be evaluated using (18), and the integrals with this denominator can be written using (25), leading to

$$
\begin{aligned}
&J(n,m,0;\ f_{\beta 0})\\
&=\frac{z}{q_\parallel}\frac{(2\pi)n_{\beta 0}}{\pi^{3/2}}\left(\kappa_\perp u_{\beta\perp}^2\right)^m(u_{\beta\parallel})^{-1}(m!)\\
&\times\frac{\Gamma(\kappa_\perp-m-1)\Gamma(\kappa_\parallel)}{\Gamma(\kappa_\perp-1)\Gamma(\kappa_\parallel-1/2)}\\
&\times\left\{-\frac{q_\parallel}{z}\frac{u_{\beta\parallel}}{u_{\beta\perp}^2}\frac{\kappa_\perp-m-1}{\kappa_\perp}\frac{\Gamma(1/2)\Gamma(\kappa_\parallel-1/2)}{\Gamma(\kappa_\parallel)}\right.\\
&\quad+\frac{q_\parallel}{z}\frac{u_{\beta\parallel}}{u_{\beta\parallel}^2}\frac{\Gamma(1/2)\Gamma(\kappa_\parallel+1/2)}{\Gamma(\kappa_\parallel+1)}\\
&\quad+\left[\frac{1}{u_{\beta\perp}^2}-\frac{q_\parallel}{z}\frac{u_{\beta\parallel}}{u_{\beta\perp}^2}(\hat{\zeta}_\beta^n)\right]\frac{\kappa_\perp-m-1}{\kappa_\perp}\pi^{1/2}\\
&\quad\times\frac{\Gamma(\kappa_\parallel-1/2)}{\Gamma(\kappa_\parallel)}Z_{\kappa_\parallel}^{(0)}(\hat{\zeta}_\beta^n)+\frac{q_\parallel}{z}\frac{u_{\beta\parallel}}{u_{\beta\parallel}^2}(\hat{\zeta}_\beta^n)\pi^{1/2}\\
&\quad\left.\times\frac{\Gamma(\kappa_\parallel-1/2)}{\Gamma(\kappa_\parallel)}Z_{\kappa_\parallel}^{(1)}(\hat{\zeta}_\beta^n)\right\},
\end{aligned}
$$

an expression which can be simplified by some algebraic manipulation.

Proceeding with the algebraic simplification, and using the same approach to the cases of $h=1$ and $h=2$, we obtain the following expressions for the integrals $J(n,m,h;\ f_{\beta 0})$ which are necessary to the components of ε_{ij}^C,

$$
\begin{aligned}
J(n,m,0;\ f_{\beta 0})&=(2\kappa_\perp)n_{\beta 0}(\kappa_\perp u_{\beta\perp}^2)^{m-1}(u_{\beta\parallel})^0(m!)\\
&\times\frac{\Gamma(\kappa_\perp-m-1)}{\Gamma(\kappa_\perp-1)}\\
&\times\left[-\frac{\kappa_\perp-m-1}{\kappa_\perp}+\frac{u_{\beta\perp}^2}{u_{\beta\parallel}^2}\frac{\kappa_\parallel-1/2}{\kappa_\parallel}\right.\\
&\quad+\left(\zeta_\beta^0-\hat{\zeta}_\beta^n\right)\frac{\kappa_\perp-m-1}{\kappa_\perp}Z_{\kappa_\parallel}^{(0)}(\hat{\zeta}_\beta^n)\\
&\quad\left.+\frac{u_{\beta\perp}^2}{u_{\beta\parallel}^2}\hat{\zeta}_\beta^n Z_{\kappa_\parallel}^{(1)}(\hat{\zeta}_\beta^n)\right],
\end{aligned}
$$

$$
\begin{aligned}
J(n,m,1;\ f_{\beta 0})&=(2\kappa_\perp)n_{\beta 0}(\kappa_\perp u_{\beta\perp}^2)^{m-1}(u_{\beta\parallel})^1(m!)\\
&\times\frac{\Gamma(\kappa_\perp-m-1)}{\Gamma(\kappa_\perp-1)}\\
&\times\left[\zeta_\beta^0\frac{\kappa_\perp-m-1}{\kappa_\perp}+\hat{\zeta}_\beta^n\right.\\
&\quad\times\left(-\frac{\kappa_\perp-m-1}{\kappa_\perp}+\frac{u_{\beta\perp}^2}{u_{\beta\parallel}^2}\frac{\kappa_\parallel-1/2}{\kappa_\parallel}\right)\\
&\quad+\hat{\zeta}_\beta^n\left(\zeta_\beta^0-\hat{\zeta}_\beta^n\right)\frac{\kappa_\perp-m-1}{\kappa_\perp}Z_{\kappa_\parallel}^{(0)}(\hat{\zeta}_\beta^n)\\
&\quad\left.+\frac{u_{\beta\perp}^2}{u_{\beta\parallel}^2}(\hat{\zeta}_\beta^n)^2 Z_{\kappa_\parallel}^{(1)}(\hat{\zeta}_\beta^n)\right],
\end{aligned}
\tag{26}
$$

$$
\begin{aligned}
J(n,m,2;\ f_{\beta 0})&=(2\kappa_\perp)n_{\beta 0}(\kappa_\perp u_{\beta\perp}^2)^{m-1}(u_{\beta\parallel})^2(m!)\\
&\times\frac{\Gamma(\kappa_\perp-m-1)}{\Gamma(\kappa_\perp-1)}\\
&\times\left[\frac{1}{2}\frac{\kappa_\parallel}{\kappa_\parallel-3/2}\right.\\
&\quad\times\left(-\frac{\kappa_\perp-m-1}{\kappa_\perp}+\frac{u_{\beta\perp}^2}{u_{\beta\parallel}^2}\frac{\kappa_\parallel-3/2}{\kappa_\parallel}\right)\\
&\quad+\hat{\zeta}_\beta^n\zeta_\beta^0\frac{\kappa_\perp-m-1}{\kappa_\perp}+(\hat{\zeta}_\beta^n)^2\\
&\quad\times\left(-\frac{\kappa_\perp-m-1}{\kappa_\perp}+\frac{u_{\beta\perp}^2}{u_{\beta\parallel}^2}\frac{\kappa_\parallel-1/2}{\kappa_\parallel}\right)\\
&\quad+(\hat{\zeta}_\beta^n)^2\left(\zeta_\beta^0-\hat{\zeta}_\beta^n\right)\frac{\kappa_\perp-m-1}{\kappa_\perp}Z_{\kappa_\parallel}^{(0)}(\hat{\zeta}_\beta^n)\\
&\quad\left.+\frac{u_{\beta\perp}^2}{u_{\beta\parallel}^2}(\hat{\zeta}_\beta^n)^3 Z_{\kappa_\parallel}^{(1)}(\hat{\zeta}_\beta^n)\right],
\end{aligned}
$$

where $\zeta_\beta^0=z/(q_\parallel u_{\beta\parallel})$.

It is interesting at this point to consider the limiting case $\kappa_\perp \to \infty, \kappa_\parallel \to \infty$. Using $\lim_{\kappa\to\infty} Z_\kappa^{(m)}(\zeta) = Z(\zeta)$, and the Stirling formula,

$$\Gamma(az+b) \sim \sqrt{2\pi} e^{-az} (az)^{az+b-1/2}, \quad (|\arg z| < \pi, a > 0),$$

$$\lim_{\kappa_\perp\to\infty,\kappa_\parallel\to\infty} J(n,m,0;\ f_{\beta 0}) = (2)n_{\beta 0}(u_{\beta\perp}^2)^{m-1}(m!) \times\left[-1+\frac{T_{\beta\perp}}{T_{\beta\parallel}}+\left(\zeta_\beta^0-\hat{\zeta}_\beta^n\right)Z\left(\hat{\zeta}_\beta^n\right)+\frac{T_{\beta\perp}}{T_{\beta\parallel}}\hat{\zeta}_\beta^n Z\left(\hat{\zeta}_\beta^n\right)\right].$$

Proceeding in the same way with the expressions obtained for $h=1$ and $h=2$, and considering the isotropic limit, $T_{\beta\parallel} = T_{\beta\perp}$, we obtain the following expressions,

$$\lim_{\kappa_\perp\to\infty,\kappa_\parallel\to\infty} J(n,m,0;\ f_{\beta 0}) = (2m!)n_{\beta 0}(u_{\beta\perp}^2)^{m-1}(u_{\beta\parallel})^0\zeta_\beta^0 Z\left(\hat{\zeta}_\beta^n\right),$$

$$\lim_{\kappa_\perp\to\infty,\kappa_\parallel\to\infty} J(n,m,1;\ f_{\beta 0}) = (2m!)n_{\beta 0}(u_{\beta\perp}^2)^{m-1}(u_{\beta\parallel})^1\zeta_\beta^0\left[1+\hat{\zeta}_\beta^n Z(\hat{\zeta}_\beta^n)\right],$$

$$\lim_{\kappa_\perp\to\infty,\kappa_\parallel\to\infty} J(n,m,2;\ f_{\beta 0}) = (2m!)n_{\beta 0}(u_{\beta\perp}^2)^{m-1}(u_{\beta\parallel})^2\zeta_\beta^0\hat{\zeta}_\beta^n\left[1+\hat{\zeta}_\beta^n Z(\hat{\zeta}_\beta^n)\right].$$

Taking into account the difference of $\sqrt{2}$ in the definition of the thermal velocities, these are the same as equations (C2) in [45], obtained for an isotropic Maxwellian case and for a single dust population, indicating that equations (26) have the expected limiting case.

3.2 Evaluation of the Integrals $J_\nu(n,m,h;\ f_{\beta 0})$, for $f_{\beta 0} = f_{\beta,\kappa}$

From (9), using the average value of the collision frequency,

$$J_\nu(n,m,h;\ f_{\beta 0}) = (2\pi)\left(\sum_j \tilde{\nu}_\beta^j\right)\int_0^\infty du_\perp\, u_\perp^{2m}\int_{-\infty}^\infty du_\parallel \times\frac{u_\parallel^h \mathcal{L}(f_{\beta 0})}{z-nr_\beta-q_\parallel u_\parallel+i\sum_j\tilde{\nu}_\beta^j}.$$

Using the differential operator given by (21),

$$J_\nu(n,m,h;\ f_{\beta 0}) = -2(2\pi)\left(\sum_j \tilde{\nu}_\beta^j\right)\int_0^\infty du_\perp\, u_\perp^{2m+1} \times\int_{-\infty}^\infty du_\parallel\frac{u_\parallel^{h+1}}{z-nr_\beta-q_\parallel u_\parallel+i\sum_j\tilde{\nu}_\beta^j} \times\left[\frac{1}{u_{\beta\perp}^2}\left(1+\frac{u_\perp^2}{\kappa_\perp u_{\beta\perp}^2}\right)^{-1} -\frac{1}{u_{\beta\parallel}^2}\left(1+\frac{u_\parallel^2}{\kappa_\parallel u_{\beta\parallel}^2}\right)^{-1}\right] f_{\beta,\kappa}\,.$$

Using the explicit form of the distribution function and proceeding with the integration as in the case of integral $J(n,m,h;\ f_{\beta 0})$, we are lead to

$$J_\nu(n,m,h;\ f_{\beta 0}) = \frac{2}{\pi^{1/2}}n_{\beta 0}\frac{\sum_j\tilde{\nu}_\beta^j}{q_\parallel}\frac{(u_{\beta\parallel})^h(\kappa_\perp u_{\beta\perp}^2)^m}{\kappa_\parallel^{1/2}}(m!) \times\frac{\Gamma(\kappa_\parallel)}{\Gamma(\kappa_\parallel-1/2)}\frac{\Gamma(\kappa_\perp-m-1)}{\Gamma(\kappa_\perp-1)} \times\int_{-\infty}^\infty ds\,\frac{s^{h+1}}{s-\hat{\zeta}_\beta^n}\left[\frac{1}{u_{\beta\perp}^2}\frac{\kappa_\perp-m-1}{\kappa_\perp}-\frac{1}{u_{\beta\parallel}^2}\left(1+\frac{s^2}{\kappa_\parallel}\right)^{-1}\right] \times\left(1+\frac{s^2}{\kappa_\parallel}\right)^{-\kappa_\parallel}. \tag{27}$$

We proceed by considering particular values of h. For $h=0$, $h=1$ and $h=2$, we obtain

$$J_\nu(n,m,0;\ f_{\beta 0}) = (2\kappa_\perp)n_{\beta 0}\frac{\sum_j\tilde{\nu}_\beta^j}{q_\parallel}(u_{\beta\parallel})^0(\kappa_\perp u_{\beta\perp}^2)^{m-1}(m!) \times\frac{\Gamma(\kappa_\perp-m-1)}{\Gamma(\kappa_\perp-1)} \times\left\{\left[\frac{\kappa_\perp-m-1}{\kappa_\perp}-\frac{u_{\beta\perp}^2}{u_{\beta\parallel}^2}\frac{\kappa_\parallel-1/2}{\kappa_\parallel}\right] +\hat{\zeta}_\beta^n\left[\frac{\kappa_\perp-m-1}{\kappa_\perp}Z_{\kappa_\parallel}^{(0)}(\hat{\zeta}_\beta^n)-\frac{u_{\beta\perp}^2}{u_{\beta\parallel}^2}Z_{\kappa_\parallel}^{(1)}(\hat{\zeta}_\beta^n)\right]\right\},$$

$$J_v(n,m,1;\ f_{\beta0})$$
$$= (2\kappa_\perp) n_{\beta0} \frac{\sum_j \tilde{\nu}_\beta^j}{q_\parallel} (u_{\beta\parallel})^1 (\kappa_\perp u_{\beta\perp}^2)^{m-1} (m!) \times \frac{\Gamma(\kappa_\perp - m - 1)}{\Gamma(\kappa_\perp - 1)} \hat{\zeta}_\beta^n \times \left\{ \left[\frac{\kappa_\perp - m - 1}{\kappa_\perp} - \frac{u_{\beta\perp}^2}{u_{\beta\parallel}^2} \frac{\kappa_\parallel - 1/2}{\kappa_\parallel} \right] + \hat{\zeta}_\beta^n \left[\frac{\kappa_\perp - m - 1}{\kappa_\perp} Z_{\kappa_\parallel}^{(0)}(\hat{\zeta}_\beta^n) - \frac{u_{\beta\perp}^2}{u_{\beta\parallel}^2} Z_{\kappa_\parallel}^{(1)}(\hat{\zeta}_\beta^n) \right] \right\},$$

$$J_v(n,m,2;\ f_{\beta0})$$
$$= (2\kappa_\perp) n_{\beta0} \frac{\sum_j \tilde{\nu}_\beta^j}{q_\parallel} (u_{\beta\parallel})^2 (\kappa_\perp u_{\beta\perp}^2)^{m-1} (m!) \frac{\Gamma(\kappa_\perp - m - 1)}{\Gamma(\kappa_\perp - 1)} \times \left\{ \frac{1}{2} \frac{\kappa_\parallel}{\kappa_\parallel - 3/2} \left[\frac{\kappa_\perp - m - 1}{\kappa_\perp} - \frac{u_{\beta\perp}^2}{u_{\beta\parallel}^2} \frac{\kappa_\parallel - 3/2}{\kappa_\parallel} \right] + (\hat{\zeta}_\beta^n)^2 \times \left[\frac{\kappa_\perp - m - 1}{\kappa_\perp} - \frac{u_{\beta\perp}^2}{u_{\beta\parallel}^2} \frac{\kappa_\parallel - 1/2}{\kappa_\parallel} \right] + (\hat{\zeta}_\beta^n)^3 \times \left[\frac{\kappa_\perp - m - 1}{\kappa_\perp} Z_{\kappa_\parallel}^{(0)}(\hat{\zeta}_\beta^n) - \frac{u_{\beta\perp}^2}{u_{\beta\parallel}^2} Z_{\kappa_\parallel}^{(1)}(\hat{\zeta}_\beta^n) \right] \right\}. \quad (28)$$

In the limit $\kappa_\parallel \to \infty, \kappa_\perp \to \infty$,

$$J_v(n,m,0;\ f_{\beta0}) = (2m!) n_{\beta0} \frac{\sum_j \tilde{\nu}_\beta^j}{q_\parallel} (u_{\beta\parallel})^0 (u_{\beta\perp}^2)^{m-1} \times \left\{ \left[1 - \frac{u_{\beta\perp}^2}{u_{\beta\parallel}^2} \right] + \hat{\zeta}_\beta^n \left[Z(\hat{\zeta}_\beta^n) - \frac{u_{\beta\perp}^2}{u_{\beta\parallel}^2} Z(\hat{\zeta}_\beta^n) \right] \right\},$$

$$J_v(n,m,1;\ f_{\beta0}) = (2m!) n_{\beta0} \frac{\sum_j \tilde{\nu}_\beta^j}{q_\parallel} (u_{\beta\parallel})^1 (u_{\beta\perp}^2)^{m-1} \hat{\zeta}_\beta^n \times \left\{ \left[1 - \frac{u_{\beta\perp}^2}{u_{\beta\parallel}^2} \right] + \hat{\zeta}_\beta^n \left[Z(\hat{\zeta}_\beta^n) - \frac{u_{\beta\perp}^2}{u_{\beta\parallel}^2} Z(\hat{\zeta}_\beta^n) \right] \right\},$$

$$J_v(n,m,2;\ f_{\beta0}) = (2m!) n_{\beta0} \frac{\sum_j \tilde{\nu}_\beta^j}{q_\parallel} (u_{\beta\parallel})^2 (u_{\beta\perp}^2)^{m-1} \times \left\{ \frac{1}{2} \left[1 - \frac{u_{\beta\perp}^2}{u_{\beta\parallel}^2} \right] + (\hat{\zeta}_\beta^n)^2 \left[1 - \frac{u_{\beta\perp}^2}{u_{\beta\parallel}^2} \right] + (\hat{\zeta}_\beta^n)^3 \left[Z(\hat{\zeta}_\beta^n) - \frac{u_{\beta\perp}^2}{u_{\beta\parallel}^2} Z(\hat{\zeta}_\beta^n) \right] \right\}.$$

It is easy to see that these quantities vanish for the isotropic situation, confirming what has been obtained for isotropic distributions in [45]. That means that (28) have the expected limit.

3.3 Evaluation of a Integral Appearing in the Expression for $e_{zz}(f_{\beta0})$, for $f_{\beta0} = f_{\beta,\kappa}$

From the equations defined with (7), it is seen that e_{zz} features three terms. Two of these terms can be evaluated with the use of J and J_v, which have already been obtained. Regarding the other term, for a kappa distribution it can be evaluated with the use of the differential operator (21),

$$\frac{2}{z^2} \sum_\beta \frac{\omega_{p\beta}^2}{\Omega_*^2} \frac{1}{n_{\beta0}} \int d^3u\, u_\parallel^2 \times \left[\frac{1}{u_{\beta\perp}^2} \left(1 + \frac{u_\perp^2}{\kappa_\perp u_{\beta\perp}^2} \right)^{-1} - \frac{1}{u_{\beta\parallel}^2} \left(1 + \frac{u_\parallel^2}{\kappa_\parallel u_{\beta\parallel}^2} \right)^{-1} \right] f_{\beta,\kappa}\,.$$

Using the explicit form of the distribution function, and proceeding as in the previous sub-sections, the integral is readily performed and the quantity e_{zz} becomes as follows,

$$e_{zz} = \frac{1}{z^2} \sum_\beta \frac{\omega_{p\beta}^2}{\Omega_*^2} \frac{\kappa_\parallel}{\kappa_\parallel - 3/2} u_{\beta\parallel}^2 \left[\frac{1}{u_{\beta\perp}^2} \frac{\kappa_\perp - 1}{\kappa_\perp} - \frac{1}{u_{\beta\parallel}^2} \frac{\kappa_\parallel - 3/2}{\kappa_\parallel} \right] + \frac{1}{z^2} \sum_\beta \frac{\omega_{p\beta}^2}{\Omega_*^2} \frac{1}{n_{\beta0}} \Big[J(0,0,2;\ f_{\beta0}) + i\, J_v(0,0,1;\ f_{\beta0}) \Big]. \quad (29)$$

For $\kappa_\parallel \to \infty$ and $\kappa_\perp \to \infty$, and isotropic situation, this expression corresponds to equation (A9) of [45], also in the isotropic limit.

3.4 Evaluation of the Integrals $J_U(n,m,h,l;\ f_{\beta0},k)$ for $f_{\beta0} = f_{\beta,\kappa}$

From (12), with the collision frequency replaced by the average value,

$$J_U(n,m,h,l;\ f_{\beta0},k) = z \int d^3u \left(\frac{\tilde{\nu}_\beta^k}{z} \right)^l \times \frac{f_{\beta0}}{z - nr_\beta - q_\parallel u_\parallel + i \sum_j \tilde{\nu}_\beta^j} \frac{u_\parallel^h u_\perp^{2m}}{u} H\left(u^2 + \frac{2 Z_{d0}^k e q_\beta}{a_k m_\beta v_*^2} \right).$$

Let us further approximate, by neglecting the effect of the Heaviside function in the numerator of the integrand. This is fact an approximation only in the case of electrons, being exact in the case of ions. Furthermore, we write $u \simeq u_\perp$, in the denominator,

$$J_U(n, m, h, l;\ f_{\beta 0}, k) \simeq \left(\frac{\tilde{\nu}_\beta^k}{z}\right)^l z\,(2\pi) \int_0^\infty \mathrm{d}u_\perp\, u_\perp\, u_\perp^{2m-1} \times \int_{-\infty}^\infty \mathrm{d}u_\parallel \frac{u_\parallel^h\, f_{\beta 0}}{z - nr_\beta - q_\parallel u_\parallel + i \sum_j \tilde{\nu}_\beta^j}\,.$$

Considering now the explicit form of the anisotropic kappa distribution, the integrals can be performed as in the previous sub-sections, and we obtain the following approximated form for the J_U integral,

$$J_U(n, m, h, l;\ f_{\beta 0}, k) \simeq -\frac{z}{q_\parallel} \frac{n_{\beta 0}\, (\kappa_\perp u_{\beta\perp}^2)^{m-1/2} u_{\beta\parallel}^{h-1}}{\pi^{1/2} \kappa_\parallel^{1/2}} \times \frac{\Gamma(m+1/2)\Gamma(\kappa_\perp - m - 1/2)\Gamma(\kappa_\parallel)}{\Gamma(\kappa_\perp - 1)\Gamma(\kappa_\parallel - 1/2)} \times \left(\frac{\tilde{\nu}_\beta^k}{z}\right)^l \int_{-\infty}^\infty \mathrm{d}s\, \frac{s^h}{s - \hat{\zeta}_\beta^n} \left(1 + \frac{s^2}{\kappa_\parallel}\right)^{-\kappa_\parallel}\,. \tag{30}$$

Considering the particular case of $h = 0$, $h = 1$, and $h = 2$,

$$J_U(n, m, 0, l;\ f_{\beta 0}, k) \simeq -n_{\beta 0}\, (\kappa_\perp u_{\beta\perp}^2)^{m-1/2} u_{\beta\parallel}^0 \frac{\Gamma(m+1/2)\Gamma(\kappa_\perp - m - 1/2)}{\Gamma(\kappa_\perp - 1)} \times \left(\frac{\tilde{\nu}_\beta^k}{z}\right)^l \zeta_\beta^0 Z_{\kappa_\parallel}^{(0)}(\hat{\zeta}_\beta^n)\,,$$

$$J_U(n, m, 1, l;\ f_{\beta 0}, k) \simeq -n_{\beta 0}\, (\kappa_\perp u_{\beta\perp}^2)^{m-1/2} u_{\beta\parallel}^1 \frac{\Gamma(m+1/2)\Gamma(\kappa_\perp - m - 1/2)}{\Gamma(\kappa_\perp - 1)} \times \left(\frac{\tilde{\nu}_\beta^k}{z}\right)^l \zeta_\beta^0 \left[1 + \hat{\zeta}_\beta^n Z_{\kappa_\parallel}^{(0)}(\hat{\zeta}_\beta^n)\right],$$

$$J_U(n, m, 2, l;\ f_{\beta 0}, k) \simeq -n_{\beta 0}\, (\kappa_\perp u_{\beta\perp}^2)^{m-1/2} u_{\beta\parallel}^2 \frac{\Gamma(m+1/2)\Gamma(\kappa_\perp - m - 1/2)}{\Gamma(\kappa_\perp - 1)} \times \left(\frac{\tilde{\nu}_\beta^k}{z}\right)^l \zeta_\beta^0 \hat{\zeta}_\beta^n \left[1 + \hat{\zeta}_\beta^n Z_{\kappa_\parallel}^{(0)}(\hat{\zeta}_\beta^n)\right]. \tag{31}$$

In the limit $\kappa_\parallel \to \infty$, $\kappa_\perp \to \infty$, and in the isotropic situation,

$$J_U(n, m, 0, l;\ f_{\beta 0}, k) \simeq -n_{\beta 0}\, (u_{\beta\perp}^2)^{m-1/2} u_{\beta\parallel}^0 \Gamma(m+1/2) \times \left(\frac{\tilde{\nu}_\beta^k}{z}\right)^l \zeta_\beta^0 Z(\hat{\zeta}_\beta^n)\,,$$

$$J_U(n, m, 1, l;\ f_{\beta 0}, k) \simeq -n_{\beta 0}\, (u_{\beta\perp}^2)^{m-1/2} u_{\beta\parallel}^1 \Gamma(m+1/2) \times \left(\frac{\tilde{\nu}_\beta^k}{z}\right)^l \zeta_\beta^0 \left[1 + \hat{\zeta}_\beta^n Z(\hat{\zeta}_\beta^n)\right],$$

$$J_U(n, m, 2, l;\ f_{\beta 0}, k) \simeq -n_{\beta 0}\, (u_{\beta\perp}^2)^{m-1/2} u_{\beta\parallel}^2 \Gamma(m+1/2) \times \left(\frac{\tilde{\nu}_\beta^k}{z}\right)^l \zeta_\beta^0 \hat{\zeta}_\beta^n \left[1 + \hat{\zeta}_\beta^n Z(\hat{\zeta}_\beta^n)\right].$$

3.5 Evaluation of the Integrals $J_{\nu L}(n, m, h;\ f_{\beta 0}, k)$, for $f_{\beta 0} = f_{\beta,\kappa}$

From (13), using the average value of the collision frequency,

$$J_{\nu L}(n, m, h;\ f_{\beta 0}, k) = \left(\frac{\tilde{\nu}_\beta^k}{z}\right) z \int \mathrm{d}^3 u \times \frac{u_\parallel^h u_\perp^{2(m-1)} u_\perp\, L(f_{\beta 0})}{z - nr_\beta - q_\parallel u_\parallel + i \sum_j \tilde{\nu}_\beta^j}\,.$$

Simple inspection shows that

$$J_{\nu L}(n, m, h;\ f_{\beta 0}, k) = \frac{\tilde{\nu}_\beta^k}{z} J(n, m, h;\ f_{\beta 0})\,, \tag{32}$$

where the $J(n, m, h;\ f_{\beta 0})$ are given by (26).

3.6 Evaluation of the Integrals $J_{\nu\nu}(n, m;\ f_{\beta 0}, k)$, for $f_{\beta 0} = f_{\beta,\kappa}$

From (14), using the average value of the collision frequency,

$$J_{\nu\nu}(n, m;\ f_{\beta 0}, k) = \left(\frac{\tilde{\nu}_\beta^k}{z}\right) \left(\sum_q \frac{\tilde{\nu}_\beta^q}{z}\right) z \int \mathrm{d}^3 u \times \frac{u_\perp^{2m-1}\, \mathcal{L}(f_{\beta 0})}{z - nr_\beta - q_\parallel u_\parallel + i \sum_j \tilde{\nu}_\beta^j}\,.$$

Considering a kappa distribution and using (21),

$$J_{\nu\nu}(n,m;\ f_{\beta 0},k)=-2\frac{z}{q_\parallel}\left(\frac{\tilde{\nu}^k_\beta}{z}\right)\left(\sum_q\frac{\tilde{\nu}^q_\beta}{z}\right)\frac{(2\pi)\,n_{\beta 0}}{\pi^{3/2}\kappa_\perp\kappa_\parallel^{1/2}u^2_{\beta\perp}u_{\beta\parallel}}$$
$$\times\frac{\Gamma(\kappa_\perp)\Gamma(\kappa_\parallel)}{\Gamma(\kappa_\perp-1)\Gamma(\kappa_\parallel-1/2)}$$
$$\times\int_{-\infty}^{\infty}\mathrm{d}u_\parallel\,\frac{u_\parallel}{u_\parallel-u_{\parallel,res}}\left(1+\frac{u_\parallel^2}{\kappa_\parallel u^2_{\beta\parallel}}\right)^{-\kappa_\parallel}$$
$$\times\int_0^{\infty}\mathrm{d}u_\perp\,u_\perp u_\perp^{2m}\left(1+\frac{u_\perp^2}{\kappa_\perp u^2_{\beta\perp}}\right)^{-\kappa_\perp}$$
$$\times\left[\frac{1}{u^2_{\beta\perp}}\left(1+\frac{u_\perp^2}{\kappa_\perp u^2_{\beta\perp}}\right)^{-1}-\frac{1}{u^2_{\beta\parallel}}\left(1+\frac{u_\parallel^2}{\kappa_\parallel u^2_{\beta\parallel}}\right)^{-1}\right].$$

Performing the integrals as in the previous subsections, we arrive to the following,

$$J_{\nu\nu}(n,m;\ f_{\beta 0},k)$$
$$=-(2\kappa_\perp)\left(\frac{\tilde{\nu}^k_\beta}{z}\right)\left(\sum_q\frac{\tilde{\nu}^q_\beta}{z}\right)$$
$$\times n_{\beta 0}(u_{\beta\parallel})(\kappa_\perp u^2_{\beta\perp})^{m-1}(m!)\frac{\Gamma(\kappa_\perp-m-1)}{\Gamma(\kappa_\perp-1)}$$
$$\times\zeta^0_\beta\left[\left(\frac{\kappa_\perp-m-1}{\kappa_\perp}-\frac{u^2_{\beta\perp}}{u^2_{\beta\parallel}}\frac{\kappa_\parallel-1/2}{\kappa_\parallel}\right)+\hat{\zeta}^n_\beta\left(\frac{\kappa_\perp-m-1}{\kappa_\perp}Z^{(0)}_{\kappa_\parallel}(\hat{\zeta}^n_\beta)-\frac{u^2_{\beta\perp}}{u^2_{\beta\parallel}}Z^{(1)}_{\kappa_\parallel}(\hat{\zeta}^n_\beta)\right)\right]. \quad (33)$$

In the limit $\kappa_\parallel\to\infty$, $\kappa_\perp\to\infty$,

$$J_{\nu\nu}(n,m;\ f_{\beta 0},k)=-2\left(\frac{\tilde{\nu}^k_\beta}{z}\right)\left(\sum_q\frac{\tilde{\nu}^q_\beta}{z}\right)$$
$$\times n_{\beta 0}u_{\beta\parallel}(u^2_{\beta\perp})^{m-1}(m!)\zeta^0_\beta$$
$$\times\left[\left(1-\frac{u^2_{\beta\perp}}{u^2_{\beta\parallel}}\right)+\hat{\zeta}^n_\beta\left(1-\frac{u^2_{\beta\perp}}{u^2_{\beta\parallel}}\right)Z(\hat{\zeta}^n_\beta)\right].$$

For the isotropic case, this quantity vanishes, corresponding to the finding of [45]. Equation 33 therefore is seen to feature the correct limit.

3.7 Evaluation of the Integrals $J_{\nu 0}(f_{\beta 0},k)$, for $f_{\beta 0}=f_{\beta,\kappa}$

From (15), using the average value of the collision frequency,

$$J_{\nu 0}(f_{\beta 0},k)=\left(\frac{\tilde{\nu}^k_\beta}{z}\right)\int\mathrm{d}^3u\,\frac{\mathcal{L}(f_{\beta 0})}{u_\perp}.$$

Considering a kappa distribution and using (21),

$$J_{\nu 0}(f_{\beta 0},k)=-2\left(\frac{\tilde{\nu}^k_\beta}{z}\right)(2\pi)\int_0^\infty\mathrm{d}u_\perp\,u_\perp\int_{-\infty}^{\infty}\mathrm{d}u_\parallel\,u_\parallel$$
$$\times\left(\frac{\kappa_\perp}{\kappa_\perp u^2_{\beta\perp}+u^2_\perp}-\frac{\kappa_\parallel}{\kappa_\parallel u^2_{\beta\parallel}+u^2_\parallel}\right)f_{\beta,\kappa}\,.$$

It is readily seen that the integral over $u_\parallel$ vanishes in the case of a bi-kappa distribution. Indeed, it vanishes for any distribution which is even in the parallel component of the velocity, leading to the result

$$J_{\nu 0}(f_{\beta 0},k)=0\,. \quad (34)$$

3.8 Evaluation of the Integrals $J_{ch}(f_{\beta 0},k)$, for $f_{\beta 0}=f_{\beta,\kappa}$

The integral J_{ch} is peculiar because it does not depend on the collision frequency. From (16),

$$J_{ch}(f_{\beta 0},k)=\int\mathrm{d}^3u\,f_{\beta 0}\frac{1}{u}H\left(u^2+\frac{2Z^k_{d0}eq_\beta}{a_km_\beta v^2_*}\right).$$

Using the anisotropic kappa distributions, and variable $\mu=\cos\theta$,

$$J_{ch}(f_{\beta 0},k)=(2\pi)\frac{n_{\beta 0}}{\pi^{3/2}\kappa_\perp\kappa_\parallel^{1/2}u^2_{\beta\perp}u_{\beta\parallel}}\frac{\Gamma(\kappa_\perp)\Gamma(\kappa_\parallel)}{\Gamma(\kappa_\perp-1)\Gamma(\kappa_\parallel-1/2)}$$
$$\times\int_{-1}^{1}\mathrm{d}\mu\int_{u^\beta_{lim}}^{\infty}\mathrm{d}u\,u\left(1+\frac{u^2\mu^2}{\kappa_\parallel u^2_{\beta\parallel}}\right)^{-\kappa_\parallel}$$
$$\times\left(1+\frac{u^2(1-\mu^2)}{\kappa_\perp u^2_{\beta\perp}}\right)^{-\kappa_\perp},$$

where

$$u^e_{lim}=\left(\frac{2Z^k_{d0}(e^2/a_k)}{m_ev^2_*}\right)^{1/2},\qquad u^i_{lim}=0\,.$$

Performing the integral over the μ variable, we obtain the following expression in terms of the hypergeometric function of two variables,

$$J_{ch}(f_{\beta 0}, k) = 2(2\pi)\frac{n_{\beta 0}}{\pi^{3/2}\kappa_\perp \kappa_\parallel^{1/2} u_{\beta\perp}^2 u_{\beta\parallel}} \times \frac{\Gamma(\kappa_\perp)\Gamma(\kappa_\parallel)}{\Gamma(\kappa_\perp - 1)\Gamma(\kappa_\parallel - 1/2)} \times \int_{u_{lim}^\beta}^\infty \mathrm{d}u\, u \left(\frac{\kappa_\perp u_{\beta\perp}^2}{u^2 + \kappa_\perp u_{\beta\perp}^2}\right)^{\kappa_\perp} \times F_1\left(\frac{1}{2}; \kappa_\parallel, \kappa_\perp; \frac{3}{2}; -\frac{u^2}{\kappa_\parallel u_{\beta\parallel}^2}, \frac{u^2}{u^2+\kappa_\perp u_{\beta\perp}^2}\right). \quad (35)$$

4 Analysis of the Equilibrium Condition

The equilibrium condition for dust particles of population k is obtained by assuming null charging current over those particles,

$$I_0^k(q) = \sum_\beta \int \mathrm{d}^3p\, q_\beta \sigma_\beta^k(p,q) \frac{p}{m_\beta} f_{\beta 0}(\mathbf{p}) = 0\,.$$

Using equation (4) for the charging cross section,

$$\sigma_\beta^k(p,q) = \pi a_k^2 \left(1 - \frac{2qq_\beta m_\beta}{a_k p^2}\right) H\left(1 + \frac{2Z_{d0}^k e q_\beta m_\beta}{a_k p^2}\right),$$

the equilibrium condition is written as follows,

$$I_0^k(q) = \pi a_k^2 \sum_\beta q_\beta \int \mathrm{d}^3p\, \frac{p}{m_\beta}\left(1 + \frac{2Z_{d0}^k e q_\beta m_\beta}{a_k p^2}\right) \times H\left(1 + \frac{2Z_{d0}^k e q_\beta m_\beta}{a_k p^2}\right) f_{\beta 0}(\mathbf{p}) = 0\,,$$

which can be written in terms of the normalized variables,

$$I_0^k(q) = \frac{1}{n_{d0}^k}\sum_\beta q_\beta \int \mathrm{d}^3u\, \frac{\pi a_k^2 n_{d0}^k v_*}{u}\left(u^2 + \frac{2Z_{d0}^k e q_\beta}{a_k m_\beta v_*^2}\right) \times H\left(u^2 + \frac{2Z_{d0}^k e q_\beta}{a_k m_\beta v_*^2}\right) f_{\beta 0}(\mathbf{u}) = 0\,.$$

On the other hand, from equation (22) for the average collision frequency

$$\nu_\beta^k = \frac{1}{n_{\beta 0}}\int \mathrm{d}^3u\, \nu_{\beta d}^{k0}(u)\, f_{\beta 0}(u)\,,$$

and using the expression for the collision frequency, from (10), we obtain

$$\nu_\beta^k = \frac{1}{n_{\beta 0}}\int \mathrm{d}^3u\, \frac{\pi a_k^2 n_{d0}^k v_*}{u}\left(u^2 + \frac{2Z_{d0}^k e q_\beta}{a_k m_\beta v_*^2}\right) \times H\left(u^2 + \frac{2Z_{d0}^k e q_\beta}{a_k m_\beta v_*^2}\right) f_{\beta 0}\,.$$

By comparison between these two expressions, it is seen that the equilibrium condition can be written in term of the average collision frequency,

$$I_0^k(q) = \sum_\beta q_\beta \frac{n_{\beta 0}}{n_{d0}^k}\nu_\beta^k = 0\,, \;\rightarrow\; I_0^k(q) = \sum_\beta q_\beta n_{\beta 0}\tilde{\nu}_\beta^k = 0.$$

Using (23),

$$I_0^k(q) = \sum_\beta q_\beta n_{\beta 0}\frac{c^3}{\Omega_*^3}\frac{a_k^2\Omega_*^2}{c^2}\frac{v_*}{c}(\epsilon_k n_{i0})\frac{2(2\pi)}{\pi^{1/2}\kappa_\perp\kappa_\parallel^{1/2}u_{\beta\perp}^2 u_{\beta\parallel}} \times \frac{\Gamma(\kappa_\perp)\Gamma(\kappa_\parallel)}{\Gamma(\kappa_\perp - 1)\Gamma(\kappa_\parallel - 1/2)} \times \int_{u_{lim}^\beta}^\infty \mathrm{d}u\, u\left(u^2 + \frac{2Z_{d0}^k e q_\beta}{a_k m_\beta v_*^2}\right)\left(\frac{\kappa_\perp u_{\beta\perp}^2}{u^2+\kappa_\perp u_{\beta\perp}^2}\right)^{\kappa_\perp} \times F_1\left(\frac{1}{2}; \kappa_\parallel, \kappa_\perp; \frac{3}{2}; -\frac{u^2}{\kappa_\parallel u_{\beta\parallel}^2}, \frac{u^2}{u^2+\kappa_\perp u_{\beta\perp}^2}\right) = 0\,.$$

The quantities which do not depend on β can be put in evidence, and since the whole expression is equal to zero, the equilibrium condition can be reduced to the following,

$$\sum_\beta \frac{q_\beta n_{\beta 0}}{u_{\beta\perp}^2 u_{\beta\parallel}}\int_{u_{lim}^\beta}^\infty \mathrm{d}u\, u\left(u^2 + \frac{2Z_{d0}^k e q_\beta}{a_k m_\beta v_*^2}\right)\left(\frac{\kappa_\perp u_{\beta\perp}^2}{u^2+\kappa_\perp u_{\beta\perp}^2}\right)^{\kappa_\perp} \times F_1\left(\frac{1}{2}; \kappa_\parallel, \kappa_\perp; \frac{3}{2}; -\frac{u^2}{\kappa_\parallel u_{\beta\parallel}^2}, \frac{u^2}{u^2+\kappa_\perp u_{\beta\perp}^2}\right) = 0. \quad (36)$$

From the neutrality condition, $n_{i0}Ze - n_{e0}e - \sum_k n_{d0}^k Z_{d0}^k e = 0$, we obtain the equilibrium electron density,

$$n_{e0} = n_{i0}\left(Z - \sum_k \epsilon^k Z_{d0}^k\right). \quad (37)$$

It is seen that there are n equilibrium conditions which have to be satisfied simultaneously, where n is the number of sizes of dust particles, with n_{e0} given by (37).

Using explicitly the neutrality condition, considering fixed n_{i0},

$$\frac{Zen_{i0}}{u_{i\perp}^2 u_{i\parallel}} \int_0^\infty \mathrm{d}u\, u \left(u^2 + \frac{2Z_{d0}^k Ze^2}{a_k m_i v_*^2}\right) \left(\frac{\kappa_\perp u_{i\perp}^2}{u^2 + \kappa_\perp u_{i\perp}^2}\right)^{\kappa_\perp}$$
$$\times F_1\left(\frac{1}{2}; \kappa_\parallel, \kappa_\perp; \frac{3}{2}; -\frac{u^2}{\kappa_\parallel u_{i\parallel}^2}, \frac{u^2}{u^2 + \kappa_\perp u_{i\perp}^2}\right)$$
$$+ \frac{(-e)n_{i0}}{u_{e\perp}^2 u_{e\parallel}} \left(Z - \sum_j \epsilon^j Z_{d0}^j\right) \int_{u_{lim}^e}^\infty \mathrm{d}u\, u \left(u^2 - \frac{2Z_{d0}^k e^2}{a_k m_e v_*^2}\right)$$
$$\times \left(\frac{\kappa_\perp u_{e\perp}^2}{u^2 + \kappa_\perp u_{e\perp}^2}\right)^{\kappa_\perp}$$
$$\times F_1\left(\frac{1}{2}; \kappa_\parallel, \kappa_\perp; \frac{3}{2}; -\frac{u^2}{\kappa_\parallel u_{e\parallel}^2}, \frac{u^2}{u^2 + \kappa_\perp u_{e\perp}^2}\right) = 0\,. \qquad (38)$$

5 Final Remarks

In the present paper, we have presented general expressions for the components of the dielectric tensor for magnetized dusty plasmas, valid for general direction of propagation and for the case in which different populations of dust particles are present in the plasma. We have assumed the presence of n populations of dust particles, which for simplicity were assumed to be motionless and spherical, with radius a_j and charge q_j, $j = 1, \ldots, n$. As in previous formulations valid for the case of a single species of dust particles, the dielectric tensor has been divided into two parts, one which is denominated "conventional" and which is formally similar to the dielectric tensor of dustless plasmas, although with some modification due to the frequency of inelastic collisions between electrons and ions and the dust particles, and another which is non-vanishing only due to the occurrence of the inelastic collisions between electrons and ions and the dust particles, and which is denominated as the "new" contribution. Both the "conventional" and the "new" contribution were written in terms of double series, formally containing all harmonic and Larmor radius contributions and depending on a small number of integrals which depend on the distribution function. We have then considered the particular case of anisotropic kappa distributions for electrons and ions, in order to provide explicit evaluations of the integrals which appear in the dielectric tensor components. This choice was motivated by the fact that non-thermal distributions of type kappa have been observed under natural conditions, as in the solar wind. We believe that the formulation presented here can be useful for the study of wave propagation in dusty plasmas under a large variety of conditions and parameters.

As an example of a possible application of the formulation, it is possible to mention the case of kinetic Alfvén waves for propagation parallel or oblique relative to the ambient magnetic field. We have studied this type of waves for the case of large wavelength and Maxwellian distribution for the plasma species, and have obtained that both the shear Alfvén ($\omega \simeq k_z v_A$) and the compressional Alfvén ($\omega = kv_A$) modes are modified by the presence of a dust population, with both modes displaying an interval of q_z values for which the group velocity is zero. The shear Alfvén wave displays an interval for which the real frequency is vanishing, while the compressional Alfvén wave features a reversed sign of the parallel group velocity for small $q_\perp$, and can also display a region of zero real frequency. All considered, the results obtained have shown that the Alfvén waves at long wavelength feature in a dusty plasma a behavior which is completely different from the behavior obtained in the absence of the dust [42]. It is therefore of interest to investigate possible modification in the dynamics of Alfvén waves in a dusty plasma in the case of non-thermal or superthermal distributions for the plasma particles, which is made possible by the present formulation.

It must be mentioned that some effects of the presence of dusty particles on the dynamics of Alfvén waves, for the case of superthermal distribution of electrons, have been investigated with a previous and more limited version of the formulation in [43], where it was shown that the combined influence of dust particles and superthermal electrons sensibly modifies the dispersion relation at long wavelengths, and increases the absorption coefficient. The results of [43] have been obtained for the case of parallel propagation, isotropical kappa distribution for electrons and Maxwellian distribution for ions, and considering a single size of dust particles. The formulation developed in the present paper opens up the possibility of extension of these studies to the investigation of the dispersion relation of Alfvén waves in dusty plasmas containing different populations of dust particles, for oblique propagation, in plasmas with electrons and ions with superthermal and non-thermal characteristics, which appear to be very prevalent in natural systems. Instabilities related to temperature anisotropy in dusty plasmas with superthermal features, are within the grasp of the present formulation.

Other possibilities of application can be mentioned. For instance, we have investigated the effect of dust particles on Langmuir and ion-acoustic waves, considering Maxwellian distributions for electrons and ions, and found that the damping at large wavelengths can be increased due to the collisional absorption of charged plasma particles by dust particles [44]. The superthermal features related to kappa distributions shall certainly influence the collisionality due to energetic particles, an argument which offers motivation to the study of electrostatic waves in dusty plasmas in the presence of superthermal electrons, which is made possible with the present formulation. Another interesting example of application of kappa distributions to the study of space phenomena involving waves and particles can be found in the analysis of hard X-ray flare spectra [59]. This example is related to a range of frequencies very different of that of Alfvén or electrostatic waves, offering additional motivation to the study of superthermal distributions.

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