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PARTICLES AND FIELDS

Bianchi -V Space -Time with Anisotropic Dark Energy in General Relativity

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Abstract In a spatially homogeneous and anisotropic Bianchi type-V space-time the consequences of the presence of dynamically anisotropic dark energy and perfect fluid with heat-conduction are studied. We assume that dark energy is minimally interacting with matter and has an equation of state which is modified in a consistent way with the conservation of energy momentum tensor. Exact solutions of Einstein field equations are obtained by taking constant value of deceleration parameter. We find that this assumption is reasonable for the observation of the present day universe. The physical and geometrical properties of the models, the behavior of the anisotropy of dark energy and the thermodynamical relations that govern such solutions are discussed in detail.

Keywords Cosmology · Dark energy · Hubble's parameter · Deceleration parameter · Bianchi models · Perfect fluid · Heat conduction

1 Introduction

Observations of Supernova Type Ia (SN Ia) [1–3], Cosmic Microwave Background (CMB) anisotropy [4–6] and studies on large scale structure formation strongly indicate that our Universe is spatially flat, with two thirds of the energy contents resultant from dark energy, a substance with negative pressure that can make the universe expanding in an accelerating fashion. Perhaps the most natural, although definitely not unique, reason for this acceleration is the presence of cosmic vacuum of nonzero energy and pressure; because of that, investigation of various cosmological models including a positive cosmological constant becomes interesting once again. Dark energy is generally represented by a phenomenologically motivated equation of state (EoS) $p^{(de)} = w\rho^{(de)}$, where $p^{(de)}$ and $\rho^{(de)}$ are the pressure and energy density of the dark energy, interlinked by a dimensionless EoS parameter w [7]. Present observation data [8] constrain the range of the EoS of dark energy as $-1.38 < w < -0.82$, which indicates the possibility of dark energy with $w < -1$, debuted as Phantom [9]. In order to produce the accelerated expansion, it requires $w < -1/3$. In recent years, several theoretical models have been proposed to understand the nature and dynamics of dark energy, however almost all these models either require tuning of their model parameters or yield quantum or gravitational instabilities that are needed to be removed.

Most prominent dark energy proposals are based on cosmological constant Λ [10, 11], quintessence [12, 13], k-essence [14, 15], phantom energy [16], quintom model [17, 18], geometric dark energy [19], holographic dark energy [20] and tachyons [21], to name a few. One of the favorite candidates among the dark energy models is related to dynamic cosmological constant Λ. In fact, the concept of dark energy and the physics of accelerating Universe appears to be inherent in the Λ -term of Einstein's field equations. The cosmological model is the simplest case with $w = -1$. Models with

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cosmological constant predict a transition to an accelerated expansion of the Universe. Quintessence models have an equation of state parameter w greater than -1. Observation constraints on w do not exclude the possibility of w being less than -1 at present epoch. This leads to the possibility that the present cosmic acceleration may be driven by a phantom field ($w < -1$). The energy density of phantom field increases as the Universe expands. The phantom energy with a constant w lead to a singularity (Big Rip) after finite time in the future [9, 22]. Akarsu and Kilinc [23] have studied LRS Bianchi type I models with anisotropic dark energy and cosntant deceleration parameter where the dark energy is minimally interacting and has dynamical energy density.

It is certainly of interest to study cosmologies with a richer structure, both geometrically and physically, than the standard perfect fluid Friedmann-Robertson-Walker(FRW) or LRS Bianchi models. Bianchi type V models are of particular interest since they are sufficiently complex as the Einstein tensor has off-diagonal terms while at the same time are a simple generalization of the negative curvature FRW models. The cosmological models of this type have a significant role in the description of the Universe in the early stages of its evolution.

The purpose of this work is to study dark energy cosmology in a Bianchi type -V spatially homogeneous and anisotropic model where the source of the gravitational field is the perfect fluid with heat flow and dark energy. We assume that dark energy is minimally interacting with matter and has an equation of state which is modified in a consistent way with the conservation of energy momentum tensor.

The paper is organised as follows. In Section 2 of this work, we present the model of the Universe and the expressions for basic equations. We discuss, in Section 3, the basic assumption on deceleration parameter and two types of solutions for the average scale factor of the model, one is of power-law type and other of exponential form for Bianchi type-V metric. Using these two forms of the average scale factor, two classes of exact solutions of Einstein field equations in the presence of a perfect fluid with heat flow and dark energy, which correspond to singular and non-singular cosmological models, are presented. We also discuss the physical and geometrical behaviors of the models in each cosmology. The thermodynamical relations related to the heat flow are discussed in Section 4 We conclude, in Section 5, with some final remarks.

2 Model and Field Equations

We consider the Bianchi type -V space-time with metric

$$ds^2 = dt^2 - A^2(t)dx^2 - e^{2\alpha x}\left[B^2(t)dy^2 + C^2(t)dz^2\right], \quad (1)$$

where A, B, C are functions of cosmic time t and α is a constant.
The generalized mean Hubble parameter H in anisotropic model (1) is given by

$$H = \frac{\dot{a}}{a} = \frac{1}{3}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right), \quad (2)$$

where $a = (ABC)^{1/3}$ is the average scale factor. A dot denotes a derivative with respect to cosmic time t.
The directional Hubble parameters in the directions of x, y and z may be defined as

$$H_x = \frac{\dot{A}}{A}, \quad H_y = \frac{\dot{B}}{B}, \quad H_z = \frac{\dot{C}}{C}. \quad (3)$$

An important observational quantity is the deceleration parameter q defined as

$$q = -\frac{a\ddot{a}}{\dot{a}^2}. \quad (4)$$

The physical quantities of observational interest in cosmology are the expansion scalar θ, the shear scalar σ and the average anisotropy parameter $\bar{A}(\geq 0)$, which are defined as

$$\theta = \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right), \quad (5)$$

$$\sigma^2 = \frac{1}{2}\sigma_{\mu\nu}\sigma^{\mu\nu} = \frac{1}{2}\left[\left(\frac{\dot{A}}{A}\right)^2 + \left(\frac{\dot{B}}{B}\right)^2 + \left(\frac{\dot{C}}{C}\right)^2\right] - \frac{\theta^2}{6}, \quad (6)$$

$$\bar{A} = \frac{1}{3}\sum_{\mu=1}^{3}\left(\frac{H_\mu - H}{H}\right)^2. \quad (7)$$

The Einstein's field equations (in the unit $8\pi G = c = 1$) read as

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -T^{(m)}_{\mu\nu} - T^{(de)}_{\mu\nu}, \quad (8)$$

where $T^{(m)}_{\mu\nu}$ and $T^{(de)}_{\mu\nu}$ are the energy momentum tensors of perfect fluid with heat flow and dark energy, respectively.

The energy-momentum tensor of a perfect fluid with heat flow has the form [24–26]:

$$T^{(m)}_{\mu\nu} = \left(\rho^{(m)} + p^{(m)}\right) u_\mu u_\nu - p^{(m)} g_{\mu\nu} + Q_\mu u_\nu + Q_\nu u_\mu. \tag{9}$$

where $\rho^{(m)}$ is the energy density, $p^{(m)}$ the thermodynamic pressure, u_μ the four-velocity of the fluid and Q_μ is the heat flow vector satisfying

$$Q^\mu u_\mu = 0, \quad Q^\mu Q_\mu > 0. \tag{10}$$

we assume that heat flow is in x-direction only so that $Q_\mu = (0, Q_1, 0, 0)$, Q_1 being a function of time.

The energy momentum tensor for dark energy (DE) is given by [23]

$$\begin{aligned} T^{(de)}_{\mu\nu} &= diag[\rho^{(de)}, -p^{(de)}_x, -p^{(de)}_y, -p^{(de)}_z] \\ &= diag[1, -w_x, -w_y, -w_z]\rho^{(de)} \\ &= diag[1, -(w+\delta), -(w+\gamma), -(w+\gamma)]\rho^{(de)}, \end{aligned} \tag{11}$$

where $\rho^{(de)}$ is the energy density of the DE component; p_x, p_y and p_z are the pressures and w_x, w_y and w_z are the directional EoS parameters of the DE along the coordinate axes x, y and z, respectively. w is the deviation -free EoS parameter of $p^{(de)} = w\rho^{(de)}$ in the case of DE; δ and γ are skewness parameters that are the deviations from w, respectively, along x, y and z and may be considered as constant or functions of the cosmic time t.

In a co-moving coordinate system, the field equations (8), in case of model (1) and energy momentum tensors (9) and (11), yield

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} - \frac{\alpha^2}{A^2} = -p^{(m)} - (w+\delta)\rho^{(de)} \tag{12}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} - \frac{\alpha^2}{A^2} = -p^{(m)} - (w+\gamma)\rho^{(de)} \tag{13}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} - \frac{\alpha^2}{A^2} = -p^{(m)} - (w+\gamma)\rho^{(de)} \tag{14}$$

$$\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} - \frac{3\alpha^2}{A^2} = \rho^{(m)} + \rho^{(de)} \tag{15}$$

$$\alpha\left(2\frac{\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right) = Q_1. \tag{16}$$

Taking into account the conservation equation, we have

$$\begin{aligned} &\dot{\rho}^{(m)} + 3\left(\rho^{(m)} + p^{(m)}\right)H + \dot{\rho}^{(de)} + 3(1+w)H\rho^{(de)} \\ &\quad + \left(\delta\frac{\dot{A}}{A} + \gamma\frac{\dot{B}}{B} + \gamma\frac{\dot{C}}{C}\right)\rho^{(de)} = \frac{2\alpha}{A^2}Q_1, \end{aligned} \tag{17}$$

We assume that the perfect fluid and dark energy components interact minimally. Hence, the energy momentum tensor of the each component is to be conserved separately for minimally interaction.

The law of energy-conservation equation $T^{(m)\mu\nu}{}_{;\nu} = 0$, of the perfect fluid gives

$$\dot{\rho}^{(m)} + \left(\rho^{(m)} + p^{(m)}\right)\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = \frac{2\alpha}{A^2}Q_1, \tag{18}$$

whereas the energy conservation equation of the dark energy component $T^{(de)\mu\nu}{}_{;\nu} = 0$ yields

$$\begin{aligned} &\dot{\rho}^{(de)} + (1+w)\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right)\rho^{(de)} \\ &\quad + \left(\delta\frac{\dot{A}}{A} + \gamma\frac{\dot{B}}{B} + \gamma\frac{\dot{C}}{C}\right)\rho^{(de)} = 0. \end{aligned} \tag{19}$$

Once again we may split (19) into two parts: deviation free part and the term arises due to deviation from w. Thus, we have

$$\dot{\rho}^{(de)} + (1+w)\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right)\rho^{(de)} = 0, \tag{20}$$

which also results

$$\left(\delta\frac{\dot{A}}{A} + \gamma\frac{\dot{B}}{B} + \gamma\frac{\dot{C}}{C}\right)\rho^{(de)} = 0, \tag{21}$$

where $\rho^{(de)} \neq 0$. The field equations (12)–(15) can also be written as

$$p^{(m)} + \frac{1}{3}(3w + \delta + 2\gamma)\rho^{(de)} = H^2(2q-1) - \sigma^2 + \frac{\alpha^2}{A^2}, \tag{22}$$

$$\rho^{(m)} + \rho^{(de)} = 3H^2 - \sigma^2 - \frac{3\alpha^2}{A^2}, \tag{23}$$

The Raychaudhuri equation is found to be

$$\dot{\theta} = -\frac{1}{2}(\rho^{(m)} + 3p^{(m)}) - \frac{1}{3}\theta^2 - 2\sigma^2 - \frac{1}{2}(3w + \delta + 2\gamma + 1)\rho^{(de)}. \tag{24}$$

We have a system of six equations (12)–(16) and (19) with ten unknown variables, namely, A, B, C, $p^{(m)}$, $\rho^{(m)}$, $\rho^{(de)}$, w, δ, γ and Q_1. In order to solve the system completely we need four extra relations among these variables.

To get general solutions, we assume that the deviation parameters δ and γ are function of the cosmic time t. Therefore, a special dynamics of δ and γ, which is consistence with (21) are assumed to be

$$\delta(t) = \beta\left(\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right), \qquad \gamma(t) = -\beta\frac{\dot{A}}{A}, \tag{25}$$

where β is an arbitrary constant, which parameterizes the anisotropy consistently with the energy momentum conservation and $\beta = 0$ implies that the DE is isotropic.

We also take the deviation free EoS parameter w as a constant. Therefore, (20) gives

$$\rho^{(de)} = \rho_0^{(de)} a^{-3(1+w)}, \tag{26}$$

where $\rho_0^{(de)}$ is a constant of integration.

Subtracting (12) from (14), (12) from (13) and (14) from (13), we get the following three relations, respectively [27]

$$\frac{A}{B} = d_1 \exp\left[k_1 \int a^{-3}dt - \frac{\beta\rho_0^{(de)}}{w}\int a^{-3(1+w)}dt\right] \tag{27}$$

$$\frac{A}{C} = d_2 \exp\left(k_2 \int a^{-3}dt - \frac{\beta\rho_0^{(de)}}{w}\int a^{-3(1+w)}dt\right] \tag{28}$$

$$\frac{B}{C} = d_3 \exp\left(k_3 \int a^{-3}dt\right), \tag{29}$$

where d_1, d_2, d_3 and k_1, k_2, k_3 are constants of integration. Using $a = (ABC)^{1/3}$, the quadrature form of the metric functions can be written explicitly as

$$A(t) = l_1 a \exp\left[X_1 \int a^{-3}dt - \frac{2\beta\rho_0^{(de)}}{3w}\int a^{-3(1+w)}dt\right], \tag{30}$$

$$B(t) = l_2 a \exp\left[X_2 \int a^{-3}dt + \frac{\beta\rho_0^{(de)}}{3w}\int a^{-3(1+w)}dt\right], \tag{31}$$

$$C(t) = l_3 a \exp\left[X_3 \int a^{-3}dt + \frac{\beta\rho_0^{(de)}}{3w}\int a^{-3(1+w)}dt\right], \tag{32}$$

where

$$l_1 = \sqrt[3]{d_1 d_2},\ l_2 = \sqrt[3]{d_1^{-1} d_3},\ l_3 = \sqrt[3]{(d_2 d_3)^{-1}},$$

$$X_1 = \frac{k_1 + k_2}{3},\ X_2 = \frac{k_3 - k_1}{3},\ X_3 = \frac{-(k_2 + k_3)}{3}.$$

The constants X_1, X_2, X_3 and l_1, l_2, l_3 satisfy the following two relations:

$$X_1 + X_2 + X_3 = 0, \quad l_1 l_2 l_3 = 1. \tag{33}$$

Thus the metric functions are found explicitly in terms of the average scale factor a.

3 Solution of Field Equations

For any physically relevant model, the Hubble H and deceleration q parameters are the most important observational quantities in cosmology. The first quantity sets the present time scale of the expansion while the second one is telling us that the present stage is speeding up instead of slowing down as expected before the Supernovae type Ia observations. Berman [28] and Berman and Gomide [29] proposed a law of variation for the Hubble parameter in the FRW model that yields a constant value of the deceleration parameter and gives two forms of the scale factor in terms of cosmic time t. Recently, Singh et al. [30] extended this approach to anisotropic Bianchi-V models and obtained some exact solutions. In the following we extend this work to anisotropic dark energy models.

Suppose that the deceleration parameter q is constant:

$$q = -\frac{\ddot{a}a}{\dot{a}^2} = n - 1 = constant, \tag{34}$$

where $n(\geq 0)$ is a constant.

In the present anisotropic model, the assumption (34) amounts to

$$a = (nlt)^{1/n}, \qquad (n \neq 0), \tag{35}$$

and for $n = 0$ we have

$$a = c_0 \exp(lt) \tag{36}$$

where $l(> 0)$ is a constant and c_0 is the constant of integration. It is however possible to have $l = 0$ for which we would have a static universe. But $l > 0$ is consistent with observation for which the universe must be expanding. Hence we disregard a static universe.

Now we solve the quadrature equations (30)–(32) in the following two cases depending on the values of n as defined in (35) and (36).

Case I: Solution with $n \neq 0$

Using the power-law form of a from (35) into (30)–(32), the solution of the metric functions can be written as

$$A(t) = l_1 (nlt)^{\frac{1}{n}} \exp\left[\frac{X_1}{l(n-3)}(nlt)^{\frac{(n-3)}{n}} - \frac{2\beta\rho_0^{(de)}}{3wl(n-3w-3)}(nlt)^{\frac{(n-3w-3)}{n}}\right], \quad (37)$$

$$B(t) = l_2 (nlt)^{\frac{1}{n}} \exp\left[\frac{X_2}{l(n-3)}(nlt)^{\frac{(n-3)}{n}} + \frac{\beta\rho_0^{(de)}}{3wl(n-3w-3)}(nlt)^{\frac{(n-3w-3)}{n}}\right], \quad (38)$$

$$C(t) = l_3 (nlt)^{\frac{1}{n}} \exp\left[\frac{X_3}{l(n-3)}(nlt)^{\frac{(n-3)}{n}} + \frac{\beta\rho_0^{(de)}}{3wl(n-3w-3)}(nlt)^{\frac{(n-3w-3)}{n}}\right], \quad (39)$$

where $n \neq 3$ and $3(1+w)$.
Substituting (37)–(39) in (16), the heat flow is given by

$$Q_1 = 3\alpha X_1 (nlt)^{-\frac{3}{n}} - \frac{2\alpha\beta\rho_0^{(de)}}{w}(nlt)^{-\frac{3(1+w)}{n}}. \quad (40)$$

The directional Hubble parameters are given by

$$H_x = (nt)^{-1} + X_1 (nlt)^{-\frac{3}{n}} - \frac{2\beta\rho_0^{(de)}}{3w}(nlt)^{-\frac{3(1+w)}{n}}, \quad (41)$$

$$H_y = (nt)^{-1} + X_2 (nlt)^{-\frac{3}{n}} + \frac{\beta\rho_0^{(de)}}{3w}(nlt)^{-\frac{3(1+w)}{n}}, \quad (42)$$

$$H_z = (nt)^{-1} + X_3 (nlt)^{-\frac{3}{n}} + \frac{\beta\rho_0^{(de)}}{3w}(nlt)^{-\frac{3(1+w)}{n}}, \quad (43)$$

whereas the mean Hubble parameter is given by

$$H = (nt)^{-1}. \quad (44)$$

Using (41)–(43) into (25), we get the following expressions for skewness parameters of DE:

$$\delta(t) = \beta\left[2(nt)^{-1} - X_1 (nlt)^{-\frac{3}{n}} + \frac{2\beta\rho_0^{(de)}}{3w}(nlt)^{-\frac{3(1+w)}{n}}\right], \quad (45)$$

$$\gamma(t) = -\beta\left[(nt)^{-1} + X_1 (nlt)^{-\frac{3}{n}} - \frac{2\beta\rho_0^{(de)}}{3w}(nlt)^{-\frac{3(1+w)}{n}}\right]. \quad (46)$$

The directional EoS parameters of the dark energy are given by

$$w_x = w + \beta\left[2(nt)^{-1} - X_1 (nlt)^{-\frac{3}{n}} + \frac{2\beta\rho_0^{(de)}}{3w}(nlt)^{-\frac{3(1+w)}{n}}\right], \quad (47)$$

$$w_y = w_z = w - \beta\left[(nt)^{-1} + X_1 (nlt)^{-\frac{3}{n}} - \frac{2\beta\rho_0^{(de)}}{3w}(nlt)^{-\frac{3(1+w)}{n}}\right]. \quad (48)$$

The kinematical parameters θ, σ^2 and $\bar{A}$ have the following expressions:

$$\theta = 3\,(nt)^{-1}, \quad (49)$$

$$\sigma^2 = \lambda (nlt)^{-\frac{6}{n}} - \frac{X_1\beta\rho_0^{(de)}}{w}(nlt)^{-\frac{3(2+w)}{n}} + \frac{\beta^2\rho_0^{(de)2}}{3w^2}(nlt)^{-\frac{6(1+w)}{n}}, \quad (50)$$

$$\bar{A} = \frac{1}{3l^2}\left[\lambda (nlt)^{\frac{2(n-3)}{n}} - \frac{2X_1\beta\rho_0^{(de)}}{w}(nlt)^{\frac{2n-3w-6}{n}} + \frac{2\beta^2\rho_0^{(de)2}}{3w^2}(nlt)^{\frac{2(n-3w-3)}{n}}\right], \quad (51)$$

where $\lambda = X_1^2 + X_2^2 + X_1 X_2$.
The energy-density and pressure of the DE have the expressions

$$\rho^{(de)} = \rho_0^{(de)}(nlt)^{-\frac{3(1+w)}{n}}, \quad (52)$$

$$p^{(de)} = w\rho_0^{(de)}(nlt)^{-\frac{3(1+w)}{n}}. \quad (53)$$

The energy density and pressure are given by

$$\rho^{(m)} = 3(nt)^{-2} - \lambda (nlt)^{-\frac{6}{n}} + \frac{X_1\beta\rho_0^{(de)}}{w}(nlt)^{-\frac{3(2+w)}{n}} - \frac{\beta^2\rho_0^{(de)2}}{3w^2}(nlt)^{-\frac{6(1+w)}{n}} - \rho_0^{(de)}(nlt)^{-\frac{3(1+w)}{n}} - 3\alpha^2 A^{-2}, \quad (54)$$

$$p^{(m)} = (2n-3)(nt)^{-2} - \lambda(nlt)^{-\frac{6}{n}} + \frac{(1+w)X_1\beta\rho_0^{(de)}}{w}(nlt)^{-\frac{3(2+w)}{n}} + \frac{(2w-1)\beta^2\rho_0^{(de)2}}{3w^2}(nlt)^{-\frac{6(1+w)}{n}} - w\rho_0^{(de)}(nlt)^{\frac{-3(1+w)}{n}} + \alpha^2 A^{-2}. \quad (55)$$

Using the above solutions, it can easily be verified that the energy conservation equation (18) and Raychaudhuri equation (24) are identically satisfied. Therefore, we have obtained exact solutions of Bianchi type -V perfect fluid model with heat flow and dark energy components.

We observe that the physical parameters $\rho^{(m)}$, $p^{(m)}$, $\rho^{(de)}$, $p^{(de)}$ skewness and directional EoS parameters, shear scalar and Q_1 tend to infinity at $t = 0$. Therefore, the universe starts evolving from initial singularity $t = 0$ with infinite density, infinite internal pressure and infinite internal heat flow. The three scale factors are monotonically increasing function of time. The deceleration parameter is constant. We observe that the spatial scale factors are zero at the initial moment $t = 0$. The model has a point of singularity. The scale factors tend to infinity whereas energy density and pressure of matter and DE tend to zero as $t \to \infty$. The dynamics of the mean anisotropy parameter depends on the value of n. For $n < 3$, $\bar{A}$ has singular state, with infinite energy density and zero scale factors. For small time, $\bar{A}$ is increasing and in the large time limits, it ends in a homogeneous and isotropic state. The model approaches isotropic during the late time of its evolution. All physical and kinematical parameters tend to zero as t tends to infinity. We also find that $\lim_{t\to\infty} \sigma^2/\theta = 0$, which means that this model of the universe approaches isotropy during the late time of its evolution.

For $0 < n < 1$, we have $-1 < q < 0$ and therefore the model exhibits accelerating expansion, while $n > 1$, i.e., $q > 0$ is the condition for decelerating expansion of the universe. The mean Hubble parameter and the expansion scalar decrease as t increases and vanish as $t \to \infty$. Thus the rate of expansion slows down as t increases. The heat conduction is a decreasing function of cosmic time t and is maximum at the initial epoch. It decreases at a faster rate due to the dark energy component and diminishes as $t \to \infty$.

The solutions for the scale factors have a combination of a power-law term and exponential term in the product form. The DE term appears in exponential form and thus effects their evolution significantly. For $\beta < 0$ and $n > 3(1+w)$, the DE contributes to the expansion of $A(t)$ while opposing to the expansion of $B(t)$ and $C(t)$. Likewise, for $\beta > 0$ and $n > 3(1+w)$, the anisotropic DE opposes the expansion of $A(t)$ while contributing to the expansion of $B(t)$ and $C(t)$. When the power-law term is significant in the scale factors, each of them expands with almost same rate, but when the exponential term dominates, the DE term plays an important role in the spatial geometry of universe depending on the nature of constants appearing in the scale factors. For instance, if $\beta > 0$, $n > 3(1+w)$, $l_2 = l_3$ and $X_2 = X_3 < 0$, then $A(t) > B(t) = C(t)$ and thus the universe has a prolate geometry. It becomes more and more prolate as t increases due to the significant contribution of DE. If the constants are such that $A(t) < B(t) = C(t)$, the spatial geometry of universe is oblate.

The difference between the directional EoS parameters and hence the pressures of the DE, along x-axis and y-axis (or z-axis) is $3\beta(nt)^{-1}$, which decreases as t increases. Therefore, the anisotropy of the DE decreases as t increases and finally drops to zero as $t \to \infty$. The behavior of DE density has been shown in Fig. 1. We observe that $\rho^{(de)}$ increases in the phantom energy model ($w < -1$), decreases in the quintessence region ($w > -1$) and is constant when $w = -1$. Further, in the quintessence model, the pressure and energy density of the perfect fluid become negligible at late times as may be observed from (54) and (55). This is acceptable in view of the present observations of the universe. Fig. 2 suggests that the overall anisotropy parameter $\bar{A}$ decreases sharply with time and tends to zero as $t \to \infty$ for $w > -1$. Thus the observed isotropy of the universe can be achieved in the quintessence model. The shear scalar also tends to zero in this model. Thus, in our analysis, the quintessence model is turning out as

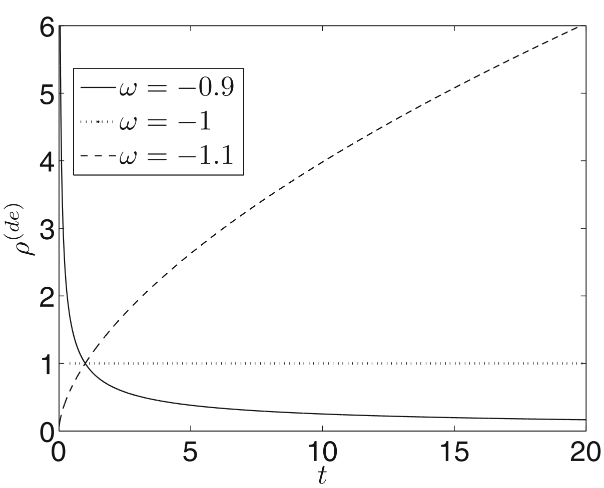

Fig. 1 Plot of $\rho^{(de)}$ versus t with $n = 0.5$, $l = 2$ and $\rho_0^{(de)} = 1$

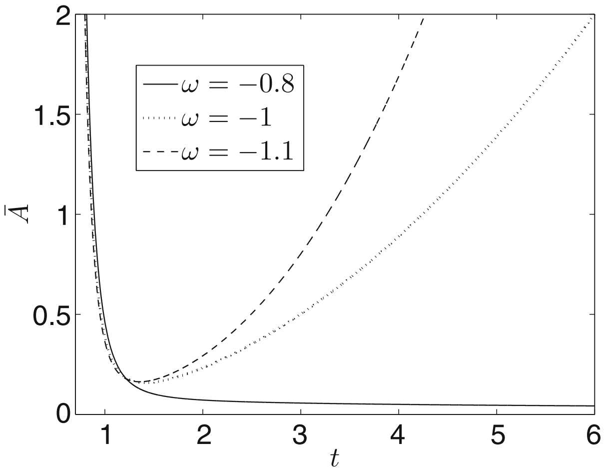

Fig. 2 Plot of $\bar{A}$ versus t with $l = 2$, $X_1 = 1$, $c_0 = 1$, $\rho_0^{(de)} = 1$ and $\beta = \lambda = 1$

a suitable model for describing the present evolution of the universe.

Case II. Solution with $n = 0$

In this case, using (36) into the quadrature equations (30)–(32), we obtain

$$A(t) = l_1 c_0 \exp\left[lt - \frac{X_1}{3lc_0^3}e^{-3lt} + \frac{2\beta\rho_0^{(de)}}{9wl(1+w)c_0^{3(1+w)}}e^{-3l(1+w)t}\right], \quad (56)$$

$$B(t) = l_2 c_0 \exp\left[lt - \frac{X_2}{3lc_0^3}e^{-3lt} - \frac{\beta\rho_0^{(de)}}{9wl(1+w)c_0^{3(1+w)}}e^{-3l(1+w)t}\right], \quad (57)$$

$$C(t) = l_3 c_0 \exp\left[lt - \frac{X_3}{3lc_0^3}e^{-3lt} - \frac{\beta\rho_0^{(de)}}{9wl(1+w)c_0^{3(1+w)}}e^{-3l(1+w)t}\right], \quad (58)$$

where $w \neq -1$ and 0.

$$Q_1 = \frac{3\alpha X_1}{c_0{}^3}\exp(-3lt) - \frac{2\alpha\beta\rho_0^{(de)}}{3wc_0^{3(1+w)}}e^{-3l(1+w)t}. \quad (59)$$

The directional Hubble parameters are given by

$$H_x = l + \frac{X_1}{c_0^3}e^{-3lt} - \frac{2\beta\rho_0^{(de)}}{3wc_0^{3(1+w)}}e^{-3l(1+w)t}, \quad (60)$$

$$H_y = l + \frac{X_2}{c_0^3}e^{-3lt} + \frac{\beta\rho_0^{(de)}}{3wc_0^{3(1+w)}}e^{-3l(1+w)t}, \quad (61)$$

$$H_z = l + \frac{X_3}{c_0^3}e^{-3lt} + \frac{\beta\rho_0^{(de)}}{3wc_0^{3(1+w)}}e^{-3l(1+w)t}, \quad (62)$$

whereas the mean Hubble parameter is $H = l$. The kinematical parameters θ, σ^2 and $\bar{A}$ are

$$\theta = 3l \quad (63)$$

$$\sigma^2 = \frac{\lambda}{c_0^6}e^{-6lt} - \frac{X_1\beta\rho_0^{(de)}}{wc_0^{3(2+w)}}e^{-3l(2+w)t} + \frac{\beta^2\rho_0^{(de)2}}{3w^2c_0^{6(1+w)}}e^{-6l(1+w)t}, \quad (64)$$

$$\bar{A} = \frac{1}{3l^2}\left[\frac{\lambda}{c_0^6}e^{-6lt} - \frac{2X_1\beta\rho_0^{(de)}}{wc_0^{3(2+w)}}e^{-3l(2+w)t} + \frac{2\beta^2\rho_0^{(de)2}}{3w^2c_0^{6(1+w)}}e^{-6l(1+w)t}\right]. \quad (65)$$

The skewness and the directional EoS parameters of DE, respectively, are given by

$$\delta(t) = \beta\left[2l - \frac{X_1}{c_0^3}e^{-3lt} + \frac{2\beta\rho_0^{(de)}}{3wc_0^{3(1+w)}}e^{-3l(1+w)t}\right], \quad (66)$$

$$\gamma(t) = -\beta\left[l + \frac{X_1}{c_0^3}e^{-3lt} - \frac{2\beta\rho_0^{(de)}}{3wc_0^{3(1+w)}}e^{-3l(1+w)t}\right], \quad (67)$$

$$\omega_x = w + \beta\left[2l - \frac{X_1}{c_0^3}e^{-3lt} + \frac{2\beta\rho_0^{(de)}}{3wC_0^{3(1+w)}}e^{-3l(1+w)t}\right], \quad (68)$$

$$\omega_y = w_z = w - \beta\left[l + \frac{X_1}{c_0^3}e^{-3lt} - \frac{2\beta\rho_0^{(de)}}{3wc_0^{3(1+w)}}e^{-3l(1+w)t}\right]. \quad (69)$$

The energy-density and pressure of the DE have expressions

$$\rho^{(de)} = \rho_0^{(de)}c_0^{-3}e^{-3l(1+w)t}, \quad (70)$$

$$p^{(de)} = w\rho_0^{(de)}c_0^{-3}e^{-3l(1+w)t}. \quad (71)$$

The energy density and pressure of the matter component are given by

$$\rho^{(m)} = 3l^2 - \frac{\lambda}{c_0^6}e^{-6lt} + \frac{X_1\beta\rho_0^{(de)}}{wc_0^{3(2+w)}}e^{-3l(2+w)t} - \frac{\beta^2{\rho_0^{(de)}}^2}{3w^2c_0^{6(1+w)}}e^{-6l(1+w)t} - \frac{\rho_0^{(de)}}{c_0^3}e^{-3l(1+w)t} - 3\alpha^2A^{-2}, \tag{72}$$

$$p^{(m)} = -3l^2 - \frac{\lambda}{c_0^6}e^{-6lt} + \frac{(1+w)X_1\beta\rho_0^{(de)}}{wc_0^{3(2+w)}}e^{-3l(2+w)t} + \frac{(2w-1)\beta^2{\rho_0^{(de)}}^2}{3w^2c_0^{6(1+w)}}e^{-6l(1+w)t} - \frac{\rho_0^{(de)}}{c_0^3}e^{-3l(1+w)t} + \alpha^2A^{-2}. \tag{73}$$

It can easily be seen that the equations (18) and (24) are identically satisfied. We observe that the physical and kinematical quantities are all constants at $t = 0$. This shows that the universe starts evolving with constant volume and expands exponentially. We find that the directional Hubble parameters are time-dependent while the mean Hubble parameter is constant. The expansion scalar is constant throughout the time of evolution. The heat flow is constant at $t = 0$ and then decreases at faster rate due to DE component. We observe that the DE term appears in exponential form in the scale factors and thus affects their evolution significantly. In the phantom region for $\beta < 0$, the DE contributes to the expansion of $A(t)$ while opposing to the expansion of $B(t)$ and $C(t)$, whereas in the quintessence region, it opposes the expansion of $A(t)$ while contributing to the expansion of $B(t)$ and $C(t)$. Thus, the spatial geometry of the universe is affected by the anisotropic DE. The difference between the directional EoS parameters of the DE and hence the pressures of the DE, along x-axis and y-axis (or z-axis) is $3\beta l$, which is constant throughout the evolution of the universe. Therefore the anisotropy of the DE does not vanish during the evolution of universe.

As $t \to \infty$, the scale factors and volume of the universe become infinitely large whereas the skewness and directional EoS parameters, directional Hubble parameters, anisotropy parameter and shear scalar become constants. The flow diminishes as $t \to \infty$. The pressure and energy density of the perfect fluid component also become constants and show the relation $p^{(m)} = -\rho^{(m)}$. The density of DE (Fig. 3) and anisotropy parameter shown in Fig. 4 exhibit a behavior consistent with the present observations in the quintessence model.

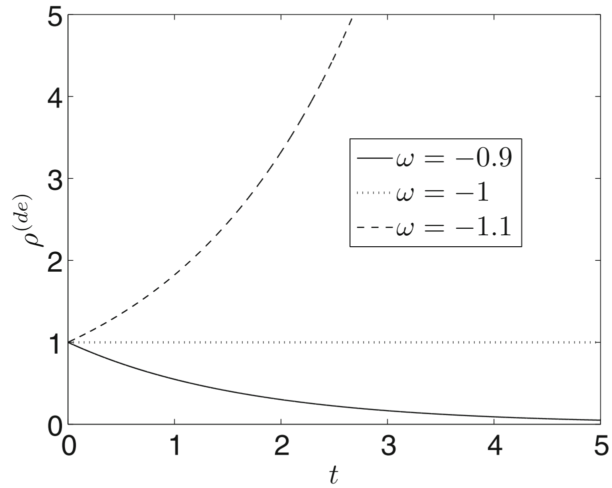

Fig. 3 Plot of $\rho^{(de)}$ versus t with $l = 2$, $c_0 = 1$ and $\rho_0^{(de)} = 1$

For $n = 0$, we get $q = -1$; incidentally this value of deceleration parameter leads to $\frac{dH}{dt} = 0$, which implies the greatest value of Hubble's parameter and the fastest rate of expansion of the universe. Therefore the universe exhibits the fastest possible rate of the expansion among the all possible values of n.

4 Thermodynamics Relations

Baryon conservation law In standard cosmology, conservation of total particle number gives

$$N^{\mu}_{;\mu} = 0, \tag{74}$$

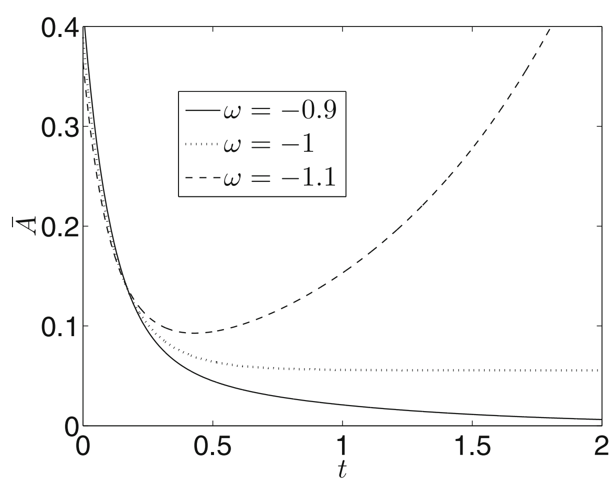

Fig. 4 Plot of $\bar{A}$ versus t with $l = 2$, $X_1 = 1$, $c_0 = 1$, $\rho_0^{(de)} = 1$ and $\beta = \lambda = 1$

where $N^\mu = \chi u^\mu$ is the particle flux and χ is the particle number density, which is given by

$$\frac{d\chi}{dt} = -\chi\theta. \quad (75)$$

For the power-law solutions, the particle number density is

$$\chi = b_1 (nlt)^{-3/n}, \quad (76)$$

and for the exponential solutions ($n = 0$), we get

$$\chi = \frac{b_2}{c^3} \exp(-3lt), \quad (77)$$

where b_1 and b_2 are constants of integration. We observe that the particle number density is large at $t = 0$ in the case of $n \neq 0$ and for $n = 0$, the particle density is constant at $t = 0$.

Temperature gradient law The usual expression for heat conduction [31] is

$$Q^\mu = -\kappa \left(g^{\mu\nu} + u^\mu u^\nu\right)\left(T_{,\nu} + T u_{\nu;\alpha} u^\alpha\right), \quad (78)$$

where $\kappa \geq 0$ is the heat conduction coefficient, i.e., thermal conductivity, T is the temperature, and $u_{\nu;\alpha}u^\alpha$ is the acceleration. Since in our case only the x-component of heat flux is retained, from above equation we obtain $Q_1 = \kappa T_{;1}$.

The temperature distribution for $n \neq 0$ and $n = 0$ respectively are given in the forms:

$$T = \frac{3\alpha X_1 (nlt)^{-\frac{3}{n}} - \frac{2\alpha\beta\rho_0^{(de)}}{w}(nlt)^{-\frac{3(1+w)}{n}}}{\kappa(t)} x + \eta_1(t), \quad (79)$$

$$T = \frac{\frac{3\alpha X_1}{{c_0}^3}\exp(-3lt) - \frac{2\alpha\beta\rho_0^{(de)}}{3wc_0^{3(1+w)}} e^{-3l(1+w)t}}{\kappa(t)} x + \eta_2(t), \quad (80)$$

where $\eta_1(t)$ and $\eta_2(t)$ appear as integration functions, which may either be an arbitrary functions of time or constants. We observe that T diverges at the initial epoch as long as the coefficient of thermal conductivity remains finite. At the final stage of expansion $t \to \infty$, we have $T \to \eta_1(t)$ in case of $n \neq 0$ and $T \to \eta_2(t)$ in case of $n = 0$, which implies that the universe will be in thermal equilibrium at the final stage of evolution.

5 Conclusion

We have studied anisotropic Bianchi type -V space -time with dynamically anisotropic dark energy and perfect fluid with heat flow in general relativity. We have assumed that the DE is minimally interacting, has dynamical energy density and anisotropic equation of state. A more general EoS parameter has been introduced for DE, and the isotropic DE can be recovered by choosing β to be null. The field equations have been solved by taking the constant deceleration parameter. Exact solutions of Einstein field equations have been obtained for Bianchi type V space-time in two types of cosmologies. In the cosmology with the power-law variation of the average scale factor, the solution corresponds to a cosmological model which starts expanding from the singular state with positive deceleration parameter. In the case of exponential variation of the average scale factor we have presented an accelerating non-singular model of the Universe. We have also discussed the physical behaviors of the models and the dynamics of the anisotropic DE in two types of cosmologies.

We have observed that the ultimate spatial geometry of the Universe is determined by the DE component. In all the models the anisotropic DE contributes to the expansion of one (or two) of the scale factors while it opposes the expansion of the other two (or one) scale factors, leading to the Universe to have prolate or oblate geometry. We have found that the quintessence model is consistent with the present and expected future evolution of the Universe. The other models, viz., phantom and dark energy vacuum models possibly represent relatively earlier epochs of the Universe. We can examine the possibilities that arise due to (22) and (23) with the vacuum energy (which is mathematically equivalent to the cosmological constant Λ and can be represented with EoS parameter $p^{(de)} = -\rho^{(de)}$ by taking $w = -1$). We found that energy density of DE component is constant, i.e., $\rho^{(de)} = const.$ and we can get the solutions accordingly from field equations by similar method. For $\rho^{(de)} = 0$, i.e., in the absence of DE, we get the solutions presented in our earlier work [30] on general relativity.

References

1. A.G. Riess, et al, Astron. J. **568**, 38 (2002)
2. S. Perlmutter, et al, Astrophys. J. **517**, 565 (1999)
3. J.L. Tonry, et al, Astrophys. J. **594**, 1 (2003)
4. C.L. Bennett, et al, Astrophys. J. Suppl. Ser. **148**, 1 (2003)
5. C.B. Netterfield, et al, Astrophys. J. **571**, 604 (2002)
6. N.W. Halverson, et al, Astrophys. J. **568**, 38 (2002)
7. S. Carroll, et al, Phys. Rev. **D68**, 023509 (2003)
8. A. Melchiorri, L.C.J. Mersini Odman, M. Trodden, Phys. Rev. **D68**, 043509 (2003)
9. R.R. Caldwell, Phys. Lett. **B545**, 23 (2002)
10. P.J. Steinhardt, N. Turok, Science **312**, 1180 (2006)
11. M. Ishak, Found. Phys. **37**, 1470 (2007)
12. Q-guo Haung, Phys. Rev. **D77**, 103518 (2008)
13. B. Ratra, P.J.E. Peebles, Ap. J. Lett. **325**, 117 (1998)
14. R. Das, T.W. Kephart, R.J. Scherrer, Phys. Rev. **D74**, 103515 (2006)

15. C. Armendariz-Picon, V. Mukhanov, P.J. Steinhardt, Phys. Rev. **D63**, 103510 (2001)
16. R.R. Caldwell, et al, Phys. Rev. Lett. **97**, 071301 (2003)
17. Y.-F. Cai, H. Li, Y.-S. Piao, X. Zhang, Phys. Lett. **B646**, 141 (2007)
18. Y.-F. Cai, M. Li, J.-X. Lu, Y.-S. Piao, T. Qiu, X. Zhang, Phys. Lett. **B651**, 1 (2007)
19. E.V. Linder, New Astron. Rev. **49**, 93 (2005)
20. M.R. Setare, Eur. Phys. J. **C50**, 991 (2007)
21. T. Padmanabhan, Phys. Rev. **D66**, 021301 (2002)
22. J.D. Barrow, Class. Quantum Gravity **28**, L79 (2004)
23. *Ö*. Akarsu, C.B. Kilinc, Gen. Relativ. Gravit. **42**, 119 (2010)
24. A.A. Coley, Gen. Relativ. Gravit. **22**, 3 (1990)
25. A.A. Coley, R.J. Hoogan, J. Math. Phys. **35**, 4117 (1994)
26. A.A. Coley, K. Dunn, J. Math. Phys. **33**, 1772 (1992)
27. B. Saha, V. Rikhvitsky, Physica D **219** 168 (2006)
28. M.S. Berman, IL Nuovo Cimento B **74**, 182 (1983)
29. M.S. Berman, F.M. Gomide, Gen. Relativ. Gravit. **20** 191 (1988)
30. C.P. Singh, ShriRam, Mohd. Jeyauddin, Astrophys. Space Sci. **315**, 181 (2008)
31. C. Eckart, Phys. Rev. **58**, 919 (1940)