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PARTICLES AND FIELDS

Two Moving-Angled 1-Branes with Electric Fields in a Partially Compact Spacetime

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Abstract In this article, we consider two $m1$-branes at angle in the presence of the background electric fields, in a partially compact spacetime. The branes have motions along a common direction that is perpendicular to both of them. Using the boundary state formalism, we calculate their interaction amplitude. Some special cases of this interaction will be studied in detail.

Keywords Moving-angled branes · Background fields · Compactification · Interaction

1 Introduction

In 1995 it is realized that open strings with Dirichlet boundary conditions can end on D-branes [1]. Two D-branes which open string is stretched between them can interact. It is known that one of the best methods for finding some properties is calculation of the amplitude of interaction. This amplitude was obtained by one loop open string diagram. However, this is equivalent to a tree-level diagram in the closed string exchange [2]. A closed string is generated from the vacuum, propagates for a while, and then annihilates again in the vacuum. The state which describes the creation (annihilation) of closed string from (in) the vacuum is called boundary state [3]. So the boundary state formalism is a strong tool for calculating the amplitude of interaction of the branes, e.g., see [4–13] and references therein.

In the case of Dp-branes with nonzero background and internal gauge fields, the boundary state formalism is an effective method for calculating the amplitude of their interaction. For a closed string emitted (absorbed) by a Dp-brane in the presence of the background field $B_{\mu\nu}$ and $U(1)$ gauge field A_α (which lives on the brane), there are mixed boundary conditions. This Dp-brane is called mp-brane [10–13].

Previously we studied the interaction of two stationary $m1$-branes at angle [12]. In addition, we considered moving mixed branes [13]. For both cases, spacetime is compact. Now we are motivated to study the effects of both cases simultaneously on the interaction amplitude of the branes. For example, we find some situations in which the branes do not interact. Therefore, we consider a system of moving and angled $m1$-branes in the partially compacted spacetime on a torus. At first the boundary state associated with a moving $m1$-brane, which makes an angle with the X^1-direction, will be obtained. It is parallel to the X^1X^2-plane and contains an electrical field along itself. Then the interaction amplitude of the system $m1$–$m1'$ branes will be obtained. The angle between the branes is ϕ. The branes move along the X^3-direction with the velocities V_1 and V_2. Various properties of the interaction amplitude of this system will be analyzed. The large distance behavior of the amplitude, which reveals the contribution of the closed string massless states on the interaction, will be obtained.

This paper is organized as follows: In Section 2, we obtain the boundary state corresponding to an oblique moving $m1$-brane. In Section 3, we obtain the amplitude of interaction via the overlapping of two boundary states. In Section 4, we suppose these $m1$-branes are located at large distance. Thus, the contribution of

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the massless states on the interaction will be studied. Section 5 is devoted to the conclusions.

2 Boundary State of an Oblique Moving $m1$-Brane

We suppose that an $m1$-brane with the electric field E_2 along it moves with the velocity V_2 along the direction X^3, while makes an angle θ_2 with the X^1-direction. It is parallel to the X^1X^2-plane. In our notations, the index 2 in $E_2, V_2, \cdots$, refers to the second $m1$-brane. Similarly, we consider $E_1, V_1, \cdots$, for the first $m1$-brane. Note that in this article the signature of the metric is $\eta_{\mu\nu} = diag(-1, 1, \cdots, 1)$.

Previously we obtained the boundary state for a moving mp-brane [13]. In the corresponding boundary state equations, we consider $p = 1$ and then rotate the $m1$-brane to make an angle θ_2 with the X^1-direction. After this process, the boundary state equations, associated with the moving-angled $m1$-brane, take the form

$$\left[\partial_\tau X^0 - V_2\partial_\tau X^3 - E_2\cos\theta_2\partial_\sigma X^1 - E_2\sin\theta_2\partial_\sigma X^2\right]_{\tau_0} \times \left|B_x{}^2, \tau_0\right\rangle = 0, \quad (1)$$

$$\left[\cos\theta_2\partial_\tau X^1 + \sin\theta_2\partial_\tau X^2 - E_2\partial_\sigma X^0\right]_{\tau_0}\left|B_x{}^2, \tau_0\right\rangle = 0, \quad (2)$$

$$\left[X^3 - V_2X^0 - y^3{}_{(2)}\right]_{\tau_0}\left|B_x{}^2, \tau_0\right\rangle = 0, \quad (3)$$

$$\left[-\left(X^1 - y^1_{(2)}\right)\sin\theta_2 + \left(X^2 - y^2_{(2)}\right)\cos\theta_2\right]_{\tau_0}\left|B_x{}^2, \tau_0\right\rangle = 0, \quad (4)$$

$$\left(X^j - y^j_{(2)}\right)_{\tau_0}\left|B^2_x, \tau_0\right\rangle = 0, \qquad j \neq 0, 1, 2, 3. \quad (5)$$

The mode expansion of $X^\mu(\sigma, \tau)$ is

$$X^\mu(\sigma,\tau) = x^\mu + 2\alpha' p^\mu \tau + 2L^\mu\sigma + \frac{i}{2}\sqrt{2\alpha'} \times \sum_{m\neq 0}\frac{1}{m}\left(\alpha^\mu_m e^{-2im(\tau-\sigma)} + \tilde{\alpha}^\mu_m e^{-2im(\tau+\sigma)}\right), \quad (6)$$

where L^μ is zero for the non-compact directions. For a compact direction, there are $L^\mu = N^\mu R^\mu$ and $p^\mu = \frac{M^\mu}{R^\mu}$, where N^μ and M^μ are winding number and momentum number of the emitted (absorbed) closed string from the brane, respectively. R^μ also is the radius of compactification of the compact direction X^μ.

After replacing the mode expansion of X^μ into the (1)–(5), these equations will be written in terms of the oscillators. The zero mode part of the boundary state equations become

$$\left[p^0 - V_2p^3 - \frac{1}{\alpha'}E_2\left(L^2\sin\theta_2 + L^1\cos\theta_2\right)\right]_{op} \times \left|B_x{}^2, \tau_0\right\rangle = 0, \quad (7)$$

$$\left[p^1\cos\theta_2 + p^2\sin\theta_2 - \frac{1}{\alpha'}E_2L^0\right]_{op}\left|B_x{}^2, \tau_0\right\rangle = 0, \quad (8)$$

$$\left[-\left(x^1 - y^1_{(2)} + 2\alpha'\tau_0p^1\right)\sin\theta_2 + \left(x^2 - y^2_{(2)} + 2\alpha'\tau_0p^2\right)\cos\theta_2\right]_{op}\left|B_x{}^2, \tau_0\right\rangle = 0, \quad (9)$$

$$\left[L^2\cos\theta_2 - L^1\sin\theta_2\right]_{op}\left|B_x{}^2, \tau_0\right\rangle = 0, \quad (10)$$

$$\left[x^3 + 2\alpha'\tau_0p^3 - y^3_{(2)} - V_2\left(x^0 + 2\alpha'\tau_0p^0\right)\right]_{op} \times \left|B_x{}^2, \tau_0\right\rangle = 0, \quad (11)$$

$$\left(L^3 - V_2L^0\right)_{op}\left|B_x{}^2, \tau_0\right\rangle = 0, \quad (12)$$

$$\left[x^j + 2\alpha'p^j\tau_0 - y^j_{(2)}\right]_{op}\left|B_x{}^2, \tau_0\right\rangle = 0, \quad (13)$$

$$(L^j)_{op}\left|B_x{}^2, \tau_0\right\rangle = 0. \quad (14)$$

For the oscillating part, the equations of the boundary state are as in the following:

$$\left[\left(\alpha^0_m - V_2\alpha^3_m + E_2\left(\alpha^2_m\sin\theta_2 + \alpha^1_m\cos\theta_2\right)\right)e^{-2im\tau_0} + \left(\tilde{\alpha}^0_{-m} - V_2\tilde{\alpha}^3_{-m} - E_2\left(\tilde{\alpha}^2_{-m}\sin\theta_2 + \tilde{\alpha}^1_{-m}\cos\theta_2\right)\right)e^{2im\tau_0}\right] \times \left|B_x{}^2, \tau_0\right\rangle = 0, \quad (15)$$

$$\left[\left(E_2\alpha^0_m + \alpha^1_m\cos\theta_2 + \alpha^2_m\sin\theta_2\right)e^{-2im\tau_0} + \left(-E_2\tilde{\alpha}^0_{-m} + \tilde{\alpha}^1_{-m}\cos\theta_2 + \tilde{\alpha}^2_{-m}\sin\theta_2\right)e^{2im\tau_0}\right] \times \left|B_x{}^2, \tau_0\right\rangle = 0, \quad (16)$$

$$\left[\left(-\alpha_m^1 \sin\theta_2 + \alpha_m^2 \cos\theta_2\right) e^{-2im\tau_0} + \left(\tilde{\alpha}_{-m}^1 \sin\theta_2 - \tilde{\alpha}_{-m}^2 \cos\theta_2\right) e^{2im\tau_0}\right] \left|B_x{}^2, \tau_0\right\rangle = 0, \tag{17}$$

$$\left[\left(-V_2\alpha_m^0 + \alpha_m^3\right) e^{-2im\tau_0} + \left(V_2\tilde{\alpha}_{-m}^0 - \tilde{\alpha}_{-m}^3\right) e^{2im\tau_0}\right] \left|B_x{}^2, \tau_0\right\rangle = 0, \tag{18}$$

$$\left[\alpha_m^j e^{-2im\tau_0} - \tilde{\alpha}_{-m}^j e^{2im\tau_0}\right] |B_x{}^2, \tau_0\rangle = 0, \quad j \in \{4, \cdots, d-1\}. \tag{19}$$

These equations can be collected in a single equation, i.e.,

$$\left(\alpha_m^\mu e^{-2im\tau_0} + S_{(2)}{}^\mu{}_\nu \tilde{\alpha}_{-m}^\nu e^{2im\tau_0}\right) |B_x{}^2, \tau_0\rangle = 0, \tag{20}$$

where the matrix $S_{(2)}{}^\mu{}_\nu$ is defined by

$$S_{(2)}{}^\mu{}_\nu = \begin{pmatrix} \Omega_{(2)}{}^p{}_q & 0 \\ 0 & -I_{(d-4)\times(d-4)} \end{pmatrix}, \quad p, q \in \{0, 1, 2, 3\}. \tag{21}$$

The matrix $\Omega_{(2)}{}^p{}_q$ also has the definition

$$\Omega_{(2)}{}^p{}_q = \frac{1}{1 - V_2^2 - E_2^2} \begin{bmatrix} 1 + V_2^2 + E_2^2 & -2E_2\cos\theta_2 & -2E_2\sin\theta_2 & -2V_2 \\ -2E_2\cos\theta_2 & \left(1 - V_2^2\right)\cos 2\theta_2 + E_2^2 & \left(1 - V_2\right)\sin 2\theta_2 & 2V_2E_2\cos\theta_2 \\ -2E_2\sin\theta_2 & \left(1 - V_2^2\right)\sin 2\theta_2 & -\left[\left(1 - V_2^2\right)\cos 2\theta_2 - E_2^2\right] & 2V_2E_2\sin\theta_2 \\ 2V_2 & -2V_2E_2\cos\theta_2 & -2V_2E_2\sin\theta_2 & -\left(1 - E_2^2 + V_2^2\right) \end{bmatrix}. \tag{22}$$

According to $(\Omega_{(2)}{}^T)^p{}_q = \eta^{pp}\eta_{qq}\Omega_{(2)}{}^q{}_p$, the matrix $\Omega_{(2)}$ is orthogonal, and hence, $S_{(2)}$ also is an orthogonal matrix.

By solving the (7)–(14) and (20), the boundary state will be obtained

$$\begin{aligned} |B_x{}^2, \tau_0\rangle &= \frac{T}{2}\sqrt{1 - V_2^2 - E_2^2} \\ &\times \exp\left[i\alpha'\tau_0\left(\gamma_2^2\left(p_{op}^3 - V_2p_{op}^0\right)^2 + \left(-p_{op}^1\sin\theta_2 + p_{op}^2\cos\theta_2\right)^2 + \sum_{j=4}^{d-1}\left(p_{op}^j\right)^2\right)\right] \\ &\times \delta\left[-\left(x^1 - y_{(2)}^1\right)\sin\theta_2 + \left(x^2 - y_{(2)}^2\right)\cos\theta_2\right] \\ &\times \delta\left(x^3 - y_{(2)}^3 - V_2x^0\right)\prod_{j=4}^{d-1}\delta\left(x^j - y_{(2)}^j\right) \\ &\times \sum_{p^0}\sum_{p^1}\sum_{p^2}|p^0\rangle|p^1\rangle|p^2\rangle\prod_{j=4}^{d-1}|p_L^j = p_R^j = 0\rangle \\ &\times |p_L^3 = p_R^3 = \frac{1}{2}V_2p^0\rangle \\ &\times \exp\left[-\sum_{m=1}^{\infty}\left(\frac{1}{m}e^{4im\tau_0}\alpha_{-m}^\mu S_{\mu\nu}^{(2)}\tilde{\alpha}_{-m}^\nu\right)\right]|0\rangle, \end{aligned} \tag{23}$$

where $\gamma_2 = 1/\sqrt{1 - V_2^2}$ and $T = \frac{\sqrt{\pi}}{2^{(d-10)/4}}(4\pi^2\alpha')^{(d-6)/4}$ is tension of the $m1$-brane which lives in the d-dimensional spacetime. The momentum components of the closed string that are appeared in (23) are given by

$$p^0 = \frac{\gamma_2^2}{\alpha'}E_2\left(\ell^2\sin\theta_2 + \ell^1\cos\theta_2\right), \tag{24}$$

$$p^1 = \frac{E_2}{\alpha'}\ell^0\cos\theta_2, \tag{25}$$

$$p^2 = \frac{E_2}{\alpha'}\ell^0\sin\theta_2, \tag{26}$$

$$p^3 = \frac{\gamma_2^2V_2}{\alpha'}E_2\left(\ell^2\sin\theta_2 + \ell^1\cos\theta_2\right), \tag{27}$$

where $p^\mu = p_L^\mu + p_R^\mu$ and $\ell^\mu = \alpha'(p_L^\mu - p_R^\mu) = N^\mu R^\mu$. We should consider (24)–(26) for summing over p^0, p^1, and p^2 in (23). Therefore, these summations convert to the winding numbers N^0, N^1, and N^2. The (24) implies that energy of the closed string is quantized and depends on its winding numbers around the X^1- and X^2-directions. However, the (24)–(27) imply that the momentum numbers of the closed string M^0, M^1, M^2, and M^3 are related to its winding numbers N^0, N^1, and N^2.

Equations (10), (12), and (14) also lead to the relations

$$\ell^2 \cos\theta_2 = \ell^1 \sin\theta_2, \tag{28}$$

$$\ell^3 = V_2 \ell^0, \tag{29}$$

$$\ell^j = 0. \tag{30}$$

We can write the (28) in the form

$$N^2 R^2 \cos\theta_2 = N^1 R^1 \sin\theta_2. \tag{31}$$

This equation tells us that only when $\frac{R^1 \sin\theta_2}{R^2 \cos\theta_2}$ is rational, closed string can wrap around X^1- and X^2-directions; otherwise, $N^1 = N^2 = 0$ and closed string has no winding around X^1 and X^2. In this case, its energy also is zero. In the same way, by the (29), for having winding around X^3 and X^0, the quantity $\frac{V_2 R^0}{R^3}$ also should be rational.

The ghost part of the boundary state is independent of the electric field E_2, the velocity V_2, and the angle θ_2. It is given by

$$\begin{aligned}|B_{gh}, \tau_0\rangle &= \exp\left[\sum_{m=1}^{\infty} e^{4im\tau_0}\left(c_{-m}\tilde{b}_{-m} - b_{-m}\tilde{c}_{-m}\right)\frac{c_0 + \tilde{c}_0}{2}\right] \\ &\quad \times |q = 1\rangle |\tilde{q} = 1\rangle. \end{aligned}\tag{32}$$

3 Interaction Between Two m1-Branes

Before calculation of the interaction amplitude, let us introduce some notations for the positions of these two mixed branes. Similar to the $m1$-brane, the $m1'$-brane also is parallel to the X^1X^2-plane and makes angle θ_1 with the X^1-direction and moves with the speed V_1 along the X^3-direction. The electric field on it also is E_1. The common direction of motions is X^3, and the other directions perpendicular to the world volume of both branes are $\{X^j | j \neq 0, 1, 2, 3\}$. We use the set $\{X^{j_n}\}$ to denote the non-compact part of $\{X^j\}$, and $\{X^{j_c}\}$ is for the compact part of $\{X^j\}$.

Now we can calculate the overlap of the two boundary states to obtain the interaction amplitude of the branes. The complete boundary state for each brane is

$$|B\rangle = |B_x\rangle |B_{gh}\rangle. \tag{33}$$

These two mixed branes simply interact via exchange of closed strings so the amplitude is given by

$$\mathcal{A} = {}^{(1)}\langle B, \tau_0 = 0 | D | B, \tau_0 = 0\rangle^{(2)}, \tag{34}$$

where "D" is the closed string propagator. The calculation is straightforward but tedious. Here we only write the final result

$$\begin{aligned}\mathcal{A} = &\frac{T^2 \alpha' L}{4(2\pi)^{d-4} |\sin\phi|\, |V_1 - V_2|} \\ &\times \sqrt{\left(1 - V_1^2 - E_1^2\right)\left(1 - V_2^2 - E_2^2\right)} \\ &\times \int_0^\infty dt \Bigg\{ e^{(d-2)t/6} \left(\sqrt{\frac{\pi}{\alpha' t}}\right)^{d_{jn}} \\ &\quad \times \prod_{j_n} \exp\left(-\frac{\left(y_{(1)}^{j_n} - y_{(2)}^{j_n}\right)^2}{4\alpha' t}\right) \\ &\quad \times \prod_{j_c} \Theta_3\left(\frac{y_{(1)}^{j_c} - y_{(2)}^{j_c}}{2\pi R_{j_c}} \middle| \frac{i\alpha' t}{\pi (R_{j_c})^2}\right) \\ &\quad \times \sum_{N^0} \sum_{N^1} \sum_{N^2} \\ &\quad \times \Bigg[\exp\Bigg[-\frac{t}{\alpha'}\Big(\ell^0\ell^0 + \left(\ell^1 \cos\theta_1 + \ell^2 \sin\theta_1\right) \\ &\qquad \times \left(\ell^1 \cos\theta_2 + \ell^2 \sin\theta_2\right) \\ &\qquad + F^{(+)} F^{(-)}\Big) \\ &\qquad + \frac{i}{\alpha'}\left(\Phi(12) y_{(2)}^3 - \Phi(21) y_{(1)}^3\right)\Bigg]\Bigg] \\ &\quad \times \Theta_3(\nu|\tau) \prod_{n=1}^{\infty}\left[\det\left(1 - \Omega_1 \Omega_2^T e^{-4nt}\right)\right]^{-1} \\ &\quad \times \left(1 - e^{-4nt}\right)^{6-d} \Bigg\}, \end{aligned}\tag{35}$$

where $L = 2\pi R_0$ and $\Phi(12)$ and $F^{(\pm)}$ are defined by

$$\begin{aligned}\Phi(12) &= \frac{1}{V_2 - V_1}\Big[\gamma_1^2 E_1 \left(V_1^2 + 1\right)\left(\ell^2 \sin\theta_1 + \ell^1 \cos\theta_1\right) - \gamma_2^2 E_2 \\ &\quad \times \left(1 + V_1 V_2\right)\left(\ell^2 \sin\theta_2 + \ell^1 \cos\theta_2\right)\Big], \end{aligned}\tag{36}$$

$$\begin{aligned}F^{(\pm)} &= \frac{1}{|V_1 - V_2|} \\ &\quad \times \Big[\gamma_2^2 \left(1 \pm V_1\right)\left(1 + V_2^2\right) E_2 \left(\ell^2 \sin\theta_2 + \ell^1 \cos\theta_2\right) \\ &\quad - \gamma_1^2 \left(1 \pm V_2\right)\left(1 + V_1^2\right) \\ &\quad \times E_1 \left(\ell^2 \sin\theta_1 + \ell^1 \cos\theta_1\right)\Big]. \end{aligned}\tag{37}$$

We can obtain $\Phi(21)$ by exchanging $1 \longleftrightarrow 2$ in (36). In addition, $\phi = \theta_2 - \theta_1$ and ν and τ also have the definitions

$$\nu = \frac{R_0}{2\pi\alpha' \sin\phi} \times \left[(E_2 - E_1 \cos\phi)\, \bar{y}^2_{(1)} + (E_1 - E_2\cos\phi)\, \bar{y}^2_{(2)} \right],$$

$$\tau = \frac{itR_0^2}{\pi\alpha'} \left(\frac{E_1^2 + E_2^2 - 2E_1E_2\cos\phi}{\sin^2\phi} - 1 \right). \tag{38}$$

The set $\{\bar{y}^2_{(2)}, y^3_{(2)}, \cdots, y^{(d-1)}_{(2)}\}$ shows the position of the $m1$-brane, with $\bar{y}^2_{(2)} = -y^1_{(2)}\sin\theta_2 + y^2_{(2)}\cos\theta_2$ and $y^1_{(2)}\cos\theta_2 + y^2_{(2)}\sin\theta_2 = 0$, similarly for the $m1'$-brane. We observe that the interaction amplitude not only depends on the relative angle ϕ between the branes but also depends on the configuration angles of the branes, i.e., θ_1 and θ_2.

Because of the electric fields, this amplitude is not symmetric under the change $\phi \to \pi - \phi$. Therefore, for the angled mixed branes, ϕ and $\pi - \phi$ indicate two different configurations. From (38), we see that the electric fields and compactification of the time direction cause $\bar{y}^2_{(2)}$ and $\bar{y}^2_{(1)}$ to appear in the interaction. In addition, the amplitude (35) is symmetric with respect to the $m1$ and $m1'$- branes, i.e.,

$$\mathcal{A}\,(V_1, V_2; E_1, E_2; \theta_1, \theta_2; y_1, y_2) = \mathcal{A}^*\,(V_2, V_1; E_2, E_1; \theta_2, \theta_1; y_2, y_1)\,. \tag{39}$$

For complex conjugation, see (34). Finally, when the electric fields and the angle of the branes satisfy the equation $\tau = 0$, there is no interaction between the branes. This fact is independent of the velocities of the branes and spacetime compactification.

For non-compact spacetime, remove all factors Θ_3 from (35). In addition, use $\ell^0 = \ell^1 = \ell^2 = 0$ and change $j_n \to j$, and hence, $d_{j_n} \to d-4$. So the interaction amplitude in the non-compact spacetime is as in the following:

$$\mathcal{A}_{\text{non-compact}} = \frac{T^2\alpha' L}{4(2\pi)^{d-4}|\sin\phi||V_1 - V_2|} \times \sqrt{\left(1 - V_1^2 - E_1^2\right)\left(1 - V_2^2 - E_2^2\right)} \times \int_0^\infty dt \left\{ \left(e^{(d-2)t/6} \left(\sqrt{\frac{\pi}{\alpha' t}} \right)^{d-4} \times \exp\left(-\sum_{j=4}^{d-1} \frac{(y^j_{(1)} - y^j_{(2)})^2}{4\alpha' t} \right) \times \prod_{n=1}^{\infty} \left[\left(\det\left(1 - \Omega_1\Omega_2^T e^{-4nt}\right)\right)^{-1} \times \left(1 - e^{-4nt}\right)^{6-d} \right] \right) \right\}. \tag{40}$$

This interaction depends on the minimal distance between the branes, that is, $\sum_{j=4}^{d-1}\left(y^j_{(1)} - y^j_{(2)}\right)^2$.

4 Large Distance Branes

Now we extract the contribution of the massless states in the interaction. As the metric $G_{\mu\nu}$, anti-symmetric tensor $B_{\mu\nu}$, and dilaton Φ have zero winding and zero momentum numbers, only the term with $N^0 = N^1 = N^2 = 0$ corresponds to these massless states. By using the identity $\det M = e^{Tr(\ln M)}$ for a matrix M, we obtain the following limit for $d = 26$:

$$\lim_{q\to 0} \frac{1}{q} \prod_{n=1}^{\infty} \left(\left[\det\left(1 - \Omega_1\Omega_2^T q^n\right)\right]^{-1} \left(1 - q^n\right)^{-20} \right) = \lim_{q\to 0}\frac{1}{q} + \operatorname{Tr}\left(\Omega_1\Omega_2^T\right) + 20, \tag{41}$$

where $q = e^{-4t}$. Put away the tachyon divergence, the contribution of the massless states is given by

$$\mathcal{A}^{(0)} = \frac{T^2\alpha' L}{4(2\pi)^{22}|\sin\phi||V_1 - V_2|} \times \sqrt{\left(1 - V_1^2 - E_1^2\right)\left(1 - V_2^2 - E_2^2\right)} \times \left[\operatorname{Tr}\left(\Omega_1\Omega_2^T\right) + 20\right] G,$$

$$G \equiv \int_o^\infty dt \left\{ \left(\sqrt{\frac{\pi}{\alpha' t}} \right)^{d_{j_n}} \prod_{j_n} \exp\left(-\frac{\left(y^{j_n}_{(1)} - y^{j_n}_{(2)}\right)^2}{4\alpha' t} \right) \times \prod_{j_c} \Theta_3\left(\frac{y^{j_c}_{(1)} - y^{j_c}_{(2)}}{2\pi R_{j_c}} \middle| \frac{i\alpha' t}{\pi\left(R_{j_c}\right)^2} \right) \Theta_3(\nu|\tau) \right\}. \tag{42}$$

For the non-compact spacetime, this amplitude reduces to

$$\mathcal{A}^{(0)}_{\text{non-compact}} = \frac{T^2\alpha' L}{4(2\pi)^{22}|\sin\phi||V_1 - V_2|} \times \sqrt{\left(1 - V_1^2 - E_1^2\right)\left(1 - V_2^2 - E_2^2\right)} \times \left[\text{Tr}\left(\Omega_1\Omega_2^T\right) + 20\right] G_{22}\left(\bar{Y}^2\right), \quad (43)$$

where $\bar{Y}^2 = \sum_{j=4}^{25}\left(y_1^j - y_2^j\right)^2$ is the impact parameter and G_{22} is the Green's function of the 22-dimensional space.

When the velocities, angles, and electric fields of the branes satisfy the relation $\text{Tr}(\Omega_1\Omega_2^T) + 20 = 0$, the amplitudes (42) and (43) vanish. In this case, attractive force due to the graviton exchange is canceled by the repulsive force of the electric fields.

5 Conclusions

We obtained the boundary state, associated with an oblique moving $m1$- brane, parallel to the X^1X^2-plane. This state reveals that how electric field, velocity of the brane, obliqueness of the brane, compact part, and non-compact part of the spacetime affect the brane. For a closed string emitted (absorbed) by such brane, some of the momentum numbers have relations with the winding numbers.

We determined the interaction amplitude of two moving-angled $m1$-branes, which live in the partially compact spacetime. This amplitude depends on the electric fields, velocities of the branes, obliqueness of the branes, compact part, and non-compact part of the spacetime. In addition, this interaction contains the relative angle ϕ and configuration angle of each brane, i.e., θ_1 and θ_2. The electric fields along the branes imply that the cases ϕ and $\pi - \phi$ are two different systems.

We extracted contribution of the massless states (i.e., graviton, dilaton, and Kalb–Ramond fields) on the interaction. For the non-compact spacetime, this contribution is proportional to the Green's function of the 22-dimensional space.

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