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PARTICLES AND FIELDS

Fitting Cosmological Data to the Function $q(z)$ from GR Theory: Modified Chaplygin Gas

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Abstract In the Friedmann cosmology, the deceleration of the expansion q plays a fundamental role. We derive the deceleration as a function of redshift $q(z)$ in two scenarios: ΛCDM model and modified Chaplygin gas (MCG) model. The function for the MCG model is then fitted to the cosmological data in order to obtain the cosmological parameters that minimize χ^2. We use the Fisher matrix to construct the covariance matrix of our parameters and reconstruct the $q(z)$ function. We use Supernovae Ia, WMAP5, and BAO measurements to obtain the observational constraints. We determined the present acceleration as $q_0 = -0.65 \pm 0.19$ for the MCG model using the Union2 dataset of SNeIa, BAO, and CMB and $q_0 = -0.67 \pm 0.17$ for the Constitution dataset, BAO and CMB. The transition redshift from deceleration to acceleration was found to be around 0.80 for both datasets. We have also determined the dark energy parameter for the MCG model: $\Omega_{X0} = 0.81 \pm 0.03$ for the Union2 dataset and $\Omega_{X0} = 0.83 \pm 0.03$ using the Constitution dataset.

Keywords Modified Chaplygin Gas · Observational constraints

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1 Introduction

During the last decade, the observation of type Ia supernovae (SNeIa) and the cosmic microwave background radiation permitted the determination of the cosmological parameters with ever increasing precision. The reported results of the 7-year analysis of WMAP [1] are $\Omega_\Lambda = 0.734 \pm 0.029$ and $\Omega_m = 0.266 \pm 0.029$; these values were obtained assuming a flat geometry ($\Omega_{k0} = 0$). From measured luminosity distances to SNeIa, Riess et al. [2, 3] determined the redshift of the transition from decelerated to accelerated expansion to be $z_t = 0.46 \pm 0.13$; this value was obtained assuming a linear expansion for the deceleration parameter, that is, $q(z) = q_0 + q_1 z$. The problem with this linear expansion is that it works well for small redshifts but the transition redshift is not so small. Using another parametrization for $q(z)$, Shapiro and Turner [4] concluded that the present SNeIa data cannot rule out the possibility that the universe has been decelerating since $z = 0.3$. In other references [5, 6], it was shown that the value of the transition redshift depends on the adopted parametrization for $q(z)$, as well as on the data sample. In particular, a parametrization that has the transition redshift as free parameter has been presented in [7]. This parametrization could be used to study the kinematics of the expansion regardless of the matter content of the Universe. However, the statistical properties of data are still not good enough to produce strong constraints. In general, the parametrizations are helpful by their phenomenological properties, since they can serve to study the accelerated expansion in different contexts, such as the structure formation, that would be difficult to study into a fundamental theory.

Among all possible candidates to explain the accelerated expansion, the Chaplygin gas is a strong candidate; it is the best known proposal of a unification of dark matter (DM) with dark energy into a single fluid. The idea is: an equation of state leads to a component which behaves as dust at early stage and as cosmological constant at later stage. Following this idea, it was considered the so-called generalized Chaplygin gas [8]. This model has been analyzed many times in the literature; see, for example, [9]. Subsequently, this model has been modified to include an initial phase of radiation and is called the modified Chaplygin gas (MCG). From the theoretical point of view, this scenario can also be restated as a Friedmann–Lemaître–Robertson–Walker (FLRW) cosmological model containing a scalar field ϕ with its self-interacting potential [10].

In this paper, we have as main aim to study the MCG model using several data to constraint as much as possible the parameters of the model. In the literature, the generalized Chaplygin gas has been studied in the context of statefinder diagnostic, stochastic gravitational waves, observational constraints using gamma ray bursts, strong lensing, Supernovae Ia, etc.; see [11–15] for references. We studied the deceleration parameter in the MCG and used recent observational data to constraint its free parameters. We used two sets of data of type Ia Supernovae: the Constitution set and the Union2 set. We also use the CMB and BAO data. In order to make comparisons, we derive the function $q(z)$ for the standard cosmological model, that is, the ΛCDM model. The derivation was performed with the minimal assumptions: GR theory is valid and the universe is homogeneous and isotropic, that is, the cosmological metric is the FLRW one. No assumption was made about the spatial curvature of the universe; however, the analysis on the observational limits of the free parameters was restricted to flat models. We leave for a future paper the models with spatial curvature.

The outline of this paper is as follows: In Section 2, we present the equations of the models, ΛCDM and MCG, and determine the Hubble parameter and the deceleration parameter, respectively. In Section 3, we describe the method used to obtain the confidence regions and the reconstruction of the function $q(z)$. Finally, Section 4 is devoted to the discussion of our results.

2 Equations of Our Models

2.1 ΛCDM Model

Let us assume that the universe is described by the FLRW metric and the energy content is a pressureless fluid and a cosmological constant. The first Friedmann equation for the scale factor $a(t)$ reads

$$H^2 = \frac{\dot{a}^2}{a^2} = \frac{8\pi G\rho}{3} + \frac{\Lambda}{3} - \frac{k}{a^2} \tag{1}$$

which can be written as

$$\Omega_m + \Omega_\Lambda + \Omega_k = 1 \tag{2}$$

with the definitions

$$\Omega_m = \frac{8\pi G\rho}{3H^2} \qquad \Omega_\Lambda = \frac{\Lambda}{3H^2} \qquad \Omega_k = \frac{-k}{a^2H^2}$$

The Bianchi identity for a pressureless fluid can be integrated to give

$$\rho a^3 = \rho_0 a_0^3 \tag{3}$$

where the subscript zero denotes present values of the quantities. Inserting (3) into (1) and writing $-k$ as

$$-k = H^2a^2\Omega_k = H_0^2a_0^2\Omega_{k0}$$

we obtain

$$\dot{a}^2 = H_0^2a_0^2\left[\Omega_{m0}\,\frac{a_0}{a} + \Omega_{\Lambda 0}\,\frac{a^2}{a_0^2} + \Omega_{k0}\right] \tag{4}$$

In terms of the redshift $z = \frac{a_0}{a} - 1$, (4) reads

$$\dot{a}^2 = \frac{H_0^2a_0^2}{(1+z)^2}\,E^2(z) = H_0^2a^2(z)E^2(z) \tag{5}$$

where

$$E^2(z) = \Omega_{m0}\,(1+z)^3 + \Omega_{k0}\,(1+z)^2 + \Omega_{\Lambda 0} \tag{6}$$

Using (5) we can relate $\frac{da}{dz}$ with $\dot{a} = \frac{da}{dt}$:

$$\frac{da}{dz} = -\frac{a(z)}{1+z} = -\frac{1}{H_0(1+z)E(z)}\,\frac{da}{dt} \tag{7}$$

from which we infer the relation between dt and dz:

$$\frac{dz}{dt} = -H_0(1+z)E(z) \tag{8}$$

Now, using (7) and calculating $\ddot{a}$, we obtain for the deceleration parameter

$$q = -\frac{\ddot{a}}{H^2a}$$

the expression

$$q(z) = \frac{\left[\frac{\Omega_{m0}}{2}(1+z)^3 - \Omega_{\Lambda 0}\right]}{\left[\Omega_{m0}(1+z)^3 + \Omega_{k0}(1+z)^2 + \Omega_{\Lambda 0}\right]} \tag{9}$$

For $z >> 1$, we see that $q(z) = 0.5$ as expected. The transition redshift, where $q(z_t) = 0$, is given by

$$(1 + z_t)^3 = \frac{2\Omega_{\Lambda 0}}{\Omega_{m0}}$$

for any value of the curvature k.

In Fig. 1, we reconstruct the evolution of $q(z)$ using the Constitution dataset and the Union dataset. From this figure, we find that both datasets predict similar transition redshifts, $z_t \approx 0.5$. Our reconstruction was made with a 1σ confidence level.

2.2 Modified Chaplygin Gas Model Without Cosmological Constant

The generalized Chaplygin gas model has been proposed as a source term in Einstein's field equations in order to unify the concepts of cold dark matter and dark energy [16–19]. We will consider the energy content of the universe as a fluid that behaves like a perfect fluid of non-zero pressure at early times, like a pressureless fluid at intermediate times, and like dark energy at present.

The equation of state of the MCG is given by [10]

$$p = B\rho - \frac{A}{\rho^{\alpha}}, \tag{10}$$

where A, B, and α are non-negative constants. For $B = 0$, we have the pure generalized Chaplygin gas and for $A = 0$ a perfect fluid. The MCG behaves as radiation (when $B = 1/3$) or dust-like matter (when $B = 0$) at early stage, while as a cosmological constant at later stage. On the other hand, the Bianchi identity

$$3\frac{\dot{a}}{a}(p + \rho) + \dot{\rho} = 0 \tag{11}$$

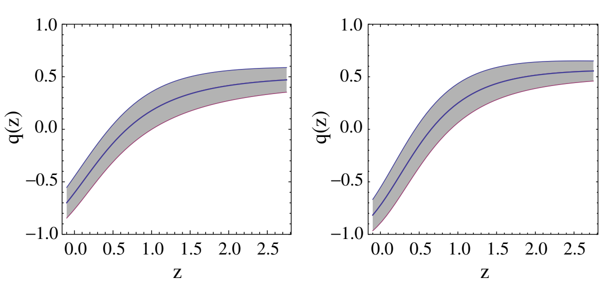

Fig. 1 We show the observational constraints for the ΛCDM model using only SNeIa with $\Omega_{b0} = 0.042 \pm 0.027$. *Left panel*: Constitution set; *right panel*: Union set. The regions correspond to $1 - \sigma$

yields after integration (see [20] for details):

$$\rho(a) = \rho_0 \left[\Omega_X + (1 - \Omega_X)\left(\frac{a_0}{a}\right)^{3R}\right]^{1/(\alpha+1)}, \tag{12}$$

where

$$R = (B + 1)(\alpha + 1)$$

ρ_0 is the present energy density, and we define the dimensionless parameter

$$\Omega_X = \frac{A}{(B + 1)\,\rho_0^{\alpha+1}}. \tag{13}$$

The same analysis of the previous section can be done, leading to the following expression for the deceleration parameter:

$$q(z) = \frac{E^{-2}(z)}{2}\left[\Omega_X + (1 - \Omega_X)(1 + z)^{3R}\right]^{-\alpha/(1+\alpha)} \times \left\{(1 - \Omega_X)(3B + 1)(1 + z)^{3R} - 2\Omega_X\right\} \tag{14}$$

where

$$E^2(z) = [\Omega_X + (1 - \Omega_X)(1 + z)^{3R}]^{1/(1+\alpha)} + \Omega_{k0}(1 + z)^2 \tag{15}$$

The transition redshift for the MCG model is given by:

$$(1 + z_t)^{3R} = \frac{2\Omega_X}{(3B + 1)(1 - \Omega_X)}. \tag{16}$$

The dimensionless parameter Ω_X represents the fraction of dark energy in the content of the universe, thus taking the value $\Omega_X \approx 0.7$. If we chose $B = 1/3$ in order to describe the evolution of the universe since the radiation era and consider $0 < \alpha < 0.5$, which is required from thermodynamical considerations (see [21]), we find $z_t \approx 0.2$. If we consider a pure Chaplygin gas ($B = 0$) and $\Omega_X \approx 0.7$, the value $z_t = 0.46$ can be achieved if $\alpha \approx 0.4$. This result is compatible with the analysis done in [22] for a Chaplygin gas in the flat ($k = 0$) case.

Now, in order to carry out our analysis of observational constraints, we consider as components of the Universe: baryons plus MCG. Thus, in the flat case, the Hubble parameter is given by:

$$E^2(z) = \left(\frac{H(z)}{H_0}\right)^2 = \Omega_{b0}(1 + z)^3 + (1 - \Omega_{b0}) \times \left[(1 - \Omega_{X0})(1 + z)^{3R} + \Omega_{X0}\right]^{1/(1+\alpha)} \tag{17}$$

This expression will be used in our analysis of observational constraints in the following section. As can be seen, we have four free parameters (Ω_{b0}, Ω_{X0}, α, B).

3 Some Observational Constraints

In the present section, we consider some observational constraints of SNeIa, BAO, and CMB for our models. In this context, it is important to consider the comoving distance to an object at redshift z,

$$r(z) = cH_0^{-1} \int \frac{dz'}{E(z')}, \tag{18}$$

where we consider only the flat case; $E(z) = H(z)/H_0$ is given for the ΛCDM model by (6) and for the MCG model by (15). Using the equation above, the luminosity distance in the flat case is then given by $d_L = (1+z)r(z)$.

3.1 Constraints from Supernovae Data

The supernovae Ia data give us the distance modulus (μ) to each supernova that is given by

$$\mu \equiv m_{\rm obs}(z_i) - M = 5\log\left[\frac{d_L}{\rm Mpc}\right] + 25 \tag{19}$$

where M is the absolute magnitude. The distance modulus can also be written as

$$\mu = 5\log_{10} D_L(z) + \mu_0 \tag{20}$$

where $D_L = \frac{H_0 d_L}{c}$ is the Hubble-free luminosity distance and μ_0 is the zero point offset (which is an additional model-independent parameter) defined by

$$\mu_0 = 5\log_{10}\left(\frac{cH_0^{-1}}{\rm Mpc}\right) + 25 = 42.38 - 5\log_{10} h \tag{21}$$

In the present paper, we used the Union2 dataset including 557 data of Amanullah et al. [23] that includes the intermediate z data observed during the first season of the Sloan Digital Sky Survey (SDSS)-II supernova survey [24] and the high z data from the Union compilation [25]. We also used the so-called Constitution set of Hicken et al. [26] including 397 data, out of which 100 come from the new low z CfA3 sample and the rest from the Union set. Both samples have a redshift range of $0.015 \leq z \leq 1.55$. The main improvement of the Constitution sample is the inclusion of a larger number of nearby ($z < 0.2$) SNeIa; their inclusion helps to reduce the statistical uncertainty [26].

The statistic χ^2 is a useful tool for estimating goodness-of-fit and confidence regions on parameters. In our case, the $\chi^2_{\rm SNIa}$ is given by

$$\chi^2_{\rm SNIa}(p_i) = \sum_{i=1}^{n} \frac{(\mu_{\rm the}(p_i, z_i) - \mu_{{\rm obs},i}(z_i))^2}{\sigma^2_{{\rm obs},i}} \tag{22}$$

where $p_i = (\Omega_{b0}, \Omega_{X0}, \alpha, B)$. The χ^2 function can be minimized with respect to the μ_0 parameter, as it is independent of the data points and the dataset. Expanding the equation above with respect to μ_0, we obtain:

$$\chi^2(p_i)_{\rm SNIa} = A(p_i) - 2\mu_0 B(p_i) + \mu_0^2 C(p_i) \tag{23}$$

which has a minimum for $\mu_0 = B(p_i)/C(p_i)$, giving

$$\chi^2_{\rm SNIa,min} = \bar{\chi}^2_{\rm SNIa} = A(p_i) - \frac{B^2(p_i)}{C(p_i)} \tag{24}$$

where

$$A(p_i) = \sum_i^n \frac{(\mu_{\rm th} - \mu_{\rm obs}(p_i, \mu_0 = 0))^2}{\sigma_i} \tag{25}$$

$$B(p_i) = \sum_i^n \frac{\mu_{\rm th} - \mu_{\rm obs}(p_i, \mu_0 = 0)}{\sigma_i} \tag{26}$$

$$C(p_i) = \frac{1}{\sigma_i^2} \tag{27}$$

Now this new $\bar{\chi}^2_{\rm SNIa}$ is independent of μ_0 and can be minimized with respect to the parameters of the theoretical model.

3.2 Baryon Acoustic Oscillations

The primordial baryon–photon acoustic oscillations leave a signature in the correlation function of the SDSS luminous red galaxies as observed by Eisenstein et al. [27], Percival et al. [32], Reid [33]. This signature provides us with a standard ruler which can be used to constrain the following quantity

$$A = \sqrt{\Omega_m}\, E(z_1)^{-1/3}\left[\frac{1}{z_1}\int_0^{z_1} \frac{dz}{E(z)}\right]^{2/3}, \tag{28}$$

where $E(z) = H(z)/H_0$, the observed value of A is $A_{\rm obs} = 0.469 \pm 0.017$ and $z_1 = 0.35$. For the BAO, the best fit parameters can also be determined by a likelihood analysis, based on the calculation of

$$\chi^2_{\rm BAO} = \frac{(0.469 - A_{\rm the})^2}{0.017^2} \tag{29}$$

3.3 CMB: Shift Parameter

We use the CMB data to impose constraints on the parameter space. We use the shift parameter R, which relates the angular diameter distance to the last scattering surface with the angular scale of the first acoustic peak in the WMAP7 power spectrum, that is given by (for $k = 0$) [28]

$$R = \sqrt{\Omega_m} \int_0^{1,090} \frac{\mathrm{d}z}{E} = 1.725 \pm 0.018. \qquad (30)$$

Also, note that in order to include the CMB shift parameter into the analysis, it is needed to integrate up to the matter-radiation decoupling ($z \simeq 1{,}090$), so that radiation is no longer negligible and it was properly taken into account. The best fit values of the model parameters are determined by minimizing

$$\chi^2_{\mathrm{CMB}} = \frac{(1.725 - R_{\mathrm{the}})^2}{0.018^2} \qquad (31)$$

3.4 Combining the Datasets

We considered that the observational data are independent, so we defined the χ^2_{total} as

$$\chi^2_{\mathrm{total}} = \bar{\chi}^2_{\mathrm{SNIa}} + \chi^2_{\mathrm{BAO}} + \chi^2_{\mathrm{CMB}} \qquad (32)$$

The best fit values of the model can be determined by minimizing the total χ^2. For Gaussian distributed measurements, the χ^2 function is directly related to the maximum likelihood estimator. Moreover, if we want to impose a Gaussian prior to one of the parameters being measured, p_i, centered around p_{i0}, with variance $\sigma^2_{p_i}$, we can use the Bayes theorem and write the expression

$$L = \exp\left(-\frac{\chi^2_{\mathrm{total}}}{2}\right) \exp\left[-\frac{(p_i - p_{i0})^2}{2\sigma^2_{p_i}}\right] \qquad (33)$$

In order to constraint the parameters of our interest, we marginalize over the other parameters. To account for the uncertainty of the Hubble parameter, we treat it as a free parameter and then fix it by using the best fit value of the data. For the reconstruction of the $q(z)$ function, we used type Ia Supernovae and BAO data. We followed the standard methodology using the Fisher matrix for generating errors. For details of the method of propagation of errors, see [29, 30].

4 Results and Discussion

The results we obtained for the parameters of the modified Chaplygin equation of state $P = B\rho - A\rho^{-\alpha}$ are presented in Table 1. We can see that, for both datasets, the exponent α is of order 0.1; the value $\alpha = 0$ is included inside the error bars. This shows that the negative pressure component $A\rho^{-\alpha}$ does not differ too much from a cosmological constant. The parameter B, related to the dark matter component with positive pressure, is also less than 0.1 for both datasets. The value $B = 0$ is also included inside the error bars. Theoretically, B would be zero for dust-like matter and

Table 1 The best fit parameters of the MCG model

Parameter	Const + CMB + BAO	Union2 + CMB + BAO
B	0.04 ± 0.15	0.02 ± 0.16
α	0.04 ± 0.09	0.11 ± 0.13
Ω_{X0}	0.83 ± 0.03	0.81 ± 0.03
q_0	-0.67 ± 0.17	-0.65 ± 0.195
z_t	0.80	0.80
χ^2_{min}	472.07	544.85

The error bars are obtained by marginalized likelihood analysis that can be obtained from (33)

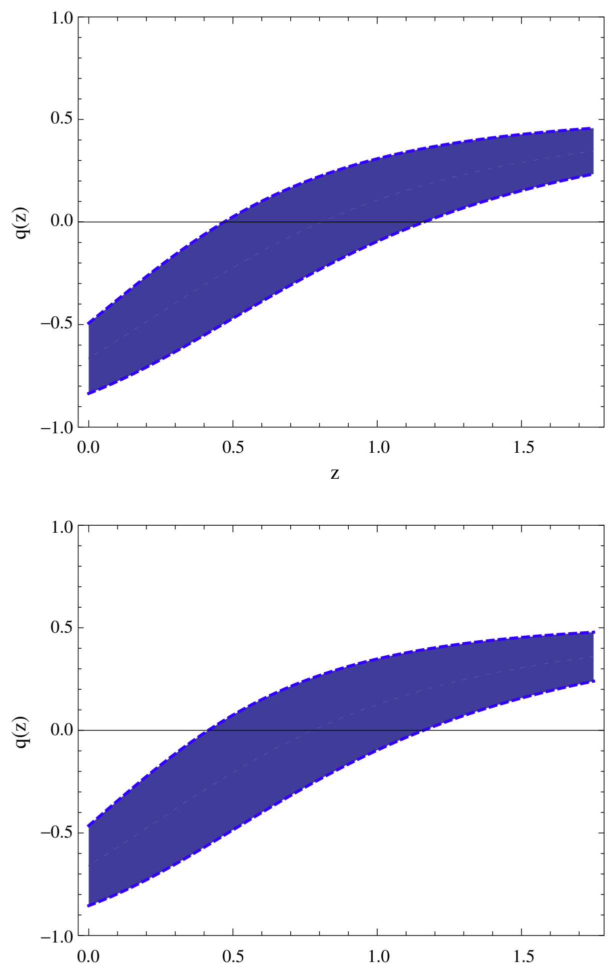

Fig. 2 In the *upper panel*, we present the best fit and reconstruct the errors for the Constitution dataset + BAO + CMB, using $B = 0.04$ (best fit), and in the *lower panel* for the Union dataset + BAO + CMB, using $B = 0.02$ (best fit). The regions corresponds to $1 - \sigma$ of confidence level

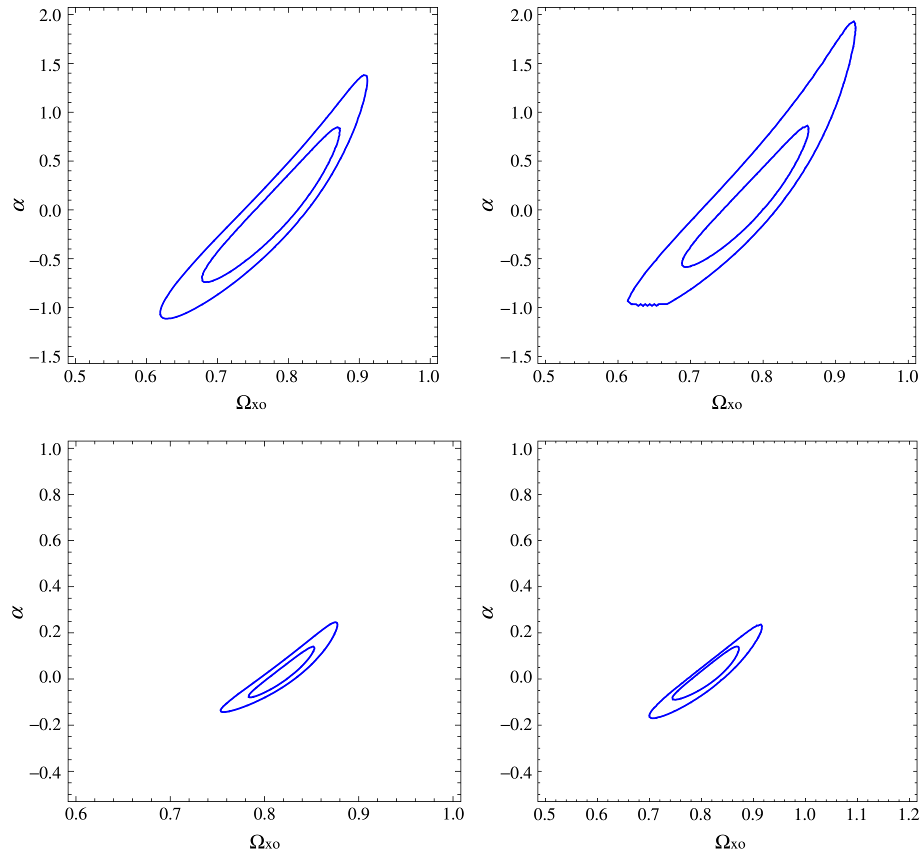

Fig. 3 In the *top left*, we show observational constraints of SNeIa Constitution and in the *top right* for SNeIa Union2. Each *bottom panel* shows the contours with $1-\sigma$ and $2-\sigma$ of errors including SNeIa + CMB + BAO. In all cases, we use $B = 0.02$

1/3 for radiation. The dark energy parameter Ω_{X0} was found to be approximately 0.8. Finally, the kinematical parameter q_0 is of order -0.7 for both datasets, while the transition redshift is $z_t \approx 0.8$. These results for q_0 and z_t are consistent with constraints obtained recently with different methods [6, 31].

In Fig. 2, we show the behavior of the reconstruction of the $q(z)$ function for the MCG model. In the upper panel, we used the Constitution dataset + BAO + CMB data, and in the lower panel, we used the Union dataset + BAO + CMB data. In the upper panels of Fig. 3, we show observational constraints using only type Ia Supernovae data. These results change with the inclusion of the BAO data and WMAP7 data; thus, we obtain results which strongly reduce the space of parameters. These results are displayed in Fig. 3 (lower), in which we clearly see that for both samples, the $\alpha = 0$ value is included within the space parameters

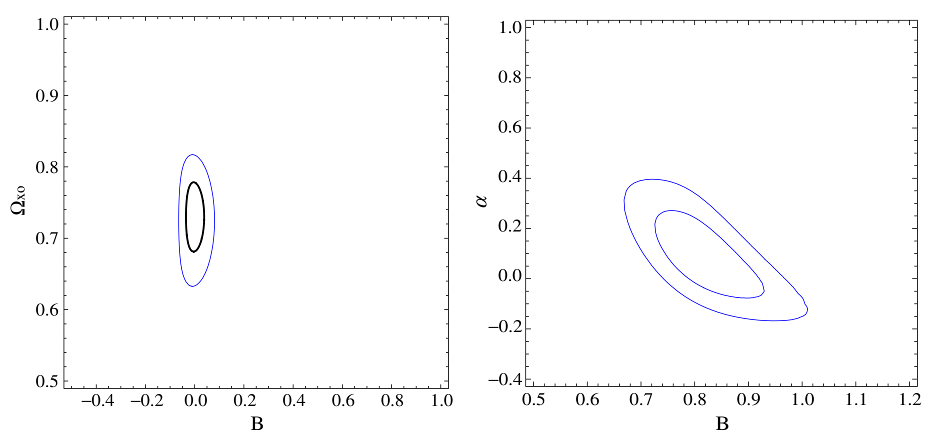

Fig. 4 In the *left panel*, we show observational constraints of SNeIa Union2 + CMB + BAO for $\alpha = 0.11$. In the *right panel*, we show the constraints for α and B, using SNeIa Union2 + CMB + BAO, where $\Omega_{X0} = 0.81$

with $1 - \sigma$ and $2 - \sigma$ of confidence regions. Finally, in Fig. 4, we present the constraints for Ω_{X0} and B (left) with $\alpha = 0.11$, obtained from Union2 + CMB + BAO data. The right panel of Fig. 4 shows the contours for α and B, with $\Omega_{X0} = 0.81$, also from UNion2 + CMB + BAO data.

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