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NUCLEAR PHYSICS

Elastic α-Nucleus Scattering at 36 to 60 Mev/Nucleon

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Abstract Working within the framework of the Coulomb modified Glauber model and using the optical limit approximation to evaluate the elastic S-matrix, we use a parameterized effective nucleon-nucleon phase shift function instead of the frequently applied Gaussian parameterization of the nucleon-nucleon scattering amplitude to compute elastic differential cross sections for alpha particles. Our phenomenological ansatz contains three parameters which are adjusted in order to reproduce the alpha nucleus elastic scattering data for one nucleus at each of three beam energies. It is found that once the nucleon-nucleon phase shift function is so calibrated, our model very nicely reproduces elastic alpha scattering data on other nuclei at the same energy.

Keywords Optical and diffraction models · Alpha elastic scattering · Nucleon–nucleon phase shift

1 Introduction

The first "heavy ion" scattering data on complex nuclei were obtained with alpha particles (^{4}He). At energies near 10 MeV per nucleon, these elastic scattering data showed striking diffractive oscillations, indicating a black nucleus with a very short mean free path for the alpha particles within the nuclei. These oscillations could be fitted by assuming a sharp cutoff in the partial wave amplitudes [1]. The oscillations with angle were similar to those from optical Fraunhofer diffraction, and the minima could be matched with a simple Bessel function. The details of the oscillations could give a sensitive measure of a nuclear radius parameter [2].

At higher beam energies, the deep diffraction minima are filled in, reflecting a longer mean free path for the ion beam in complex nuclei. These and other data have been addressed with a wide range of computational methods. The availability of precise data and the use of these models can give insights into the fundamental nucleon–nucleon interaction within the nuclei, where it might differ from that in free space. In this sense, the alpha particle scattering at suitable beam energies can be used as the model for other reactions seeking to investigate the issue of "medium" modifications. It is of value to investigate a wide range of theoretical models, since the strong absorption may lead to ambiguities of parameter choices made, for instance, with optical model methods. The use of models which enable specific insights into the sensitivities of the assumptions can be particularly valuable.

Generally, two approaches have been used for analyzing α-nucleus elastic scattering differential cross section data. One often used at low and medium energies is the phenomenological optical model potential. At these energies, the behavior of α-nucleus scattering cross sections is dominated by strong absorption in the nuclear surface region. More explicitly, the cross section depends mainly on a small number of phase shifts (δ_l), i.e., to those where l values correspond to impact parameters in the region of the nuclear surface. Since a different interior wave function can generate the same phase shifts, the optical model parameterization of the interaction, as has been explained above, leads to ambiguous information regarding the optical potential. This has been found to give the existence of discrete and continuous ambiguities as well as significant uncertainty in the general shapes of the real and imaginary potentials [3–8].

The other approach which has frequently been used is the eikonal approximation of the Glauber multiple scattering theory [9], which provides us with an excellent framework to describe reactions at high and intermediate energies in various fields of

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physics. Glauber theory is a semiclassical model picturing the nuclei moving in a straight path along their center of mass collision direction. The model gives the nucleus–nucleus interaction [10, 11] in terms of an interaction between the constituent nucleons [nucleon–nucleon (NN) cross sections] and the nuclear density distributions. Since the analytic evaluation of the full Glauber amplitude for a realistic description of two colliding nuclei is a prohibitively complex task, in most of these applications the optical limit approximation (OLA) has been employed to evaluate the Glauber model elastic *S*-matrix elements [11–16]. As described below, it is found that the OLA gives a reasonably good account of the experimental data at lower energies (as low as about 25 MeV/nucleon), but does less well at higher energies, especially at large momentum transfers (see, e.g., [12]). The rapid decrease of the NN total cross sections $\sigma_{\rm NN}$ at higher energies implies that nuclear collisions become more and more transparent, filling in the diffraction minima. Consequently, the effect of higher order terms in the expansion of the nucleus–nucleus phase shift function is expected to be important at these energies [12]; this has been neglected in the OLA. Several analyses of elastic nucleus–nucleus scattering differential cross section data have also been made by evaluating the higher order terms in the expansion of the Glauber amplitude [14, 17–19]. In all these studies, some improvements over the OLA results have been achieved, but a large discrepancy between theory and experiments still exists, especially at higher momentum transfers.

The NN amplitude, which is a basic input in the Glauber model calculation, has generally been parameterized with a one-term Gaussian form, the parameters of which are obtained from NN scattering experiments. Recently [20], α-nucleus elastic scattering data have been studied extensively over a wide energy range considering the higher momentum transfer components of the NN amplitude in addition to the normally used one-term Gaussian NN amplitude. In order to fit the α-nucleus elastic scattering angular distribution data, these authors varied not only the parameters responsible for the large momentum transfer but also the values of the parameters (except the NN total cross section) of the small q behavior of the NN amplitudes, which are generally fixed from the NN scattering experiments. Moreover, the trends of energy dependence of these parameters are not in accordance with earlier analysis [21]. This, together with the use by different authors of different parameter values (later we will discuss this in detail) in nucleus–nucleus elastic scattering studies, shows that conventional Glauber model analyses are not completely parameter free. This calls for a theoretical study to see if the situation could be improved by adopting some other method, but with a tinge of phenomenology [22] for analyzing nucleus–nucleus elastic scattering data at intermediate energies.

It has already been stated earlier that several optical model studies at low and medium energies have been made to give fits to the α-nucleus elastic scattering data. The adjustable parameters obtained by the fits are not measurable directly by the experiments and also are not unique. It is, therefore, undoubtedly necessary to further develop an effective method by which one can avoid parameter ambiguities, while keeping high accuracy. In this paper, we use a recently proposed [22] phenomenological method of analysis for α-nucleus elastic scattering data under the optical limit approximation. The essential point of the proposed method is to calculate the input NN amplitude from the phenomenological NN phase shift function, the parameters of which are varied to fit the elastic α-nucleus scattering data. A similar approach was undertaken by Alvi et al. [23] in a different context. That work used an effective N-α amplitude with one adjustable parameter to calculate α-^{58}Ni, ^{116}Sn, and ^{197}Au elastic differential cross sections at a beam energy 240 MeV.

In an attempt to test the applicability of the proposed method, we extensively analyze a number of α-nucleus elastic scattering data [7, 24, 25] at energies spanning 36–60 MeV/nucleon within the framework of the Coulomb-modified Glauber model. It is found, as we will see shortly, that once the parameters of the NN phase shift function are fixed for one scattering system, the method very nicely reproduces the available elastic α scattering data on other nuclei at the same beam energy.

2 Theoretical Formulation

According to the Glauber multiple scattering theory [9], the elastic S-matrix element $S_{\rm el}(b)$ in the impact parameter space $\mathbf{b}$ for the scattering of a projectile nucleus of mass number B from a target nucleus of mass number A is described by the intrinsic ground state wave functions Φ_O and Ψ_O respectively, and may be expressed as

$$S_{\rm el}(b) = \left\langle \Psi_o\Phi_o \left| \prod_{i=1}^{A}\prod_{j=1}^{B}\left[1-\Gamma_{\rm NN}\left(\mathbf{b}-\mathbf{s}_i+\mathbf{s}_{\rm j}^{'}\right)\right]\right| \Phi_o\Psi_o \right\rangle. \quad (1)$$

In Eq. 1, $\mathbf{s}_i$ ($\mathbf{s}'_j$) are the projections of the target (projectile) nucleon coordinates on the impact parameter plane and $\Gamma_{\rm NN}(\mathbf{b})$ is the NN profile function which is the two-dimensional Fourier transform of the NN scattering amplitude $f_{\rm NN}(q)$

$$\Gamma_{\rm NN}(\mathbf{b}) = \frac{1}{2\pi ik}\int e^{-iq.b} f_{\rm NN}(q) d^2q, \quad (2)$$

where $\mathbf{q}$ is the momentum transfer and k is the nucleon momentum corresponding to the projectile energy/nucleon. The nucleus–nucleus elastic scattering amplitude is written as

$$F(q) = F_{\rm c}(q) + \frac{i}{2K} \times \sum_{l=0}^{\infty}(2l+1)e^{2i\sigma_l}[1-S_l]P_l(\cos\vartheta), \quad (3)$$

where $F_c(q)$ is the usual point Coulomb scattering amplitude, K is the c.m. momentum of the system, σ_l is the point Coulomb phase shift and P_l $(\cos\vartheta)$ is the Legendre polynomial. Following Refs. [19] and [23], we evaluate S_l approximately from the relation

$$S_l \approx e^{i\chi_c(b)} S_{\text{el}}(b)\big|_{Kb=l+1/2}, \tag{4}$$

where $\chi_c(b)$ is the difference between the phase shift functions of the Coulomb potential due to the extended charge distributions of the interacting nuclei and the corresponding point charges.

It is well-known that the evaluation of $S_{\text{el}}(b)$ as given by Eq. 1 is a difficult task for realistic descriptions of the two colliding nuclei. So, because of its simplicity, the OLA of the Glauber theory has successfully been used to describe the nucleus–nucleus scattering data. In this approximation, the total eikonal phase shift function $\chi(b)$ which plays a basic role in the Glauber theory is related to the elastic S-matrix element $S_{\text{el}}(b)$, through the relation

$$S_{\text{el}}(b) = e^{i\chi(b)} \tag{5}$$

with

$$\chi(b) = \frac{AB}{k}\int_0^\infty q\,dq\,J_0(qb)\,f_{\text{NN}}(q)\,F_B(q)\,F_A(q), \tag{6}$$

where $F_A(q)$ and $F_B(q)$ are the target and projectile form factor respectively. The NN scattering amplitude $f_{\text{NN}}(q)$ has been generally parameterized as

$$f_{\text{NN}}(q) = \frac{ik\sigma_{\text{NN}}(1-i\alpha_{\text{NN}})}{4\pi}e^{-(\beta_{\text{NN}}^2+i\gamma)q^2/2}, \tag{7}$$

where σ_{NN} is the NN total cross section, α_{NN} the ratio of the real to the imaginary parts of the forward angle NN scattering amplitude, β_{NN}^2 the slope parameter of the NN elastic differential cross section and γ is the phase variation parameter. It is well-known that NN scattering measurements leave an overall phase of the amplitude undetermined. The phase factor $e^{i\gamma q^2/2}$ in Eq. 7 is to take care of this fact. The phase parameter γ may be positive or negative [20, 26, 27], and it has been shown [20, 26] that in some situations, inclusion of the phase variation significantly affects the calculated cross section. Thus, the parameterization used in Eq. 7 has at least one adjustable unknown energy-dependent parameter.

To describe NN scattering at low and intermediate energies, the Gaussian parameterization is not appropriate because at low energy the scattering is non-diffractive, and not many partial waves are involved. Perhaps this is one of the reasons that a cursory survey of the literature shows that many different values for β_{NN}^2 have been used for calculating nucleus–nucleus differential elastic cross sections at intermediate energies [11, 12, 18, 19]. Chauhan and Khan [20] have recently analyzed α-nucleus elastic scattering data in the energy range of 25–70 MeV/nucleon using a semiphenomenological NN amplitude that might preserve low q behavior, whereas the parameters responsible for large q behavior are treated as adjustable to fit the angular distribution data. Though their analysis is interesting and appears to be convincing, there is still room for further investigation. The value of the slope parameter β_{NN}^2 is found to be increasing with incident energy, whereas in practice a totally opposite trend has been observed, as tabulated recently by Abu-Ibrahim et al. [21] in the NN energy range spanning 40 to 1,000 MeV.

Apart from the uncertainties and ambiguities in some parameter values of $f_{\text{NN}}(q)$, one other point of most concern is the nuclear medium effect on the NN scattering amplitude. Several studies [28–30] lead to various prescriptions for obtaining the medium effect, especially on the NN total cross sections σ_{NN}. These lack quantitative agreement, and it is difficult to assess which one is preferable. It is, therefore, necessary to see if the theoretical situation could be improved by adopting some other effective method by which one can avoid parameter ambiguities, while keeping high accuracy and agreement with a wide range of data.

In this work, we calculate the input NN amplitude from a phenomenological NN phase shift function [22], the parameters of which are varied to fit the experimental data for elastic α-nucleus scattering. We choose the effective phase shift function to be of the form

$$\chi_{\text{NN}}(b) = a_2\left(b^2 - a_3\right)e^{-a_1 b^4} \tag{8}$$

where a_1, a_2, and a_3 are energy-dependent adjustable parameters. Later, we shall see that a_3 which determines the change in the sign of χ_{NN} (b) is almost constant as it should be, within the energy range considered in this paper. The required $f_{\text{NN}}(q)$ is obtained from the relation

$$f_{\text{NN}}(q) = i\,k\int_0^\infty b\,db\,J_0(qb)\left[1 - e^{i\chi_{\text{NN}}(b)}\right]. \tag{9}$$

The $f_{\text{NN}}(q)$ so obtained is substituted in Eq. 6 to calculate the eikonal phase shift function $\chi(b)$ and hence the elastic S-matrix element $S_{\text{el}}(b)$ can easily be evaluated.

To calculate the α-nucleus elastic scattering amplitude F (q) from Eq. 3 in the Coulomb-modified Glauber model we replace b in the elastic S-matrix element $S_{\text{el}}(b)$ by the distance of the closest approach b' [31]. The quantity b' is related to b as

$$Kb' = \eta + \sqrt{\eta^2 + K^2 b^2},$$

where for the α-nucleus system, the Sommerfield parameter is $\eta = \frac{2Z_A e^2}{\hbar v}$ with v as the velocity of the α particle in the lab frame.

Finally, the elastic differential cross sections for α-nucleus scattering can be calculated using the expression

$$d\sigma/d\Omega = |\mathrm{F}(q)|^2. \tag{10}$$

3 Results and Discussion

Following the approach outlined in Section 2, we calculate the elastic differential cross sections (solid curves) at energies 60, 43, and about 36 MeV/nucleon for the systems α-^{58}Ni, ^{116}Sn, and ^{197}Au; α-^{58}Ni, ^{27}Al, ^{24}Mg, and ^{12}C; and α-^{27}Al, ^{24}Mg, and ^{12}C, respectively, and compare them with experimental data [7, 24, 25] shown in Figs. 1, 2, 3 as functions of the momentum transfer in the center of mass frame. These results are obtained by fitting the elastic α-^{58}Ni data at 60 and 43 MeV/nucleon and α-^{27}Al data at 36 MeV/nucleon scattering data by varying the parameters a_1, a_2, and a_3 of χ_{NN} (b) and using realistic nuclear form factors of the target nuclei. For computational simplicity, in all our calculations we parameterize the required nuclear form factors as the sum of Gaussians: $F(q) = \sum b_i e^{-c_i q^2}$ of the realistic nuclear form factor. The values of the parameters b_i and c_i used in the calculation for ^{4}He and ^{12}C and are taken from [14] while those for ^{58}Ni,^{116}Sn, and ^{197}Au have been taken from Alvi et al. [23]. With regard to the values of b_i and c_i for ^{27}Al and ^{24}Mg, these have been determined by fitting the realistic ground state charge densities [32] after correcting for the finite proton charge density. The values so obtained are given in Table 1.

For Fig. 1, we fit the α-^{58}Ni elastic scattering angular distribution data at 60 MeV/nucleon [7] by varying the parameters a_1, a_2, and a_3 of the effective $\chi_{\mathrm{NN}}(b)$ and then calculate α-^{116}Sn and α-^{197}Au scattering cross sections with the same parameter values. The same procedure has also been applied for α scattering data at 43 MeV/nucleon [24, 25], and the results are displayed in Fig. 2. The values of the parameters of the effective $\chi_{\mathrm{NN}}(b)$ obtained in the fitting are reported in Table 2. The solid curves in Figs. 1 and 2 show an excellent agreement with the experimental data at each energy.

Recently, Chauhan and Khan [20] have also studied the elastic angular distribution of 60 MeV/nucleon α-

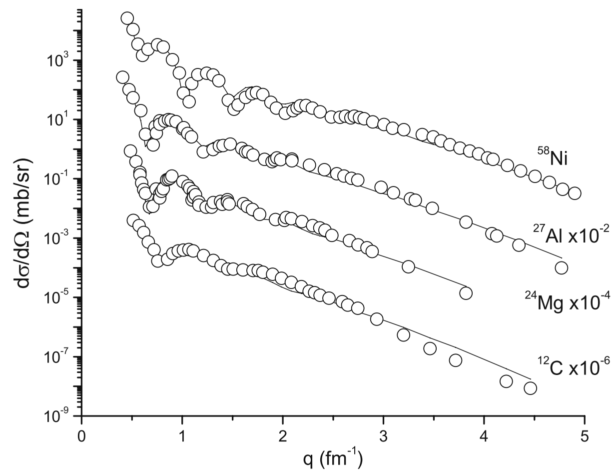

Fig. 2 Elastic differential cross sections for α+^{58}Ni, ^{27}Al, ^{24}Mg, and ^{12}C scattering at 43 MeV/nucleon. The *solid curves* represent the results with parameters of $\chi_{\mathrm{NN}}(b)$ obtained by fitting the elastic α+^{58}Ni scattering data. The *circles* are experimental data taken from Refs. [24] and [25]

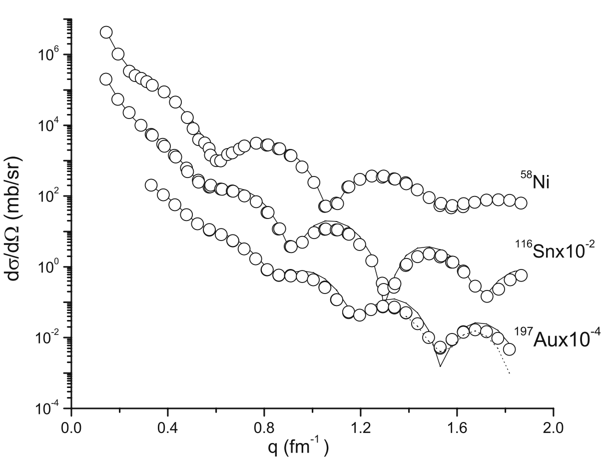

Fig. 1 Elastic differential cross sections for α+^{58}Ni, ^{116}Sn, and ^{197}Au scattering at 60 MeV/nucleon. The *solid curves* represent the results with parameters obtained by fitting the α-^{58}Ni scattering data. The *dotted curve* in ^{197}Au shows optical model fitting by Clark et al. [7]. The *circles* are experimental data taken from Ref. [7]

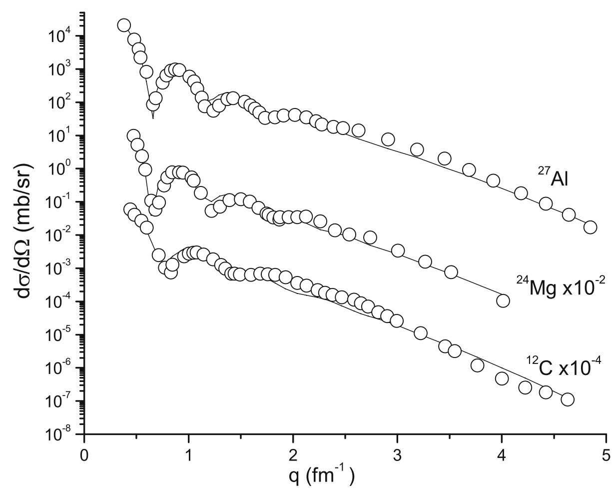

Fig. 3 Elastic differential cross sections for α+^{27}Al, ^{24}Mg, and ^{12}C scattering at 36 MeV/nucleon. The *solid curves* represent the results with the parameters of $\chi_{\mathrm{NN}}(b)$ obtained by fitting the elastic α+^{27}Al scattering data. The *circles* are experimental data taken from Ref. [24]

Table 1 Parameter values of the sum of Gaussians parameterization of the nuclear form factor

Nucleus	b_i	c_i (fm^2)	q_{max} (fm^{-1})	Nucleus	b_i	c_i (fm^2)	q_{max} (fm^{-1})
^{24}Mg	−79.11908	0.98719	3.5	^{27}Al	−20.8946	1.38944	2.0
	−7.95410	0.80392			9.7953	1.39924	
	0.26090	0.34854			4.66016	1.21006	
	87.82014	0.97656			9.03632	1.38337	
	−0.00785	0.05405			−1.59718	0.8519	

The q_{max} represent the maximum momentum transfer [32] up to which the charge form factors forming the basis of our parameters have been fit

particle scattering on ^{58}Ni, ^{116}Sn, and ^{197}Au using a semi-phenomenological NN amplitude where the parameters responsible for large q behavior are treated as adjustable to fit the angular distribution data. They achieved fairly satisfactory results up to about momentum transfer $q \approx 1$ fm^{-1} by not using only this adjustable complex parameter, but they also used some sort of average parameter values of the NN amplitude extracted from the analysis of α–α scattering at 50 and 70 Mev/nucleon. However, by varying the phase of the NN amplitude, they too achieved good fitting for each nucleus considered, but it should be mentioned here that the values of the phase variation parameters are different for different target nuclei. Clark et al. [7] also got nearly the same agreement with data in their optical model fitting for ^{58}Ni and ^{116}Sn elastic scattering data with different sets of potential parameter for each nucleus as shown in Fig. 1. In the case of α-^{197}Au at higher momentum transfer, their result (dashed curve) is not as good as we have achieved without varying the parameters of $\chi_{NN}(b)$. Similarly, for α-^{58}Ni at 43 MeV/nucleon, Ingemarsson et al. [33] used an optical model calculation which showed a sharp minimum around q=1 fm^{-1} (not found in the data) but no such minimum has been obtained in our results.

The full curves in Fig. 3 represent the calculated and experimental (circles) α-^{27}Al, ^{24}Mg, and ^{12}C elastic scattering differential cross sections at a beam energy of about 36 MeV/nucleon [24]. As before, we first fit the α-^{27}Al elastic scattering data by varying the parameters a_1, a_2, and a_3 of the effective $\chi_{NN}(b)$ and then calculated α-^{24}Mg and ^{12}C elastic scattering cross sections with the same parameter values. The parameter values so obtained are also quoted in Table 2. It is seen that the parameters of $\chi_{NN}(b)$ very nicely predict the α-^{24}Mg and ^{12}C elastic scattering data over the whole range of momentum transfer covered by the experiment. The good description of α-nucleus elastic scattering at energies considered in this work speaks of the success of our approach. It would be needless to comment on the optical model parameterization, where one frequently encounters the existence of discrete and continuous ambiguities as well as uncertainty in the general shapes of real and imaginary potentials, with many more than the three adjustable parameters used here.

We report in the sixth column of Table 2 the calculated NN total cross section σ_{cal} evaluated from the effective $\chi_{NN}(b)$ by using Eq. 9 and the optical theorem. For comparison, the corresponding free values σ_{free} are also tabulated. These are calculated from the parameterization of σ_{pp} and σ_{np} as given by Charagi and Gupta [11]. It is seen that σ_{cal} is less than σ_{free} in all cases. The smaller calculated value is understandable because of earlier microscopic studies [28, 30] which have revealed that at relatively low energies, the in-medium NN total cross section gets modified due to Pauli blocking. The microscopic in-medium NN total cross section is found to decrease with increasing density so that it has different values in different overlap regions of the colliding nuclei, but here, σ_{cal} in our analysis seems to be some sort of average over the whole nuclear volume involved in the collision process and therefore, a quantitative comparison between the two is not possible.

Table 2 The values of the parameters of $\chi_{NN}(b)$ along with NN total cross sections σ_{cal} obtained in our calculations and σ_{free} [11]

Energy/nucleon (MeV)	Nuclei	a_1 (fm^{-4})	a_2 (fm^{-2})	a_3 (fm^2)	σ_{cal} (mb)	σ_{free}	σ_R^{cal} (mb)	σ_R^P
36	^{27}Al	0.132	1.927	1.15	120	162	1,135	–
	^{24}Mg						1,111	–
	^{12}C						793	780
43	^{58}N$_i$	0.1607	2.0487	1.05	102	129	1,504	1,559
	^{27}Al						1,085	–
	^{24}Mg						1,060	–
	^{12}C						748	747
60	^{58}N$_i$	0.23	2.506	1.102	92	97	1,420	1,464.8
	^{116}S$_n$						1,803	1,859.6
	^{197}Au						2,148	2,203.2

The last column compares the predictions of α-nucleus total reaction cross sections of our approach with those of Refs. [20] and [33]

Finally, we consider our theoretical estimates of α-nucleus total reaction cross sections σ_R^{cal} calculated using the parameter values of the effective $\chi_{NN}(b)$. In the last column of Table 2, we compare σ_R^{cal} for α-nucleus scattering with those predicted (σ_R^P) by Chauhan and Khan [20] and Ingemarsson et al. [33]. It is seen that our computed σ_R^{cal} are approximately within 2–3% of those reported in Ref. [20] for α-nucleus scattering at 60 MeV/nucleon. For ^{12}C and ^{58}Ni at an α-particle energy of 43 MeV/nucleon, our results closely match those estimated by Ingemarsson et al. [33]. As the experimental data for the reaction cross sections at 36 and 43 MeV/nucleon are not available for ^{24}Mg and ^{27}Al nuclei, we have not reported them in the Table. However, for ^{12}C at an α beam energy of 36 MeV/nucleon we make a linear interpolation between the experimentally measured values [34] at 117 and 164 MeV and obtain about 780 mb which, within the statistical uncertainties, agrees with our calculated σ_R^{cal} value.

4 Concluding Remarks

The work in this paper deals with the description of elastic α-nucleus scattering data at incident alpha energies between 145 and 240 MeV by a simple and effective method, suited for generalization to other heavy ion elastic scattering calculations. We apply a Coulomb-modified Glauber model to evaluate the elastic *S*-matrix and thus the differential elastic scattering cross section. Deviating from the standard applications of Glauber theory, we use a parameterized effective NN phase shift function instead of the conventionally applied Gaussian parameterization of the NN scattering amplitude. This phenomenological ansatz contains three parameters which are adjusted in order to reproduce the α-nucleus elastic scattering data. Used at a given energy per nucleon, the same parameters of the NN phase shift function are able to reproduce the elastic differential cross sections of other nuclei at the same incident α energy per nucleon. This simplicity has made it possible to analyze elastic scattering data for α-^{12}C, ^{24}Mg, ^{27}Al, ^{58}Ni, ^{116}Sn, and ^{197}Au and leads to the conclusion that, at a given projectile energy, once the parameters of the effective $\chi_{NN}(b)$ are fixed, our model predicts nicely the available elastic α scattering data on other nuclei.

Given the success of our phenomenological approach in analyzing the α-nucleus elastic scattering data, we feel that in future applications, the method can be extended to study heavy ion collisions. One of the most attractive feature of our approach is that the parameters a_1 and a_2 of effective $\chi_{NN}(b)$ vary smoothly with energy while the parameter a_3 which determines the change in sign of $\chi_{NN}(b)$ is almost constant. Finally, the effective NN potential $V_{NN}(r)$ can be determined from the effective $\chi_{NN}(b)$ by the high energy inversion formula [9]. Thus, both the real and imaginary parts of the α-nucleus interaction are derived from the same effective $\chi_{NN}(b)$.

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References

1. J.S. Blair, Phys. Rev. **108**, 827 (1957)
2. B. Fernandez, J.S. Blair, Phys. Rev. **C1**, 523 (1970)
3. M.E. Farid, Phys. Rev. C **65**, 067303 (2002)
4. M. E. Farid, Phys. Rev. C **74**, 064616 (2006)
5. Y.-X. Yang, Q.-R. Li, Nucl. Phys. A **732**, 3 (2004)
6. K.A.G. Rao et al., Phys. Rev. C **62**, 014607 (2000)
7. H.L. Clark, Y.W. Lui, D.H. Youngblood, Nucl. Phys. A **589**, 416 (1995)
8. L. Trache et al., Phys. Rev. C **61**, 024612 (2000)
9. R.J. Glauber, in *Lectures in theoretical physics*, ed. by W.E. Brittin, L.G. Dunham, vol. 1 (Interscience, New York, 1959), p. 315
10. P.J. Karol, Phys. Rev. C **11**, 1203 (1975)
11. S.K. Charagi, S.K. Gupta, Phys. Rev. C **46**, 1982 (1992)
12. S.M. Lenzi, A. Vitturi, F. Zardi, Phys. Rev. C **40**, 2114 (1989)
13. J. Chauvin, D. Lebrun, A. Lounis, M. Buenerd, Phys. Rev. C **28**, 1970 (1983)
14. I. Ahmad, J. Phys. G: Nucl. Phys. **6**, 947 (1980)
15. V. Franco, G.K. Varma, Phys. Rev. C **18**, 349 (1978)
16. G.D. Alkhazov, S.L. Belostotsky, A.A. Vorobyov, Phys. Reports **42**, 89 (1978)
17. V. Franco, Y. Yin, Phys. Rev. C **34**, 608 (1986)
18. M.M.H. El-Gogary, A.S. Shalabay, M.Y.M. Hassan, A.M. Hegazy, Phys. Rev. C **61**, 044604 (2000)
19. I. Ahmad, M.A. Abdulmomen, M.S. Al-Enazi, Phys. Rev. C **65**, 054607 (2002)
20. D. Chauhan, Z.A. Khan, Eur. Phys. J. A **41**, 179 (2009)
21. B. Abu-Ibrahim, W. Horiuchi, A. Kohama, Y. Suzuki, Phys. Rev. C **77**, 034607 (2008)
22. M.A. Alvi, M.R. Arafah, F.S. Al-Hazmi, J. Phys. G: Nucl. and Part. Phys. **37**, 045101 (2010)
23. M.A. Alvi, J.H. Madani, A.M. Hakmi, Phys. Rev. C **75**, 064609 (2007)
24. S. Wiktor et al., Acta Phys. Pol. B **12**, 491 (1981)
25. J. Albinski et al., Nucl. Phys. A **445**, 477 (1985)
26. V. Franco, Y. Yin, Phys. Rev. Lett. **55**, 1059 (1985)
27. I. Ahmad, M.A. Alvi, Phys. Rev. C **48**, 3126 (1993)
28. G.Q. Li, R. Machleidt, Phys. Rev. C **49**, 566 (1994)
29. C. Xiangzhou, F. Jun, S. Wenqing, M. Yugang, W. Jiansong, Y. Wei, Phys. Rev. C **58**, 572 (1998)
30. C.A. Bertulani, J. Phys. G: Nucl. and Part. Phys. **27**, L67 (2001)
31. G. Faldt, H. Pilkuhn, Phys. Lett. B **46**, 337 (1973)
32. H. De Vries, C.W. De Jager, C. De Vries, At. Data Nucl. Data Tables **36**, 495 (1987)
33. A. Ingemarsson et al., Nucl. Phys. A **696**, 3 (2001)
34. A. Ingemarsson et al., Nucl. Phys. A **676**, 3 (2000)