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PARTICLES AND FIELDS

Moving Branes with Background Massless and Tachyon Fields in the Compact Spacetime

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Abstract In this article, we shall obtain the boundary state associated with a moving Dp-brane in the presence of the Kalb–Ramond field $B_{\mu\nu}$, an internal $U(1)$ gauge field A_α and a tachyon field, in the compact spacetime. According to this state, properties of the brane and a closed string, with mixed boundary conditions emitted from it, will be obtained. Using this boundary state, we calculate the interaction amplitude of two moving Dp_1 and Dp_2-branes with above background fields in a partially compact spacetime. They are parallel or perpendicular to each other. Properties of the interaction amplitude will be analyzed, and contribution of the massless states to the interaction will be extracted.

Keywords Moving branes · Boundary state · Background fields · Interaction

1 Introduction

Strings are not the only objects of string theory. Since 1995, it has been realized [1] that the theory includes extended objects carrying charges related to special antisymmetric fields and that the strings cannot be the sources of these fields. The sources are D-branes (the word "*D-brane*" is a contraction of "*Dirichlet brane*"). The coordinates of the attached strings satisfy Dirichlet boundary conditions in the directions normal to the brane and Neumann boundary conditions in the directions tangent to the brane. A Dp-brane is a p-dimensional object. In the type IIA string theory, the branes have the even dimensions $p \in \{0, 2, \ldots, 8\}$ while for the type IIB theory the branes dimensions are odd $p \in \{-1, 1, \ldots, 9\}$. For the type I theory, there is $p \in \{1, 5, 9\}$. The Dp-brane is the source of the $(p+1)$-form R-R gauge field.

The D-branes are found to be important in nonperturbative string theories because in the strong coupling they become arbitrarily light and dominate the theory at low energies [2]. They are also important because the low-energy fluctuations of D-branes are described by supersymmetric gauge theory, which is non-Abelian for coincident branes.

One of the interesting aspects of the D-branes is the interaction between them, which can be obtained through two different procedures, which are nonetheless equivalent: from the one-loop diagram of an open string and from the tree-level diagram of a closed string [3]. Since the two D-brane interaction can be described by the exchanging of closed strings, we here restrict ourselves to the second approach. The state describing the closed-string production from the vacuum is called a boundary state. The boundary-state formalism is a powerful method for studying branes properties and their interactions.

Among the achievements in the subject of boundary-state formalism is the study of the interaction of mixed branes (branes with both Neumann and Dirichlet boundary conditions), moving and angled branes in the presence of background fields such as a U(1) gauge field [4–6] and an antisymmetric field $B_{\mu\nu}$ [7–16]. The

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tachyon field has also been added as a background field in some studies [17–20].

Since D-branes are not static objects, studying their dynamics is essential to interpret them as physical objects in string theory. Considering the velocity for a D-brane, which is equivalent to taking into account the scalar fields from the worldsheet point of view [21], as well as the gauge field on the D-brane worldvolume, is very instructive, to study D-brane dynamics. In addition, progress in the study of open-string tachyon fields, due to the seminal work of Sen [18, 22–24] added to our knowledge of D-branes, their instability or stability features, and the true vacuum of tachyonic string theories, among other issues [22, 23].

The above facts motivated us to study a system of two moving Dp_1 and Dp_2-branes in the presence of the following background: tachyon field, Kalb–Ramond field $B_{\mu\nu}$, $U(1)$ gauge fields which live in the worldvolumes of the branes, and a partially compacted spacetime on tori. The brane dimensions p_1 and p_2 are arbitrary. The relative configurations of the branes are parallel and/or perpendicular. Without fixing the position of the branes, we study both configurations simultaneously. We calculate the boundary state, corresponding to the branes, and then obtain the interaction amplitude between them through exchange of closed strings. While the spacetime is allowed to have compact directions, we observe that presence of the tachyon field has some effects on wrapping of the closed string around these directions. In addition, the tachyon field also affects the interaction amplitude of the branes. For example, the behavior of the amplitude for large distance branes deviates substantially from what is expected in the conventional case; we present a simple interpretation for this deviation.

2 The Boundary State

We begin with a special sigma model for the string. This sigma-model action contains the antisymmetric field $B_{\mu\nu}$, tachyon fields, two $U(1)$ gauge fields living on the world volume of the branes, and two velocity terms corresponding to the motion of the branes

$$
\begin{aligned}
S = &-\frac{1}{4\pi\alpha'}\int_{\Sigma} \mathrm{d}^2\sigma \left(\sqrt{-g}g^{ab}G_{\mu\nu}\partial_a X^{\mu}\partial_b X^{\nu} + \varepsilon^{ab} B_{\mu\nu}\partial_a X^{\mu}\partial_b X^{\nu}\right) \\
&-\frac{1}{2\pi\alpha'}\int_{(\partial\Sigma)_1} \mathrm{d}\sigma \left(A^{(1)}_{\alpha_1}\partial_\sigma X^{\alpha_1} + V^{i_1}_1 X^0 \partial_\tau X^{i_1} + \left(T^{(1)} + \frac{1}{2}U^{(1)}_{\mu\nu}X^{\mu}X^{\nu}\right)\right) \\
&+\frac{1}{2\pi\alpha'}\int_{(\partial\Sigma)_2} \mathrm{d}\sigma \left(A^{(2)}_{\alpha_2}\partial_\sigma X^{\alpha_2} + V^{i_2}_2 X^0 \partial_\tau X^{i_2} + \left(T^{(2)} + \frac{1}{2}U^{(2)}_{\mu\nu}X^{\mu}X^{\nu}\right)\right), \quad (1)
\end{aligned}
$$

where Σ is the worldsheet of the closed string, exchanged between the branes. The boundaries of this worldsheet, i.e., $(\partial\Sigma)_1$ and $(\partial\Sigma)_2$, are at $\tau = 0$ and $\tau = \tau_0$, respectively. The $U(1)$ gauge field $A^{(2)}_{\alpha_2}$ lives in the Dp_2-brane, and $V^{i_2}_2$ is its velocity component along X^{i_2} direction. The set $\{X^{\alpha_2}\}$ specifies the directions along the Dp_2-brane worldvolume and $\{X^{i_2}\}$ shows the directions perpendicular to it. Similar variables with the index "1" refer to the Dp_1-brane.

Here, we take the background fields $G_{\mu\nu}$ and $B_{\mu\nu}$ to be constant, and the profile of the tachyon field is defined as $T^2 = T_0 + \frac{1}{2}U_{\mu\nu}X^{\mu}X^{\nu}$, with constant T_0 and constant symmetric matrix $U_{\mu\nu}$. The advantage of this profile is that the theory will be Gaussian and, therefore, exactly solvable [19, 20]. Setting the variation of this action with respect to $X^{\mu}(\sigma,\tau)$ equal to zero gives the equation of motion of $X^{\mu}(\sigma,\tau)$ and the boundary-tate equations of the emitted (absorbed) closed string from (by) the brane. For simplicity, we remove the indices "1" and "2" of the variables which refer to the Dp_1 and Dp_2-branes. We shall restore these indices in the interaction of these branes. Therefore, we have the following the mixed boundary-state equations (i.e., a combination of Dirichlet and Neumann boundary conditions) at $\tau = 0$:

$$[\partial_\tau(X^0 - V^i X^i) + \mathcal{F}^0{}_{\alpha}\partial_\sigma X^{\alpha} - U^0{}_{\nu}X^{\nu}]|B_x, \tau=0\rangle = 0, \quad (2)$$

$$[\partial_\tau X^{\bar{\alpha}} + \mathcal{F}^{\bar{\alpha}}{}_{\beta}\partial_\sigma X^{\beta} - U^{\bar{\alpha}}{}_{\nu}X^{\nu}]|B_x, \tau = 0\rangle = 0, \quad (3)$$

$$[X^i - V^i X^0 - y^i]|B_x, \tau = 0\rangle = 0, \quad (4)$$

where $\bar{\alpha}$ refers to the spatial directions of the brane (i.e., $\bar{\alpha} \neq 0$), $\{y^i\}$ denotes the initial transverse coordinates of the brane, and $\mathcal{F}$ is total field strength

$$\mathcal{F}_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha - B_{\alpha\beta}. \quad (5)$$

The first two terms define the field strength of A_α, which is assumed to be constant. For simplification, we also assumed the mixed elements of the Kalb–Ramond field to be zero, i.e., $B^{\alpha}{}_{i} = 0$.

To solve these equations, we use the general solution of the closed-string equation of motion

$$X^\mu(\sigma,\tau) = x^\mu + 2\alpha' p^\mu \tau + 2L^\mu \sigma + \frac{i}{2}\sqrt{2\alpha'} \times \sum_{m\neq 0} \frac{1}{m}\left(\alpha^\mu_m e^{-2im(\tau-\sigma)} + \tilde{\alpha}^\mu_m e^{-2im(\tau+\sigma)}\right). \tag{6}$$

In this relation, L^μ is zero for non-compact directions and $L^\mu = N^\mu R^\mu$ for compact directions, where N^μ is the winding number of the closed string and R^μ is the radius of compactification of the compact direction X^μ. Therefore, the boundary-state (2)–(4) can be written in terms of the oscillators

$$\left[\left(\alpha^0_m - V^i\alpha^i_m - \mathcal{F}^0{}_\alpha \alpha^\alpha_m - \frac{i}{2m}U^0{}_\nu \alpha^\nu_m\right) + \left(\tilde{\alpha}^0_{-m} - V^i\tilde{\alpha}^i_{-m} + \mathcal{F}^0{}_\alpha \tilde{\alpha}^\alpha_{-m} + \frac{i}{2m}U^0{}_\nu \tilde{\alpha}^\nu_{-m}\right)\right]|B_x, \tau=0\rangle = 0, \tag{7}$$

$$\left[\left(\alpha^{\bar{\alpha}}_m - \mathcal{F}^{\bar{\alpha}}{}_\beta \alpha^\beta_m - \frac{i}{2m}U^{\bar{\alpha}}{}_\nu \alpha^\nu_m\right) + \left(\tilde{\alpha}^{\bar{\alpha}}_{-m} + \mathcal{F}^{\bar{\alpha}}{}_\beta \tilde{\alpha}^\beta_{-m} + \frac{i}{2m}U^{\bar{\alpha}}{}_\nu \tilde{\alpha}^\nu_{-m}\right)\right]|B_x, \tau=0\rangle = 0, \tag{8}$$

$$\left[\left(\alpha^i_m - V^i\alpha^0_m\right) - \left(\tilde{\alpha}^i_{-m} - V^i\tilde{\alpha}^0_{-m}\right)\right]|B_x, \tau=0\rangle = 0, \tag{9}$$

and zero modes

$$\left[2\alpha'\left(p^0 - V^i p^i\right) + 2\mathcal{F}^0{}_\alpha L^\alpha - U^0{}_\nu x^\nu\right]_{\text{op}}|B_x, \tau=0\rangle = 0, \tag{10}$$

$$U^0{}_\nu L^\nu_{\text{op}}|B_x, \tau=0\rangle = 0, \tag{11}$$

$$\left(2\alpha' p^{\bar{\alpha}} + 2\mathcal{F}^{\bar{\alpha}}{}_\beta L^\beta - U^{\bar{\alpha}}{}_\nu x^\nu\right)_{\text{op}}|B_x, \tau=0\rangle = 0, \tag{12}$$

$$U^{\bar{\alpha}}{}_\nu L^\nu_{\text{op}}|B_x, \tau=0\rangle = 0, \tag{13}$$

$$\left(x^i - V^i x^0 - y^i\right)_{\text{op}}|B_x, \tau=0\rangle = 0, \tag{14}$$

$$L^i_{\text{op}}|B_x, \tau=0\rangle = 0. \tag{15}$$

$$\left(p^i - V^i p^0\right)_{\text{op}}|B_x, \tau=0\rangle = 0. \tag{16}$$

where we assumed that the time direction is non-compact, i.e., $L^0 = 0$. The index "op" identifies an operator.

From now on, we put a restriction on the velocities and consider both Dp_1 and Dp_2-branes to move along the common direction X^{i_0}, which is perpendicular to both of them. Thus, (10), (12), and (16), lead to

$$p^0 = \frac{\gamma^2}{\alpha'}\left(\frac{1}{2}U^0{}_\mu x^\mu - \mathcal{F}^0{}_{\bar{\beta}} L^{\bar{\beta}}\right),$$
$$p^{\bar{\alpha}} = \frac{1}{\alpha'}\left(\frac{1}{2}U^{\bar{\alpha}}{}_\mu x^\mu - \mathcal{F}^{\bar{\alpha}}{}_{\bar{\beta}} L^{\bar{\beta}}\right),$$
$$p^{i_0} = \frac{V\gamma^2}{\alpha'}\left(\frac{1}{2}U^0{}_\mu x^\mu - \mathcal{F}^0{}_{\bar{\beta}} L^{\bar{\beta}}\right), \tag{17}$$

where $\gamma = 1/\sqrt{1-V^2}$. For the compact direction X^μ, we also have $p^\mu = M^\mu/R^\mu$ where M^μ is the momentum number of the closed string. These equations imply that the nonzero momentum components (momentum numbers) of the closed string depend on its winding numbers around the wrapped directions of the brane, and its center-of-mass position. The former is due to the massless background fields, while the latter is the effect of the tachyon field.

Combining (11), (13), and (15) leads to

$$U^\alpha{}_{\bar{\beta}} L^{\bar{\beta}}_{\text{op}}|B_x, \tau=0\rangle = 0. \tag{18}$$

If the $p\times p$ square sub-matrix $U^{\bar{\alpha}}{}_{\bar{\beta}}$ is invertible, we obtain

$$L^{\bar{\alpha}}_{\text{op}}|B_x, \tau=0\rangle = 0. \tag{19}$$

Thus, the background tachyon field prevents the wrapping of the closed string around the compact directions of the brane. Equations (15) and (19) imply that the closed string cannot wrap around any compact direction of the spacetime. Putting aside the invertibility of the sub-matrix $U^{\bar{\alpha}}{}_{\bar{\beta}}$, the closed string can wind around the compact directions of the brane. However, we assume $U^{\bar{\alpha}}{}_{\bar{\beta}}$ to be invertible.

We now solve the boundary-state equations to obtain the boundary state. By using the coherent state method [25], (7)–(9) give the oscillating part of the boundary state as in the following expression:

$$|B_{\text{osc}}, \tau=0\rangle = \prod_{n=1}^{\infty}\left[\det M_{(n)}\right]^{-1} \times \exp\left[-\sum_{m=1}^{\infty}\left(\frac{1}{m}\alpha^\mu_{-m}\mathcal{S}_{(m)\mu\nu}\widetilde{\alpha}^\nu_{-m}\right)\right]|0\rangle, \tag{20}$$

where

$$\begin{cases} \mathcal{S}_{(m)} = S_{(m)} + ((S_{(-m)})^{-1})^T, \\ S_{(m)} = M^{-1}_{(m)} N_{(m)}. \end{cases} \tag{21}$$

The matrix $((S_{(-m)})^{-1})^T$ appears here because $S_{(m)}$ is mode dependent and, generally, not orthogonal. The matrices $M_{(m)}$ and $N_{(m)}$, which depend on $\mathcal{F}$, V, B and U, are defined by

$$M^{\mu}_{(m)\nu} = \Omega^{\mu}{}_{\nu} - \frac{i}{2m} U^{\alpha}{}_{\nu} \delta^{\mu}{}_{\alpha} \tag{22}$$

where

$$\begin{cases} \Omega^{0}{}_{\mu} = \delta^{0}{}_{\mu} - V\delta^{i_0}{}_{\mu} - \mathcal{F}^{0}{}_{\alpha}\delta^{\alpha}{}_{\mu}, \\ \Omega^{\bar{\alpha}}{}_{\mu} = \delta^{\bar{\alpha}}{}_{\mu} - \mathcal{F}^{\bar{\alpha}}{}_{\beta}\delta^{\beta}{}_{\mu}, \\ \Omega^{i}{}_{\mu} = \delta^{i}{}_{\mu} - V\delta^{i}{}_{i_0}\delta^{0}{}_{\mu}, \end{cases} \tag{23}$$

and

$$\begin{cases} N^{0}_{(m)\mu} = \delta^{0}{}_{\mu} - V\delta^{i_0}{}_{\mu} + \mathcal{F}^{0}{}_{\alpha}\delta^{\alpha}{}_{\mu} + \frac{i}{2m}U^{0}{}_{\mu}, \\ N^{\bar{\alpha}}_{(m)\mu} = \delta^{\bar{\alpha}}{}_{\mu} + \mathcal{F}^{\bar{\alpha}}{}_{\beta}\delta^{\beta}{}_{\mu} + \frac{i}{2m}U^{\bar{\alpha}}{}_{\mu}, \\ N^{i}_{(m)\mu} = -\delta^{i}{}_{\mu} + V\delta^{i}{}_{i_0}\delta^{0}{}_{\mu}, \end{cases} \tag{24}$$

where $V^{i_0} \equiv V$. The infinite factor in (20) can be regularized as

$$\prod_{n=1}^{\infty} \left[\det M_{(n)}\right]^{-1} = \sqrt{\det \Omega}\ \det \Gamma\left(\frac{U}{2i\Omega} + 1\right). \tag{25}$$

It is seen that if $U = 0$ then (25) will be the familiar DBI Lagrangian.

Solving the zero mode equations (10)–(16) by considering x^{μ} and p^{μ} as quantum mechanical operators and using their commutation relations, we find the state

$$\begin{aligned} &|B_x, \tau = 0\rangle^{(0)} \\ &= \frac{T_p}{2} \int_{-\infty}^{\infty} \prod_{\mu} dp^{\mu} \\ &\times \Bigg\{ \exp\Bigg[-i\alpha' \left(U^{00} - U^{0i_0}V\right)^{-1} \left(P^0\right)^2 \\ &\quad - 2i\alpha' \sum_{\bar{\beta}} \left(\frac{\sum_{\bar{\alpha}} \left[\left(1 - \frac{1}{2}\delta_{\bar{\alpha}\bar{\beta}}\right) U^{\bar{\beta}\bar{\alpha}} P^{\bar{\alpha}}\right]}{\sum_{\bar{\gamma}} \left(U^{\bar{\beta}\bar{\gamma}} U^{\bar{\beta}\bar{\gamma}}\right)} P^{\bar{\beta}} \right) \Bigg] \\ &\times \delta\left(x^{i_0} - Vx^0 - y^{i_0}\right) \prod_{j \neq i_0} \delta\left(x^j - y^j\right) \\ &\times \prod_{\alpha} |p^{\alpha}_L = p^{\alpha}_R\rangle \prod_{j \neq i_0} |p^j_L = p^j_R = 0\rangle |p^{i_0}_L \\ &= p^{i_0}_R = \frac{1}{2}Vp^0\rangle \Bigg\}, \end{aligned} \tag{26}$$

where $P^{\alpha} = p^{\alpha} - Vp^{i_0}\delta^{\alpha}{}_0$ and $\bar{\alpha}$, $\bar{\beta}$ and $\bar{\gamma}$ take their values from the spatial directions of the brane. The index j indicates the directions perpendicular to the brane, except i_0. Since the tachyon field prevents the closed string from wrapping around the compact directions, the left and right components of the momentum are equal. This implies that closed string has zero winding numbers, and its momentum components are not discrete. We, therefore, have to integrate over them.

3 Interaction Between the Branes

3.1 Amplitude

To calculate the amplitude of the interaction, we need the total boundary state. The result in (20) and (26) is the matter part of it. We should also take into account the boundary state associated with the conformal ghosts. This is due to the fact that we are working in the covariant formalism. So the total boundary state, with which we will calculate the amplitude, is

$$|B, \tau = 0\rangle = |B_{\text{osc}}, \tau = 0\rangle |B_x, \tau = 0\rangle^{(0)} |B_{\text{gh}}, \tau = 0\rangle, \tag{27}$$

where the ghost part is

$$\begin{aligned} |B_{\text{gh}}, \tau = 0\rangle = \exp\left[\sum_{m=1}^{\infty} \left(c_{-m}\widetilde{b}_{-m} - b_{-m}\widetilde{c}_{-m}\right)\right] \\ \times \frac{c_0 + \widetilde{c}_0}{2} |q = 1\rangle |\widetilde{q} = 1\rangle. \end{aligned} \tag{28}$$

Now, we proceed to calculate the amplitude of the interaction between the Dp_1-brane and Dp_2-brane through the closed string exchanging between them. For this purpose, we need the closed-string propagator, which is given by $D = 2\alpha' \int_0^{\infty} dt e^{-tH}$, where H is the closed-string Hamiltonian. The overlap of the two boundary states, corresponding to the branes, via this propagator defines the interaction amplitude, i.e. $\mathcal{A} = \langle B_1|D|B_2\rangle$. Before calculating the interaction amplitude, we have to define index conventions. The set $\{\bar{i}\}$ shows directions perpendicular to both branes, except i_0, $\{\bar{u}\}$ is for the directions along both branes, except 0, $\{\alpha'_1\}$ is used for the directions along the Dp_1-brane and perpendicular to the Dp_2-brane, and $\{\alpha'_2\}$ indicates the directions along the Dp_2-brane and perpendicular to the Dp_1-brane. We then have the relations

$$\begin{aligned} &\{\alpha_1\} = \{\bar{u}\} \cup \{\alpha'_1\} \cup \{0\}, \quad \{\alpha_2\} = \{\bar{u}\} \cup \{\alpha'_2\} \cup \{0\}, \\ &\{i_1\} = \{\bar{i}\} \cup \{\alpha'_2\} \cup \{i_0\}, \quad \{i_2\} = \{\bar{i}\} \cup \{\alpha'_1\} \cup \{i_0\}, \\ &\{\mu\} = \{\alpha_1\} \cup \{i_1\} = \{\alpha_2\} \cup \{i_2\}. \end{aligned} \tag{29}$$

Since the position of the branes are specified by the running indices $\{\bar{\alpha}_1\}$ and $\{\bar{\alpha}_2\}$, the Dp_1 and Dp_2-brane can be parallel or perpendicular to each other.

After a long calculation, the following interaction amplitude results

$$\mathcal{A} = \frac{\alpha' V_{\bar{u}}}{4(2\pi)^{d_{\bar{i}}}} \frac{T_{p_1} T_{p_2}}{|V_1 - V_2|} \prod_{m=1}^{\infty} \left(\det[M_{(m)1} M_{(m)2}] \right)^{-1} \int_0^\infty dt \times \Bigg\{ e^{(d-2)t/6} \prod_{m=1}^{\infty} \left(\left[\det\left(1 - \mathcal{S}_{(m)1}\mathcal{S}^T_{(m)2} e^{-4mt}\right)\right]^{-1} \times \left(1 - e^{-4mt}\right)^2 \right) \times \left(\sqrt{\frac{\pi}{\alpha' t}}\right)^{d_{\bar{i}_n}} \times \exp\left[-\frac{1}{4\alpha' t} \sum_{\bar{i}_n} \left(y_1{}^{\bar{i}_n} - y_2{}^{\bar{i}_n} \right)^2 \right] \prod_{\bar{i}_c} \Theta_3 \times \left(\frac{y_1^{\bar{i}_c} - y_2^{\bar{i}_c}}{2\pi R_{\bar{i}_c}} \middle| \frac{i\alpha' t}{\pi (R_{\bar{i}_c})^2} \right) \int_{-\infty}^{+\infty} \prod_{\alpha_1} dp_1{}^{\alpha_1} \prod_{\alpha_2} dp_2{}^{\alpha_2} \times \Big[\exp\Big(-\alpha' t \Big(f^{(+)} f^{(-)} + p_1{}^{\alpha'_1} p_1{}^{\alpha'_1} + p_2{}^{\alpha'_2} p_2{}^{\alpha'_2} + p_1{}^{\bar{u}} p_2{}^{\bar{u}} \Big) \Big) \times \exp\Big(i\Phi(12) y_2{}^{i_0} - i\Phi(21) y_1{}^{i_0} + 2i y_1{}^{\alpha'_2} p_2{}^{\alpha'_2} - 2i y_2{}^{\alpha'_1} p_1{}^{\alpha'_1} \Big) \times \exp\Big(\pi^{0T} Q \pi^0 + E^T \pi^0 + \pi_1{}^T G_1 \pi_1 + \pi_2{}^T G_2 \pi_2 \Big) \Big] \Bigg\}, \quad (30)$$

where $V_{\bar{u}}$ is the common worldvolume. $\bar{i}_c$ and $\bar{i}_n$ indicate the compact and non-compact parts of $X^{\bar{i}}$, with $d_{\bar{i}_n} = \dim\{X^{\bar{i}_n}\}$ and $d_{\bar{i}} = \dim\{X^{\bar{i}}\}$. The directions $X^{\bar{i}_c}$ and $X^{\bar{i}_n}$ are perpendicular to both branes. In addition, $R_{\bar{i}_c}$ is the radius of compactification and d is the spacetime dimension. We also defined

$$\pi^0 = \begin{pmatrix} p_1^0 \\ p_2^0 \end{pmatrix}, \quad \pi_1 = \begin{pmatrix} p_1^{\alpha'_1} \\ p_1^{\bar{u}} \end{pmatrix}, \quad \text{and} \quad \pi_2 = \begin{pmatrix} p_2^{\alpha'_2} \\ p_2^{\bar{u}} \end{pmatrix},$$

in which the elements of π_1 and π_2 are $(d_{\alpha'_1} + d_{\bar{u}})$ and $(d_{\alpha'_2} + d_{\bar{u}})$, respectively. The notations $\Phi(12)$ and $f^{(+)}$ are defined by

$$\Phi(12) = \frac{1}{V_2 - V_1} \left[\left(1 + V_1 V_2\right) p_2^0 - \left(1 + V_1^2\right) p_1^0 \right], \quad (31)$$

$$f^{(+)} = \frac{1}{|V_2 - V_1|} \times \left[\left(1 + V_1\right)\left(1 + V_2^2\right) p_2^0 - \left(1 + V_2\right)\left(1 + V_1^2\right) p_1^0 \right]. \quad (32)$$

By exchanging $2 \leftrightarrow 1$ in (31) we find $\Phi(21)$, and by letting $V_1 \longrightarrow -V_1$ and $V_2 \longrightarrow -V_2$ in (32) we find $f^{(-)}$. The matrix elements of Q, G_1 and G_2 and of the doublet

$$E = \begin{pmatrix} E_1 \\ E_2 \end{pmatrix}$$

are defined through by the following expressions:

$$Q : \begin{cases} Q_{11} = \frac{\alpha' t}{\left(V_2 - V_1\right)^2} \left(1 + V_1{}^2\right)\left(1 - V_2{}^2\right) + 2i\alpha' \left(U_1^{00} - U_1^{0,i_0} V_1\right)^{-1} \left(1 - V_2{}^2\right)^2, \\ Q_{22} = \frac{\alpha' t}{\left(V_2 - V_1\right)^2} \left(1 + V_2{}^2\right)\left(1 - V_1{}^2\right) - 2i\alpha' \left(U_2^{00} - U_2^{0,i_0} V_2\right)^{-1} \left(1 - V_1{}^2\right)^2, \\ Q_{12} = Q_{21} = \frac{\alpha' t}{\left(V_2 - V_1\right)^2} \left(1 + V_1{}^2\right)\left(1 + V_2{}^2\right) \times \left(1 - V_1 V_2\right), \end{cases} \quad (33)$$

$$E : \begin{cases} E_1 = \frac{i}{V_2 - V_1} \left[y_2{}^{i_0} \left(1 + V_1{}^2\right)^2 - y_1{}^{i_0} \left(1 + V_1 V_2\right) \right], \\ E_2 = \frac{i}{V_2 - V_1} \left[y_1{}^{i_0} \left(1 + V_2{}^2\right)^2 - y_2{}^{i_0} \left(1 + V_1 V_2\right) \right], \end{cases} \quad (34)$$

$$G : \begin{cases} G_1^{\alpha'_1 \beta'_1} = 4i\alpha' \frac{(1 - \frac{1}{2}\delta_{\alpha'_1 \beta'_1}) U_1^{\alpha'_1 \beta'_1}}{\sum_{\bar{\gamma}_1} (U_1^{\bar{\gamma}_1 \beta'_1} U_1^{\bar{\gamma}_1 \beta'_1})} - \alpha' t \delta^{\alpha'_1 \beta'_1}, \\ G_1^{\alpha'_1 \bar{u}} = 4i\alpha' \frac{U_1^{\alpha'_1 \bar{u}}}{\sum_{\bar{\gamma}_1} (U_1^{\bar{\gamma}_1 \bar{u}} U_1^{\bar{\gamma}_1 \bar{u}})}, \\ G_1^{\bar{u}\alpha'_1} = 4i\alpha' \frac{U_1^{\bar{u}\alpha'_1}}{\sum_{\bar{\gamma}_1} (U_1^{\bar{\gamma}_1 \alpha'_1} U_1^{\bar{\gamma}_1 \alpha'_1})}, \\ G_1^{\bar{u}\bar{v}} = 4i\alpha' \frac{(1 - \frac{1}{2}\delta_{\bar{u}\bar{v}}) U_1^{\bar{u}\bar{v}}}{\sum_{\bar{\gamma}_1} (U_1^{\bar{v}\bar{\gamma}_1} U_1^{\bar{v}\bar{\gamma}_1})} - \frac{1}{2}\alpha' t \delta^{\bar{u}\bar{v}}. \end{cases} \quad (35)$$

To define G_2, we exchange $1 \longleftrightarrow 2$ and $i \rightarrow -i$ in each element of G_1. For parallel D-branes with the same dimension, the terms containing α'_1 and α'_2 disappear. The effects of compactification are in the product of the Θ_3-functions. The amplitude in the non-compact spacetime can, therefore, be obtained as follows: remove the Θ_3-functions and let $\bar{i}_n \to \bar{i}$ and $d_{\bar{i}_n} \to d_{\bar{i}}$.

Integration over the momenta simplifies the expressions for the amplitude. After introducing the regularization (25), we find that

$$\mathcal{A} = \frac{\alpha' V_{\bar{u}}}{4(2\pi)^{d_{\bar{i}}}} \frac{T_{p_1} T_{p_2}}{|V_1 - V_2|} \sqrt{\det(\Omega_1 \Omega_2)} \det \times \left[\Gamma\left(\frac{U_1}{2i\Omega_1}+1\right) \Gamma\left(\frac{U_2}{2i\Omega_2}+1\right)\right] \int_0^\infty dt \times \left\{ e^{(d-2)t/6} \prod_{m=1}^{\infty} \left(\left[\det\left(1 - \mathcal{S}_{(m)1} \mathcal{S}^T_{(m)2} e^{-4mt}\right)\right]^{-1} \times \left(1 - e^{-4mt}\right)^2 \right) \left(\sqrt{\frac{\pi}{\alpha' t}}\right)^{d_{\bar{i}_n}} \times \exp\left[-\frac{1}{4\alpha' t} \sum_{\bar{i}_n} \left({y_1}^{\bar{i}_n} - {y_2}^{\bar{i}_n}\right)^2 \right] \prod_{\bar{i}_c} \Theta_3 \times \left(\frac{y_1^{\bar{i}_c} - y_2^{\bar{i}_c}}{2\pi R_{\bar{i}_c}} \middle| \frac{i\alpha' t}{\pi (R_{\bar{i}_c})^2} \right) \frac{1}{\sqrt{\det Q \;\det G_1 \;\det G_2}} \times \exp\left[-\frac{1}{4} \left(E^T Q^{-1} E + \sum_{\alpha'_1, \beta'_1} \left(y_2^{\alpha'_1} y_2^{\beta'_1} (G_1^{-1})_{\alpha'_1 \beta'_1}\right) + \sum_{\alpha'_2, \beta'_2} \left(y_1^{\alpha'_2} y_1^{\beta'_2} (G_2^{-1})_{\alpha'_2 \beta'_2}\right) \right)\right] \right\}. \tag{36}$$

The tachyon, Kalb–Ramond and gauge fields are collected in the matrices Ω_1, Ω_2, $\mathcal{S}_1$, $\mathcal{S}_2$, Q, G_1, G_2, and the doublet E. As expected, the amplitude is symmetric under the exchange $1 \longleftrightarrow 2$.

The constant factors multiplying the integral show the strength of the interaction. The third and fourth lines in (36) reflect the portion of oscillators and conformal ghosts in the interaction. The exponential factor in the fifth line is a damping factor with respect to the distance of the branes. If all directions $\{X^{\bar{i}}\}$ are compact, then $d_{\bar{i}_n} = 0$, and this exponential factor disappears. Similarly, if they are non-compact then the Θ_3-factor will be eliminated.

3.2 Behavior of the Interaction Amplitude for Large Distances

In any interaction theory, one should verify behavior of the amplitude at large distances, which yields the associated long-range force. In our case, it is related to the contribution of the closed-string tachyon and massless states to the interaction. We now want to verify this statement for our system, which contains a special tachyon field. In other words, we intend to study the effect of the background fields on the interaction amplitude after long times. For this purpose, we should take the limit $\lim_{t\to\infty} \mathcal{A}$. Since the matrices Q, G_1, and G_2 are functions of time, for $d = 26$ the following limit results:

$$\lim_{t\to\infty} \left\{ e^{4t} \prod_{m=1}^{\infty} \left(\left[\det\left(1 - \mathcal{S}_{(m)1} \mathcal{S}^T_{(m)2} e^{-4mt}\right)\right]^{-1} \left(1 - e^{-4mt}\right)^2 \right) \times \frac{1}{\sqrt{\det Q \;\det G_1 \;\det G_2}} \exp\left(-\frac{1}{4} E^T Q^{-1} E\right) \right\} = \frac{i 2^{d_{\bar{u}}+1/2} (-1)^{(p_1+p_2)/2}}{\alpha'^{(p_1+p_2)/2}} \frac{|V_1 - V_2|}{(1+{V_1}^2)(1+{V_2}^2)} \times \lim_{t\to\infty} \left\{ \frac{e^{4t}}{t^{1+(p_1+p_2)/2}} + \frac{\mathrm{Tr}(\mathcal{S}_{(1)1} \mathcal{S}^T_{(1)2}) - 2}{t^{1+(p_1+p_2)/2}} \right\}. \tag{37}$$

After substitution of the limit (37) into the amplitude (36), the massless states and tachyon contributions to the interaction amplitude for $d = 26$ become

$$\mathcal{A}_0 = \frac{V_{\bar{u}} T_{p_1} T_{p_2}}{4(2\pi)^{d_{\bar{i}}}} \sqrt{\det(\Omega_1 \Omega_2)} \times \det \left[\Gamma\left(\frac{U_1}{2i\Omega_1}+1\right) \Gamma\left(\frac{U_2}{2i\Omega_2}+1\right)\right] \times \frac{i(-1)^{(p_1+p_2)/2}\, 2^{d_{\bar{u}}+1/2}}{\alpha'^{(p_1+p_2)/2}} \frac{1}{(1+{V_1}^2)(1+{V_2}^2)} \times \int^{\infty} \mathrm{d}t \left\{ \left(\sqrt{\frac{\pi}{\alpha' t}}\right)^{d_{\bar{i}_n}} \exp\left[-\frac{1}{4\alpha' t} \sum_{\bar{i}_n} ({y_1}^{\bar{i}_n} - {y_2}^{\bar{i}_n})^2 \right] \times \prod_{\bar{i}_c} \Theta_3 \left(\frac{y_1^{\bar{i}_c} - y_2^{\bar{i}_c}}{2\pi R_{\bar{i}_c}} \middle| \frac{i\alpha' t}{\pi (R_{\bar{i}_c})^2} \right) \lim_{t\to\infty} \times \left[\frac{e^{4t}}{t^{1+(p_1+p_2)/2}} + \frac{\mathrm{Tr}(\mathcal{S}_{(1)1} \mathcal{S}^T_{(1)2}) - 2}{t^{1+(p_1+p_2)/2}} \right] \right\}. \tag{38}$$

The divergent part in the last bracket corresponds to the tachyonic closed-string state. It differs from the same part in the papers by the coefficient $1/t^{1+(p_1+p_2)/2}$ which slows down this divergence. The other term of the last bracket is related to the contribution of the massless fields, which rapidly approaches zero in the large-distance limit. It is remarkable that this damping factor depends only on the two D-branes dimensions not on their relative configuration. So in the presence of the tachyon field the behavior of the interaction amplitude has changed in such a manner that in large distances the contribution of the graviton, dilaton and Kalb–Ramond fields disappears. This effect may be understood as follows.

According to the first equation of (17) the energy p^0 defines a linear potential, acting on the closed string. This potential, due to the background tachyon, slows down the motion of the closed-string. This is true for all closed-string states, including the massless ones. The exchanged closed-strings will, therefore, cease and the long-range force, vanish.

The vanishing of the D-branes interaction after sufficiently long time may also be interpreted as the rolling of the tachyon field in this limit [24]. The presence of the open-string tachyon field implicates that the D-brane is unstable, that is, in the long-time limit the tachyon rolls down to its minimum potential, and hence the D-brane decays to closed-string states and finally disappears. Thus, in the long run no D-branes are left to interact with each other.

4 Conclusions

We obtained the boundary state of a closed string, emitted (absorbed) from (by) a moving brane in the presence of the background fields $B_{\mu\nu}$, tachyon, and internal $U(1)$ gauge field. By partially compactifying the spacetime on tori, the formalism was applied to both compact and non-compact spacetimes. We observed that the closed string cannot wrap around the compact directions of the spacetime, which are perpendicular to the brane's world volume. In addition, for a special tachyon matrix, the tachyon prevents the closed string from winding around the compact directions parallel to the brane's volume.

The interaction amplitude of two D-branes with arbitrary dimensions was calculated. The D-branes are parallel or perpendicular to each other. Due to the tachyon field, the strength of the interaction between the branes depends on all mode numbers of the exchanged closed string. The background fields, specially the tachyon field, affect the interaction. In other words, the background fields define an effective tension for each D-brane. In the corresponding potential, which is related to the interaction amplitude, the product of these effective tensions define a coupling constant. The value of this coupling constant determines the strength of the interaction. By adjusting the parameters V_1, V_2, $\{U^{(1)}_{\mu\nu}, U^{(2)}_{\mu\nu}\}$, $\{\mathcal{F}^{(1)}_{\mu\nu}, \mathcal{F}^{(2)}_{\mu\nu}\}$ and $\{R^\mu|\mu \neq 0\}$ we can therefore control this strength.

If the branes are separated by large distances, the contribution of the massless states (i.e., graviton, dilaton and Kalb–Ramond fields) goes to zero and the divergence part related to the tachyonic closed-string state slows down considerably This can be understood as a decelerating potential, which acts on the exchanging closed strings, and also by the rolling of the tachyon, which leads to the instability of the D-branes and hence causes them to decay.

Although in this article, we are dealing only with bosonic string, it is worth noting that similar consideration, concerning the effects of compactification, works for the superstring case, since these effects are independent of the fermions. The addition of fermionic degrees of freedom to the present formalism is in progress.

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