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GENERAL AND APPLIED PHYSICS

# The Effects of an Induced Electric Dipole Moment due to Earth's Electric Field on the Artificial Satellites Orbit

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**Abstract** The orbits of artificial satellites are very sensitive to a large number of disturbances, whose effects add to the main force exerted by Earth's gravitational field. The most important perturbations, caused by solar radiation pressure, the Moon and the Sun gravitational fields, have been extensively discussed in the literature, and must be taken into account in order to correct the orbital motion, to prevent collisions between satellites in close orbits. In this paper we consider an additional source of acceleration arising from an electric dipole moment induced by the high altitude Earth electric field in a metallic satellite of spherical shape. The order of magnitude of such effect is estimated to be in the range of $10^{-23}$m/s$^2$. It is emphasized that the electric dipole moment effect(EDME) is dependent on the satellite shape and geometry and proportional to $E_0 v/r^4$. The Earth electric field $E_0$ is largely influenced by atmospheric electromagnetic phenomena, such as whistler waves and thunderstorms.

**Keywords** Artificial satellite · Electric dipole moment · Earth's electric field

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## 1 Introduction

Satellite orbital dynamics is primarily influenced by the Earth gravitational field but there are several other contributions which affect the trajectory and must be taken into account in order to prevent escape from the desired orbit or collision with satellites in neighboring orbits. Since the precise knowledge of the position and velocity of artificial satellites is essential to the current technologies involving satellite communications and GPS systems [1], an enormous effort to predict all the effects influencing the orbits has been made over the years. The major concern is related to calculations of the solar radiation pressure effect, the gravitational effects produced by the Sun, the Moon and other planets of the solar system, atmospheric drag, albedo, thermal radiation and relativistic effects, among others [2]. The most relevant electromagnetic effect is due to the solar radiation pressure, since the mean value of the Sun's Poynting vector has a large magnitude, of the order of $\sim 1400$W/m$^2$ and is almost independent of the altitude, albeit strongly dependent on the time of the day. The electromagnetic radiation pressure model was first considered by Bartoli already in 1885 [3]. Here we briefly sketch the main steps to estimate the order of magnitude of that perturbation, by considering the linear momentum conservation law as the starting point:

$$\sum_i \mathbf{p}_i = \sum_f \mathbf{p}_f \,,$$

where $\mathbf{p}_i$ are the initial momenta of the particles and electromagnetic fields and $\mathbf{p}_f$ the final momenta. An artificial satellite of mass $m$ has linear momentum given by $m\mathbf{v}$, where $\mathbf{v}$ is the velocity vector. From Maxwell's equations and the Lorentz force one finds the Poynting theorem, a statement of energy conservation [4], which allows the definition of Poynting's vector $\mathbf{S} = \mathbf{E} \times \mathbf{H}$ and the linear momentum density vector $\mathbf{P}_{em}$ for the electromagnetic field:

$$\mathbf{P}_{em} = \frac{1}{c^2}\mathbf{S} = \frac{1}{c^2}[\mathbf{E} \times \mathbf{H}]\,,$$

where $\mathbf{E}$ and $\mathbf{H}$ are the electric and magnetic fields of an electromagnetic wave, respectively, and $c = 2.9979 \times 10^8$m/s is the velocity of light in vacuum. Describing a solid body a mass density function, defined as $\rho = dm/dV$, we are able to rewrite the momentum conservation law in terms of momentum densities, as follows [4]:

$$\rho\mathbf{v}_i + \frac{1}{c^2}\mathbf{S}_i = \rho\mathbf{v}_f + \frac{1}{c^2}\mathbf{S}_f\,.$$

The vector equation for a satellite velocity deviation is easily shown to be:

$$\rho\Delta\mathbf{v} = -\frac{1}{c^2}\Delta\mathbf{S}\,.$$

Here, $\Delta\mathbf{v} = \mathbf{v}_f - \mathbf{v}_i$ and $\Delta\mathbf{S} = \mathbf{S}_f - \mathbf{S}_i$. Since the satellite is composed of metallic materials, it is customary to consider that the incident electromagnetic wave is totally reflected, reverting the sign of the Poynting's vector component perpendicular to the satellite surface. In this way we get an acceleration:

$$\Delta\mathbf{a} = \frac{\Delta\mathbf{v}}{\Delta t} = -\frac{2S_i A\cos(\theta_{SA})}{mc}\hat{\mathbf{n}}\,, \tag{1}$$

where $m$ is the total mass of the satellite, $S_i$ is the averaged modulus of the Poynting vector of the electromagnetic radiation, $A$ is the cross-sectional area of the satellite and $\theta_{SC}$ is the angle between the direction of the Poynting vector and the vector normal to the satellite surface, $\mathbf{n}$. For $\theta_{SA} = 0$ we find the maximum value for the solar radiation pressure on a typical satellite of mass $m = 407$kg and cross-sectional area $A \approx 1\text{m}^2$ to be $|\Delta\mathbf{a}| \sim 10^{-8}\text{m/s}^2$. Notice that despite being 7 to 9 orders of magnitude smaller than Earth's gravitational field acceleration($a_E \sim 1\text{m/s}^2$), the solar pressure must be included in orbital dynamics models to correctly predict unwanted orbit deviations. Effects of thermal re-radiation of the satellite and the reflection of solar radiation by the Earth surface have been estimated with increasing accuracy over the years. By contrast with the radiation pressure exerted by electromagnetic waves, the Lorentz force acceleration effects [5] induced by electrical charging of artificial satellites, a phenomenon predicted by Al'pert *et al* already in 1964 [6], is quite well described by electrostatics and magnetostatics.

It is our aim in this manuscript to study a novel effect related the electric field of the Earth which induces an electric dipole moment in metallic satellites moving in the magnetic field of the Earth. The nearly constant magnetic field gives rise to a Lorentz force on the moving electric dipole. A crucial prediction of our model is a force varying as $v/r^4$ which influences low-orbit satellites more pronouncedly. Satellites in the Low Earth Orbit(LEO) and Midle Earth Orbits(MEO) may feel the effects of the electric dipole moment induced by Earth's residual electric field resulting from atmospheric phenomena in the ionospheric and magnetospheric regions.

The content of this paper can be described as follows: in Section 2 we briefly review the induced electric dipole moment effect of a metallic object of spherical shape in the presence of an uniform electric field and discuss the existence of an average Earth's electric field in the range of a few mV/m to V/m due to atmospheric phenomena. In Section 3 we consider the effect of the induced electric dipole moment by the Earth's electric field and get the analytical form and order of magnitude of the Lorentz force related to this phenomenon. Finally, in the last Section a few conclusions and remarks are added.

## 2 The Electric Dipole Moment Effect

It is well known from Maxwell's equations for electrostatics that a metallic object in the presence of a uniform electric field of magnitude $E_a$ will acquire an effective electric dipole moment $\mathbf{p_e}$, in order to satisfy the boundary condition which states that the tangential component of the resulting electric field at the metallic boundary surfaces must vanish. For a metallic object of spherical shape, the volumetric charge density is given by [4]:

$$\rho_v = 3\varepsilon_0 E_a \delta(r_c - r_s)\cos\alpha, \tag{2}$$

where $\varepsilon_0 = 8.854 \times 10^{-12}$F/m is the vacuum dielectric constant, $\mathbf{r}_c = (r_c, \alpha, \beta)$ are spherical coordinates with origin in the satellite coordinate system, $\delta(...)$ is the Dirac delta function, $r_c$ is the radial coordinate measured from the center of the sphere and $r_s$ is the sphere radius, $\alpha$ is the angle relative to the uniform electric field direction ($0 \leq \alpha \leq \pi$). Notice that a positive electric charge is accumulated at the northern hemisphere of the object($\alpha < \pi/2$), while a negative electric charge is developed at the southern hemisphere($\alpha > \pi/2$). We can replace the metallic sphere by an effective electric dipole moment, whose definition for a given charge distribution is [4]:

$$\mathbf{p}_e = \int_{V'} \rho_v(\mathbf{r}')\mathbf{r}' dV',$$

where $\mathbf{r}'$ is the position vector measured from the origin of the coordinate system. It follows that:

$$\mathbf{p}_e = 3\varepsilon_0 E_a V_e \hat{\mathbf{a}}_E \ , \tag{3}$$

where $V_e = 4\pi r_s^3/3$ is the spherical volume and $\hat{\mathbf{a}}_E$ is the unit vector representing the local direction of the electric field. Since an artificial satellite is composed of metallic materials and orbits regions of non-negligible residual Earth's electric field, mainly by atmospheric phenomena in the ionosphere and magnetosphere, we must account for a Lorentz force acceleration on the electric dipole moment induced in the satellite. Thus, in the analysis of that effect we will replace the satellite by an effective dipole moment, given by (3), whenever possible. Before proceeding, however, we will briefly discuss the possible origin of the residual Earth electric field in regions where artificial satellites are allowed to orbit. According to Devandraa *et al* [7] the global atmospheric electrical circuit correlates the electric field and electric current that flows from the lower atmosphere to the ionosphere and magnetosphere forming a giant spherical capacitor continuously charged by the electrical storms. The current produces a weak electrification of clouds resulting in a vertical potential gradient near the Earth surface with resulting electric field in the range of 100 – 700V/m [8]. There is a non-linear and chaotic connection between the electric field of the Earth and storms. Above the thunderstorms the electric field is sufficiently intense to generate a current of electrons that propagate as Whistler Waves. At high altitudes the residual field is in the range of mV/m to a few V/m.

In the next Section we will discuss effects of the induced electric dipole moment to the satellite orbit, to estimate the order of magnitude of the Lorentz force acceleration.

## 3 The Electric Dipole Moment Effect for Satellite Acceleration

Considering the Lorentz force in a multipole expansion, one obtains the Lorentz force experienced by an electric dipole moment in the presence of an electric field. The result is [4]:

$$\mathbf{F} = \nabla(\mathbf{p}_e \cdot \mathbf{E}) \ . \tag{4}$$

The above expression can be written in spherical coordinates, with the center of the Earth as the origin and the angle $\theta$ being measured with respect to the $z$-axis, which is identified with the direction of the magnetic dipole moment of the Earth magnetic field. We have that

$$\mathbf{F} = \hat{\mathbf{a}}_r \frac{\partial}{\partial r}(\mathbf{p}_e \cdot \mathbf{E}) + \hat{\mathbf{a}}_\theta \frac{1}{r}\frac{\partial}{\partial \theta}(\mathbf{p}_e \cdot \mathbf{E}) + \hat{\mathbf{a}}_\varphi \frac{1}{r\sin\theta}\frac{\partial}{\partial \varphi}(\mathbf{p}_e \cdot \mathbf{E}) \ . \tag{5}$$

Since a satellite is a non-relativistic object we can make use of a gallilean transformation to obtain the electric field $\mathbf{E}$ experienced by the satellite electric dipole moment which is moving with velocity $\mathbf{v}$ relative to the Earth's magnetic field $\mathbf{B}$. The electric field is simply given by $\mathbf{E} = \mathbf{v} \times \mathbf{B}$, allowing us to rewrite (5) as:

$$\mathbf{F} = \hat{\mathbf{a}}_r \frac{\partial}{\partial r}(\mathbf{p}_e \cdot \mathbf{v} \times \mathbf{B}) + \hat{\mathbf{a}}_\theta \frac{1}{r}\frac{\partial}{\partial \theta}(\mathbf{p}_e \cdot \mathbf{v} \times \mathbf{B}) + \hat{\mathbf{a}}_\varphi \frac{1}{r\sin\theta}\frac{\partial}{\partial \varphi}(\mathbf{p}_e \cdot \mathbf{v} \times \mathbf{B}) \ . \tag{6}$$

The Earth magnetic field $\mathbf{B}$ is generated by an effective magnetic dipole moment, $m_e$. We can calculate $\mathbf{B}$ in any point above the Earth surface by the following expression:

$$\mathbf{B} = \frac{\mu_0 m_e}{4\pi r^3}(2\cos\theta \hat{\mathbf{a}}_r + \sin\theta \hat{\mathbf{a}}_\theta) \ , \tag{7}$$

where $\mu_0$ is the vacuum permeability, $m_e = 7.856 \times 10^{22}\,\mathrm{A{\cdot}m^2}$ is the Earth magnetic dipole moment [9], $r$ is the radius measured from Earth's geometric center and $\theta$ is the magnetic latitude. After the insertion of (7) into (6) it is easy to introduce an electric dipole moment perturbative force in an orbital dynamics model.

Our primary concern here is to estimate the order of magnitude of such a perturbation. In order to do this we will simplify the picture by considering only the radial component of the force on a satellite in a nearly circular motion, in the plane $\theta = \pi/2$, for which we have $\mathbf{B} = B_\theta(r)\hat{\mathbf{a}}_\theta$, $\mathbf{v} = v_\varphi(r)\hat{\mathbf{a}}_\varphi$. With the definitions:

$$B_\theta(r) = \frac{\mu_0 m_e}{4\pi r^3} \ , \tag{8}$$

$$v_\varphi(r) = \sqrt{\frac{GM}{r}} \ , \tag{9}$$

where $G$ is the Newton gravitational constant and $M$ is the Earth mass. It is straightforward to show that the radial force component is given by:

$$F_r = \frac{21}{8\pi}\frac{v_\varphi m_e}{c^2 r^4} E_a \left(\frac{4\pi r_s^3}{3}\right) \hat{\mathbf{a}}_{\mathbf{E}} \cdot \hat{\mathbf{a}}_{\mathbf{r}} \ , \tag{10}$$

with $\hat{\mathbf{a}}_{\mathbf{E}} = \mathbf{E}_a/E_a$ the unit vector in the direction of the Earth electric field, $\mathbf{E}_a$. For the Earth we have the product $GM \approx 3.9860 \times 10^{14}$ N·m$^2$/kg. For a LAGEOS satellite typical values are $m = 407$kg and radius $r_s = 30$cm, in an orbit of major semi-axis $a = 12300$km and excentricity $e \approx 0$ with respect to the Earth center(in terms of altitude it corresponds to approximately 5900km above Earth's surface). According to Ref. [10] the the magnetospheric electric field has a magnitude in the range of 4 mV/m, but we assume it can vary to a maximum value of 1 V/m. With these values in hand we get the order of magnitude of maximum electric dipole moment induced acceleration to be in the range of $\Delta a = F_r/m = 5 \times 10^{-23}$ m/s$^2$. The effect predicted here is 12 orders of magnitude smaller than expected for the perturbation due to tidal forces caused by the Sun and Moon [11], or relative to the Lorentz force on an electrically charged satellite. For a charge $Q = 10$pC [5, 12] the Lorentz acceleration magnitude, $F = QvB$ will be in the range of $10^{-13}$m/s$^2$ for the same LAGEOS satellite.

By contrast, there is an effect of radiation pressure exerted by the the so-called whistler mode waves, whose order of magnitude is practically the same as that due to the electric dipole moment. Whistler waves [13] correspond to an electromagnetic signal originated from lightning with averaged pulse duration of about 10ms propagating in the magnetized plasma of the ionosphere and magnetosphere regions in the range of very low frequency ELF (0.2 - 10 kHz). They were initially detected in 1894 by Preece [14] and confirmed in 1919 by Barkhausen [15]. The minimum detectable Poynting vector radiated by whistler modes is in the range of $10^{-13}$ W/m$^2$, but according to Smith *et al* [16] they have a value of about $2.7 \times 10^{-11}$W/m$^2$ in the range of 3-5 kHz. Applying formula (1) for an averaged Poynting vector of value $S_i = 10^{-13}$ W/m$^2$ we get an acceleration effect of order of magnitude $10^{-23}$m/s$^2$.

## 4 Conclusion

In summary, in this paper we developed a model to describe the effects of a Lorentz force acceleration on satellite orbits due to an electric dipole moment induced on the satellite by the Earth residual electric field. We demonstrated that the acceleration varies as $v/r^4$ and is very small compared the most important electromagnetic effect due to solar radiation pressure($\sim 10^{-8}$m/s$^2$) effects or even compared with charging effects($\sim 10^{-13}$m/s$^2$), but is in the same order of magnitude of secondary electromagnetic radiation emission by atmospheric phenomena, such as whistler waves. Here we have considered only the radial component of the electric dipole moment force acting on artificial satellite, but the effect of the other components are also negligible compared to radiation pressure and charging effects.

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