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Competition Among Reputations in the 2D Sznajd Model: Spontaneous Emergence of Democratic States

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Abstract We propose a modification in the Sznajd sociophysics model defined on the square lattice. For this purpose, we consider reputation—a mechanism limiting the agents' persuasive power. The reputation is introduced as a time-dependent score, which can be positive or negative. This mechanism avoids dictatorship (full consensus, all spins parallel) for a wide range of model parameters. We consider two different situations: case 1, in which the agents' reputation increases for each persuaded neighbor, and case 2, in which the agents' reputation increases for each persuasion and decreases when a neighbor keeps his opinion. Our results show that the introduction of reputation avoids full consensus even for initial densities of up spins greater than 1/2. The relaxation times follow a log-normal-like distribution in both cases, but they are larger in case 2 due to the competition among reputations. In addition, we show that the usual phase transition occurs and depends on the initial concentration d of individuals with the same opinion, but the critical points d_c in the two cases are different.

Keywords Dynamics of social systems · Phase transitions · Cellular automata

1 Introduction

Ising-type models have been reviewed and used by physicists in many different areas, such as sociology, politics, marketing, and finance [1–4]. In 2000, an agent-based model proposed by Sznajd-Weron et al. [5] was successfully applied to the dynamics of a social system. In particular, the model reproduced certain properties observed in a real community. At the focus of the Sznajd model (SM) is the emergence of social collective (macroscopic) behavior due to the interactions among individuals, which constitute the microscopic level of a social system.

This model has been extensively studied since the introduction of the original one-dimensional version in 2000. Many modifications have been proposed, e.g., square [4], triangular [6], and cubic lattices [7]; increased interaction range [8] and number of states of the variable [9–11]; and diffusion of the agents [11, 12].

The original SM consists of a chain with periodic boundary conditions. Each site (an individual opinion) can have one of two states (opinions) represented by Ising spins (“yes” or “no”). A pair of parallel spins on sites i and $i + 1$ forces its two neighbors, $i - 1$ and $i + 2$, to have the same orientation (opinion), while for an antiparallel pair ($i, i + 1$), the left-hand neighbor ($i - 1$) takes the opinion of spin $i + 1$, while the right-hand neighbor ($i + 2$) takes the opinion of spin i . In this simplest formulation of the SM, two types of steady states are always reached: complete consensus (ferromagnetic state) or stalemate, in which every site has an opinion

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that is different from that of its neighbors (antiferromagnetic state). The transient nonetheless displays an interesting behavior, as pointed by Stauffer et al. [4]. Those authors considered a square lattice and not a pair of neighbors, but a 2×2 plaquette with four neighbors. Making each fully polarized plaquette convince its eight neighbors—we will call this Stauffer's rule—they found a phase transition for an initial density of up spins $d = 1/2$.

It is more realistic to associate a probability of persuasion to each site. The Sznajd model is robust with respect to this choice: If one convinces the neighbors with some probability p and leaves them unchanged with probability $1 - p$, a consensus is still reached after a long time [2]. Models allowing many different opinions (Potts's spins, for example) or defined on small-world networks were studied to better reproduce the behavior of real communities (see [2] and references therein). In another extension, in order to avoid full consensus and to make the model more realistic, Schneider introduced opportunists and persons in opposition, who are unconvinced by their neighbors [13].

In the real world, however, the dynamics of social relationships is more complex. Even when such more structured topologies as small-world networks are adopted to bring the SM closer to reality, a large number of details is often neglected. In order to advance toward realism, we recently considered a reputation mechanism [14]. We believe that the reputation of agents who hold the same opinion is an important factor in opinion propagation across the community. In other words, it is realistic to believe that the individuals will change their opinions under the influence of highly respected persons. The reputation limits the agents' power of persuasion, and we can expect the model in [14] to be more realistic than the standard one [4]. In fact, we showed that simple microscopic rules are sufficient to generate a democracy-like state, ferromagnetically ordered with only partial polarization [14].

In this work, we revise and extend our previous results by allowing reputations to increase and decrease, depending on whether the agents are or are not persuaded. This generalization is based on the behavior of real social networks: Certain persons tend to be skeptical if the persuaders have low reputation, in which case their best strategy is to keep their opinions. In this sense, including reputation makes the SM more realistic. To be thorough, we will consider two different protocols. In the first case, the agents' reputations increase for each persuaded neighbor, whereas in the second case, the agents' reputations rise in case of persuasion and decrease whenever the agents fail to convince their neighbors.

This paper is organized as follows: In Section 2, we present the model and define the microscopic rules. The numerical results as well as the finite-size scaling analysis are discussed in Section 3. Finally, in Section 4, we summarize our conclusions.

2 Model

We have considered a square lattice with $L \times L$ agents and periodic boundary conditions. Similar to Stauffer's rule (rule Ia of [4]), we choose at random a 2×2 plaquette with four neighbors. If all central spins are parallel, the neighbors may change their opinions. The difference in our model is that the neighboring spins will be flipped depending on the plaquette reputation. An integer number (R) labels each player and represents its reputation across the community, in analogy to the Naming game model considered by Brigatti [15]. The reputation is introduced as a score for each player and is time dependent. The agents start with a random distribution of R values, and as time evolves, the reputation of each agent changes according to its persuasive power. We will let the initial reputation of the agents follow a Gaussian distribution centered at 0 with given standard deviation σ .

Two different situations are studied in this work: In the first case, the reputations increase following predefined rules, whereas in the second case the reputations may rise and fall. A time step is defined by the following microscopic rules:

Case 1

1. We randomly choose a 2×2 plaquette of four neighbors.
2. If not all four center spins are parallel, leave its eight neighbors unchanged.
3. On the other hand, if the four center spins are fully polarized, we calculate the average reputation \bar{R} of the plaquette,

$$\bar{R} = \frac{1}{4} \sum_{i=1}^4 R_i, \quad (1)$$

where each R_i represents the reputation of one of the agents.

4. We compare the reputations of each of the eight neighbors of the plaquette with the average reputation. If the reputation of a neighbor is less than the average, this neighbor follows the plaquette orientation. On the other hand, if the neighbor's reputation exceeds \bar{R} , no action is taken.

5. For each persuasion, the reputation of the plaque-tte agents is incremented by 1, so that the average plaque-tte reputation is increased by 1.

Case 2

In this case, steps 1–4 are as described above. Step 5, by contrast, is changed to the following rule:

- For each persuasion, the reputation of the plaque-tte agents is incremented by 1. On the other hand, for each failure, the reputations within the plaque-tte are decremented by 1.

Thus, even in the case of fully polarized plaquettes, different numbers of agents may be convinced, namely 8, 7, 6, ..., 1, or 0. As pointed by Stauffer in [2], we can imagine that each agent in the Sznajd model carries an opinion that can either be up (e.g., Republican) or down (e.g., Democrat), which represents one of two possible opinions on any question. The objective of the agents in the game is to convince their neighbors. One can expect that, if a certain group of agents convince many others, their persuasive power grows. On the other hand, the persuasive powers may drop if the agents fail to convince other individuals. The inclusion of reputation in our model captures this feature of the real world.

3 Numerical Results

3.1 Case 1: Emergence of Consensus

In all simulations, $\sigma = 5$. Following previous work on the SM, we start with the time evolution of the magnetization per site,

$$m = \frac{1}{N} \sum_{i=1}^N s_i, \quad (2)$$

where $N = L^2$ is the total number of agents and $s_i = \pm 1$. In the standard SM defined on the square lattice [4], the enforcement of the Stauffer's rule, according to which a fully polarized 2×2 plaquette convinces its eight neighbors, an initial density of up spins $d = 1/2$ drives the system to the fixed points with all up or all down spins with equal probability. For $d < 1/2$ ($> 1/2$), the system goes to a ferromagnetic state with all spins down (up) in all samples. Clearly, in the limit of large L , a phase transition arises at $d = 1/2$. Fixed points with all spins parallel describe the public opinion in a dictatorship [4]—not a common situation nowadays.

By contrast, ferromagnetism with partial polarization corresponds to a democracy—a very common situation in our world.

Figure 1 shows the magnetization as a function of the simulation time in our model, for case 1. Figure 1a shows $d > 1/2$ ($< 1/2$); total consensus with all spins up (down) is then not be achieved in any sample. On the other hand, Fig. 1b shows situations where the consensus is obtained with all up (for $d = 0.9$) or down spins (for $d = 0.1$). These results indicate that (a) a democracy-like situation is possible in the model even without a mixture of different rules [4], or special agents such as contrarians and opportunists [13], and (b) if a phase transition also occurs in our case, the transition point will be located somewhere at $d > 1/2$.

We have also studied the relaxation times of the model, i.e., the time needed to find all the agents with the same opinion. The number of sweeps through the lattice, averaged over 10^4 samples, needed to reach the fixed point is shown in Fig. 2a. The relaxation-time distribution is compatible with a log-normal one, which yields a parabola in the log-log plot. The same behavior was observed in other studies of the SM [4, 13, 16].

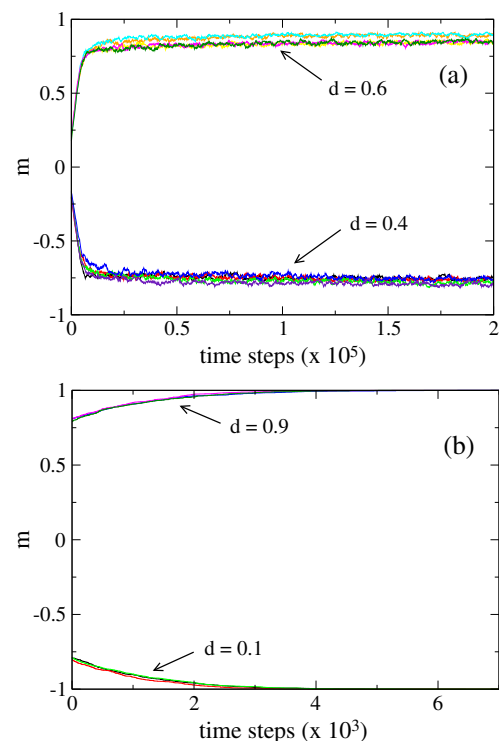


Fig. 1 Time evolution of the magnetization (case 1) for $L = 53$ and different samples. **a** Initial densities of up spins $d = 0.4$ and 0.6 . The steady states show only partial consensus, in contrast with the standard Sznajd model on the square lattice [4]. **b** Results for $d = 0.1$ and 0.9 . The system now reaches consensus in all samples

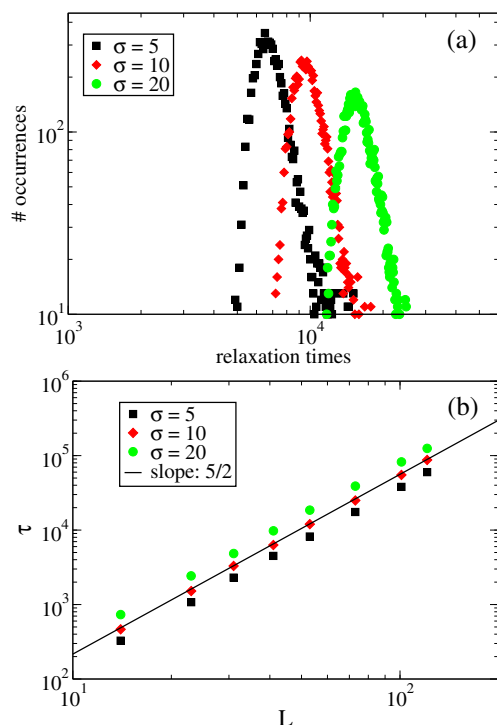


Fig. 2 **a** Log-log histogram of relaxation times (case 1) for $L = 53$ and $d = 0.8$ obtained from 10^4 samples, with agents' initial reputations following a Gaussian distribution with the indicated standard deviations σ . For all σ , the distribution is compatible with a log-normal one, which corresponds to the observed parabolas in the log-log plots. **b** Average relaxation time τ , over 10^4 samples, as a function of lattice size L (log-log scale). The slope of the straight line is $5/2$. The result is robust with respect to variations of σ

Figure 2b shows the average relaxation time τ , also over 10^4 samples, considering the relaxation times of Fig. 2a, as a function of lattice size L . The two quantities are related by a power law, $\tau \sim L^{5/2}$, for large L and all standard deviations, an indication that this result is robust with respect to changes in σ . Power laws relating τ and L were also found in a previous work on the SM [16].

We now focus on the phase transition. To this end, we simulate the system for different lattice sizes L , and we measure the fraction of samples which show all spins up when the initial density of up spins d is varied in the range $0.4 \leq d \leq 1.0$. In other words, this quantity f gives us the probability that the population reaches consensus, for a given value of d . We consider 1,000 samples for $L = 31$ and 53 , 500 samples for $L = 73$ and 101 , and 200 samples for $L = 121$. The results are shown in Fig. 3a. The transition point is located somewhere in the region $d > 1/2$, as pointed out above. In order to locate the critical point, we performed a finite-

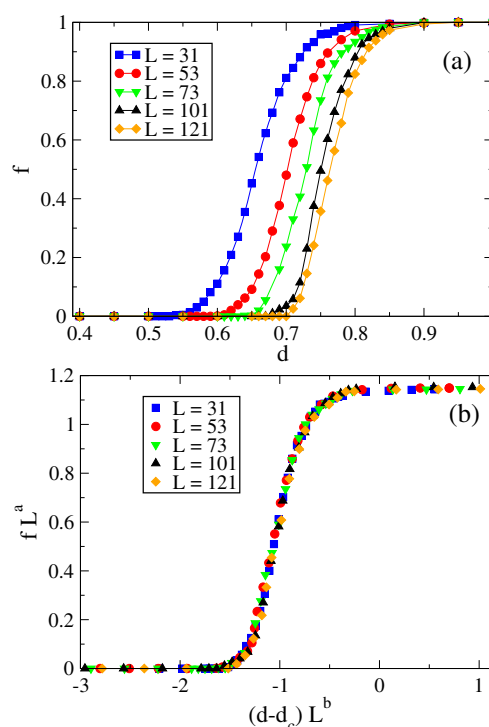


Fig. 3 **a** Fraction f of samples (case 1) showing all spins up when the initial density of up spins d is varied in the range $0.4 \leq d \leq 1.0$, for various lattice sizes L . The total number of samples are 1,000 (for $L = 31$ and 53), 500 (for $L = 73$ and 101), and 200 (for $L = 121$). **b** Scaling plot of f . The best collapse of data was obtained for $a = 0.03$, $b = 0.47$, and $d_c = 0.88$

size scaling (FSS) analysis, on the basis of standard FSS equations [14, 16],

$$f(d, L) = L^{-a} \tilde{f}((d - d_c) L^b), \quad (3)$$

where \tilde{f} is a scaling function, and

$$d_c(L) = d_c + c L^{-b}. \quad (4)$$

where c is a constant.

The result is shown in Fig. 3b. We have found that

$$d_c = 0.88 \pm 0.01, \quad (5)$$

in the limit of large L . In addition, we have obtained $a = 0.030 \pm 0.005$ and $b = 0.47 \pm 0.02$. The critical point occurs at $d > 1/2$, in contrast with the SM without reputation on the square lattice. This is easily understood: At each time step, the randomly chosen 2×2 plaquette may convince 8, 7, 6, ..., 1 or 0 neighbors, even if the plaquettes' spins are parallel. In the standard model if the plaquette spins have the same orientation, eight neighbors are convinced immediately. It follows that a smaller initial density of up spins drives the system to the fixed point with all spins up. Thus, the usual

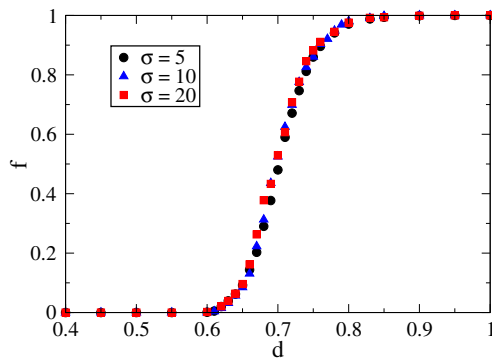


Fig. 4 Fraction f of samples (case 1) showing all spins up when the initial density d is varied in the range $0.4 \leq d \leq 1.0$, for $L = 53$, 1,000 samples and different σ . We see that f is robust against changes in σ

phase transition of the SM is also seen in our model, in case 1, and this transition is robust with respect to changes in σ (see Fig. 4).

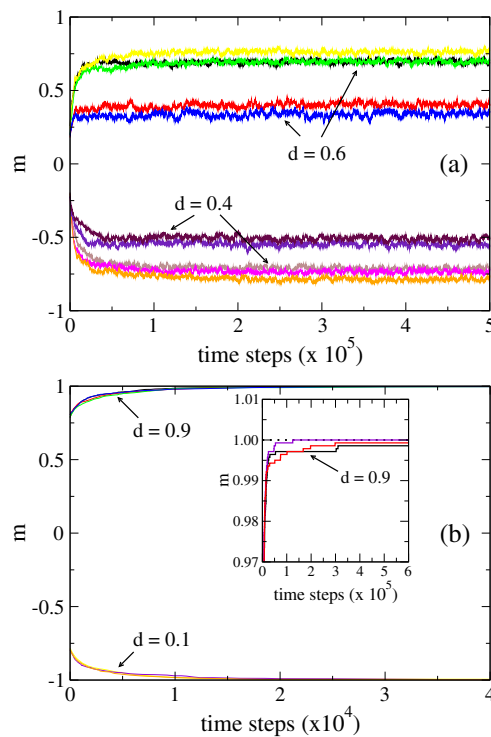


Fig. 5 **a** Time evolution of the magnetization (case 2) for $L = 53$ and different samples. Initial densities of up spins $d = 0.4$ and 0.6 . Although there are differences between these steady states and those of case 1, democracy-like situations also seen. **b** Results for $d = 0.1$ and 0.9 . The *inset* shows that even for large values of densities, such as $d = 0.9$, the system reaches consensus in only a fraction of the samples. Analogous comments apply for $d = 0.1$. The dotted line is $m = 1$ (full consensus, see the inset)

3.2 Case 2: Competition Among Reputations

As discussed in Section 2, in this second case, each agent's reputation may increase or decrease, which defines a competition among the reputations in the game. The evolution of the magnetization per site is shown in Fig. 5. For intermediary densities d , the system reaches democracy-like steady states, with $m < 1$. The results nonetheless show a variety of steady states, with different magnetizations, due to the competition among reputations, which grow and decay depending on the evolution of the average reputation of the plaquettes. Another consequence of the competition appears for large or very small initial densities d : Even for $d = 0.9$ and $d = 0.1$, the system reaches consensus only in a few realizations of the dynamics. This fact is visible in the inset of Fig. 5b, where the dotted line represents $m = 1$: Only one of the three realizations reaches consensus. Thus, in case 2, it is difficult to obtain a consensus. Nonetheless, in comparison with case 1, case 2 favors the emergence of democratic steady states and is hence more realistic.

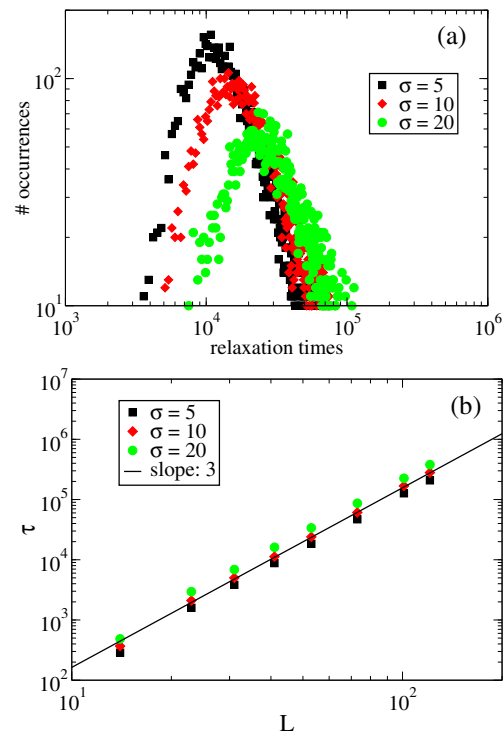


Fig. 6 **a** Log-log histogram of relaxation times in case 2 for $L = 53$ and $d = 0.99$, obtained from 10^4 samples, with the agents' initial reputations following a Gaussian distribution with the displayed standard deviations σ . For each σ , the results are compatible with a log-normal distribution, corresponding to the parabola in the plot. **b** Average relaxation time τ , over 10^4 samples, as a function of lattice size L (log-log scale). For large L , the relaxation time follows a power law, $\tau \sim L^3$, for each σ

We have also studied the relaxation times for case 2. The number of sweeps through the lattice (averaged over 10^4 samples) needed to reach the fixed point is shown in Fig. 6a for three standard deviations σ . As in case 1, for each σ , the plot is compatible with a log-normal distribution of relaxation times. Due to the competition among reputations, the relaxation times of case 2 are nevertheless larger than in case 1. Figure 6b shows the average relaxation times τ , also over 10^4 samples, considering the relaxation times of Fig. 6a, as a function of lattice size L . For large L and all σ , the log-log plot identifies a power law, $\tau \sim L^3$. In other words, the competition among reputations increases the relaxation times of the system. Since this enhancement grows with the number of agents (or the lattice size L), the power-law exponent relating τ and L is larger than the exponent in case 1.

Following the approach of Section 3.1, with the same number of samples, we have simulated the system for different lattice sizes L and measured the fraction of samples with all spins up for initial densities of up spins d in the range $0.5 \leq d \leq 1.0$. The results are depicted in Fig. 7a, which points to a transition point in the region

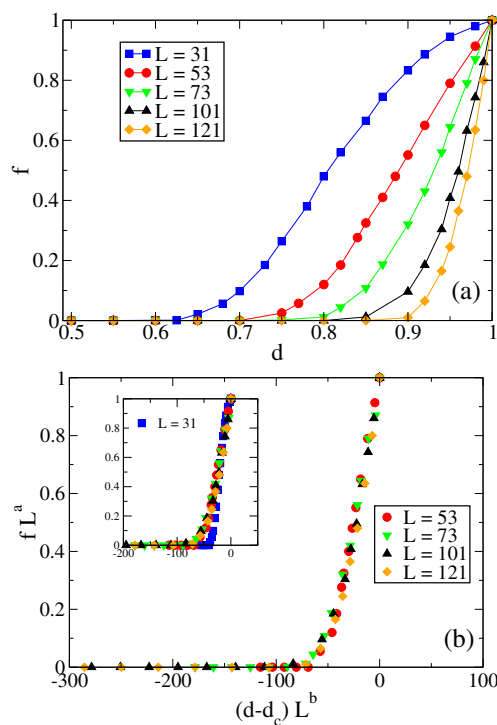


Fig. 7 **a** Fraction f of samples in case 2 with all spins up for initial density of up spins d in the range $0.5 \leq d \leq 1.0$ and the displayed lattice sizes L . The number of samples are 1,000 (for $L = 31$ and 53), 500 (for $L = 73$ and 101), and 200 (for $L = 121$). **b** Scaling plot of f resulting from the optimal choice of parameters: $a = 0.0$, $b = 1.37$, and $d_c = 1.0$. To minimize the finite-size effects, we have excluded the smallest size $L = 31$ from the inset

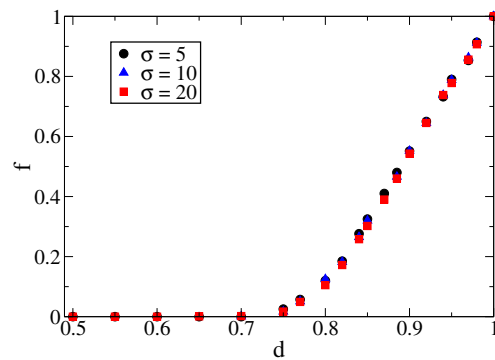


Fig. 8 Fraction f of samples in case 2 showing all spins up for initial densities d in the range $0.5 \leq d \leq 1.0$, for $L = 53$, 1,000 samples and the displayed σ . This shows that f is insensitive to increments in σ

$d > 0.88$. The critical density in case 2 is therefore larger than in case 1, as expected in view of the competition among reputations. We have obtained the critical point from Equations (3) and (4). The best collapse of the data is shown in Fig. 7b, obtained with $d_c = 1.00 \pm 0.01$, $a = 0.00 \pm 0.01$, and $b = 1.37 \pm 0.02$. In other words, in comparison with case 1, case 2 yields a different critical density and different critical exponents. However, the usual SM phase transition also occurs in case 2 and is robust with respect to changes in σ (see Fig. 8).

In order to minimize the finite-size effects, we have excluded the smallest size, $L = 31$ (see the inset of Fig. 7b), from the FSS process. In fact, as the squares in Fig. 7a shows, the $L = 31$ curve has an inflection point not found in the other plots. This inflection point is hence an outstanding finite-size effect of the case 2 model.

4 Conclusions

We have studied a modified version of the Sznajd sociophysics model. In particular, we have considered reputation, a mechanism that limits the agents' power of persuasion. Introduced as a score for each player, the reputation changes with time according to model-dependent rules. The agents start with a random distribution of reputations. As time evolves, each agent's reputation rises and falls in response to changes in the agent's persuasive power. The initial reputation of the agents follows a Gaussian distribution centered at 0 with given standard deviation σ . We have studied separately two different protocols: In case 1, the reputations increase after each persuasion; in case 2, the reputations

increase after persuasions and decrease if a group of agents fails to convince one of its neighbors.

In the first case, we obtained a log-normal-like distribution of relaxation times, i.e., of the times elapsed until all agents had the same opinion. The average relaxation times grow with the linear dimension of the lattice according to the power law, $\tau \sim L^{5/2}$. The system undergoes the usual phase transition. To identify the latter, we monitored the fraction f of samples showing all spins up when the initial density of up spins d is varied. In other words, the fraction f measures the probability of a population reaching consensus, for given d . We determined the transition point by means of a finite-size scaling analysis and found $d_c = 0.88$. It is easy to understand why d_c is greater than the value $1/2$ found by Stauffer et al. [4] in the standard formulation of the Sznajd model: At each time step, the randomly chosen 2×2 plaquette may convince 8, 7, 6, ..., 1, or 0 neighbors, even if the its own spins are parallel. In the standard case, if the plaquette is fully polarized, all of its eight neighbors are immediately convinced; a smaller initial density of up spins is therefore necessary to drive the system toward the fixed point with all spins up. The simulations indicate that the phase transition is robust with respect to changes in σ .

In case 2, the competition among reputations favors steady states with $m < 1$, and even for large densities d only in some samples a consensus ever reached. As in case 1, the relaxation times are log-normally distributed and are related to the linear dimension L by a power law; both the average relaxation times and the exponent of the power law, $\tau \sim L^3$, are larger than in case 1. The usual phase transition also occurs in case 2, but the

critical density was found to be $d_c = 1.0$. In additional contrast with case 1, case 2 displays strong finite-size effects. The differences between the two cases are due to the competition among reputations, present in case 2 and absent from case 1.

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