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PARTICLES AND FIELDS

The Influence of Hyperons and Strong Magnetic Field in Neutron Star Properties

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Abstract Neutron stars are among the most exotic objects in the universe and constitute a unique laboratory to study nuclear matter above the nuclear saturation density. In this work, we study the equation of state (EoS) of the nuclear matter within a relativistic model subject to a strong magnetic field. We then apply this EoS to study and describe some of the physical characteristics of neutron stars, especially the mass–radius relation and chemical compositions. To study the influence of the magnetic field and the hyperons in the stellar interior, we consider altogether four solutions: two different magnetic fields to obtain a weak and a strong influence; and two configurations: a family of neutron stars formed only by protons, electrons, and neutrons and a family formed by protons, electrons, neutrons, muons, and hyperons. The limit and the validity of the results found are discussed with some care. In all cases, the particles that constitute the neutron star are in β equilibrium and zero total net charge. Our work indicates that the effect of a strong magnetic field has to be taken into account in the description of magnetars, mainly if we believe that there are hyperons in their interior, in which case the influence of the magnetic field can increase the mass by more than 10 %. We have also seen that although a magnetar can reach 2.48 $M_{\odot}$, a natural explanation of why we do not know pulsars with masses above 2.0 $M_{\odot}$ arises. We also discuss how the magnetic field affects the strangeness fraction in some standard neutron star masses, and to conclude our paper, we revisit the direct Urca process related to the cooling of the neutron stars and show how it is affected by the hyperons and the magnetic field.

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1 Introduction

Neutron stars are compact objects maintained by the equilibrium of gravity and the degenerescence pressure of the fermions together with a strong nuclear repulsion force due to the high density reached in their interior. Since we do not know yet the precise and detailed structure and composition of the inner core of a neutron star, many models have been used to describe it. In the literature, we can find some standard ones: hadronic neutron stars, quark stars, strange stars, and hybrid stars [1–3].

In the present work, we study a hadronic neutron star constituted by nucleons and hyperons and subjected to a strong magnetic field. The presence of hyperons is justifiable since the constituents of neutron stars are fermions. So, according to the Pauli Principle, as the baryon density increases, so do the Fermi momentum and the Fermi energy. Ultimately, the Fermi energy exceeds the masses of the heavier baryons [1]. On the other hand, some strange objects, such as the soft gamma-ray repeaters and anomalous X-ray pulsars can be explained assuming that these objects are neutron stars subject to a strong magnetic fields on their surface. These objects are called magnetars [4]. Although the magnetic field of the magnetars do not exceed $10^{15}G$ in their surface, it is well accepted on the literature that the magnetic field in the core of the neutron stars can reach values greater than $10^{18}G$ [5, 6]. Due to the large densities in the neutron star interior, we do not expect any significant influence of the magnetic field until it reaches values on the order of $10^{18}G$. Besides, it is well established in the literature that the direct Urca process is an efficient method to cool neutron stars if the proton fraction reaches values of $11 \sim 15$ % [7]. Hence, the study of the proton fraction in the interior of the neutron stars is very important to determine how fast the cooling is.

This paper is organized as follows: we make a review of the formalism of the nonlinear Walecka model (NLWM) in the presence of a magnetic field. Then we present the numerical results showing how the presence of hyperons and a strong magnetic field affects the equations of state (EoS) and the chemical composition. We study how these terms alter the macroscopic mass–radius relation of the neutron stars and compare our results with those found in literature. To conclude our paper, we discuss how the magnetic field affects the strangeness fraction for some standard masses, and revisit the direct Urca process.

2 The Formalism

The total Lagrangian is given by [8, 9]:

$$\mathcal{L} = \sum_b \mathcal{L}_b + \mathcal{L}_m + \sum_l \mathcal{L}_l + \mathcal{L}_\mathcal{B}, \tag{1}$$

where b stands for the baryons, m for the mesons, l for the leptons, and $\mathcal{B}$ for the electromagnetic field itself. The sum in b can run over the eight lighter baryons and in l over the two lighter leptons. Explicitly, in the presence of a electromagnetic field, the Lagrangian is

$$\mathcal{L}_b = \bar{\Psi}_b \big[\gamma_u \big(i\partial^\mu - eA^\mu - g_{v,b}\omega^\mu - g_{\rho,b} I_{3b}\rho^\mu\big) - (M_b - g_{s,b}\sigma)\big]\Psi_b, \tag{2}$$

$$\mathcal{L}_m = \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}m_s^2\sigma^2 + \frac{1}{2}m_v^2\omega_\mu\omega^\mu - \frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} + \frac{1}{2}m_\rho^2(\rho_\mu\rho^\mu) - \frac{1}{4}P_{\mu\nu}P^{\mu\nu} - \frac{1}{3!}\kappa\sigma^3 - \frac{1}{4!}\lambda\sigma^4, \tag{3}$$

$$\mathcal{L}_l = \bar{\psi}_l[\gamma_u(i\partial^\mu - eA^\mu)] - m_l]\psi_l, \tag{4}$$

$$\mathcal{L}_\mathcal{B} = -\frac{1}{16\pi}F_{\mu\nu}F^{\mu\nu}, \tag{5}$$

where Ψ_b and ψ_l are the baryon and lepton Dirac fields, respectively. The baryon mass and isospin projection are denoted by M_b and I_{3b}, respectively. The masses of the leptons are m_l, and the electric charge of the particles is given by e. The antisymmetric mesonic and electromagnetic field strength tensors are given by their usual expressions: $\Omega_{\mu\nu} = \partial_u\omega_\nu - \partial_\nu\omega_\mu$, $P_{\mu\nu} = \partial_\mu\rho_\nu - \partial_\nu\rho_\mu$ and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. The γ_μ are the Dirac matrices [10]. The strong interaction couplings are denoted by g, and the meson masses by m, all with appropriate subscripts. The second subscript of the g constant is due to the distinctive coupling of hyperons with the mesons. In this work, we assume that $g_{s,H} = 0.7g_{s,N}$; $g_{v,H} = 0.783g_{v,N}$, and $g_{\rho,H} = 0.783g_{\rho,N}$ [11], where H denotes hyperons and N nucleons. The hadronic part of the Lagrangian is the called NLWM. The leptons are included in the total Lagrangian density as a non-interacting Fermi gas in order to account for the β equilibrium in the star.

To solve the equation of motion, we use the mean field approximation, where the meson fields are replaced by their expectation values, i.e.: $\sigma \to \langle\sigma\rangle = \sigma_0$, $\omega^\mu \to \delta_{0\mu}\langle\omega^\mu\rangle = \omega_0$ and $\rho^\mu \to \delta_{0\mu}\langle\rho^\mu\rangle = \rho_0$.

In this work, we use a GM1 parametrization [12], which can describe the most important properties of nuclear matter and reproduce the macroscopic properties of the neutron stars consistent with those observed in nature. The GM1 parameters are showed in Table 1.

This parametrization is fixed so that the incompressibility of nuclear matter $K = 300$ MeV and the nuclear saturation density $n_0 = 0.153\,fm^{-3}$. The masses of the baryon octet are $M_N = 939$ MeV (nucleons), $M_\Lambda =$ 1,116 MeV, $M_\Sigma =$1,193 MeV, and $M_\Xi = 1{,}318$ MeV. The meson masses are $m_s = 400$ MeV, $m_v = 783$ MeV, and $m_\rho = 770$ MeV. The masses of the leptons are $m_e = 0.511$ MeV and $m_\mu = 105.66$ MeV. Applying the Euler–Lagrange in (1) in the absence of an electric field, the equation of motion in the mean field approximation for an arbitrary baryon becomes

$$\big[\gamma_0\big(i\partial^0 - g_{v,b}\omega_0 - g_{\rho,b} I_{3b}\rho_0\big) - \gamma_j(i\partial^j - eA^j) - M_b^*\big]\Psi = 0, \tag{6}$$

where

$$M_b^* = M_b - g_{s,b}\sigma_0, \tag{7}$$

is the baryon effective mass.

For an uncharged particle, eA^μ is always zero. The quantization rules are $i\partial^0 = E$ and $i\partial^j = k^j$, where k^j is

Table 1 Values of GM1 parametrization

Set	g_s/m_s^2	g_v/m_v^2	g_ρ/m_ρ^2	κ/M_N	λ
GM1	11.785 fm^2	7.148 fm^2	4.410 fm^2	0.005894	–0.006426

the momentum in the j direction. Setting $E - g_{v,b}\omega_0 - g_{\rho,b} I_{3b}\rho_0 = E^*$, we have the following equation of motion written in a block matrix:

$$\begin{pmatrix} (E^* - M_b^*) & -\sigma.\mathbf{k} \\ \sigma.\mathbf{k} & -(E^*\rho_0 + M_b^*) \end{pmatrix} \begin{pmatrix} u_A \\ u_B \end{pmatrix} = 0. \quad (8)$$

This is an eigenvalue equation, which can be solved as the free Dirac equation for an effective mass and energy, whose solution is

$$E = \sqrt{k^2 + M_b^{*2}} + g_{v,b}\omega_0 + g_{\rho,b} I_{3b}\rho_0 = \mu, \quad (9)$$

where μ is the chemical potential.

For a charged baryon, the Dirac equation assumes the following form:

$$\begin{pmatrix} (E^* - M_b^*) & -\sigma.(\mathbf{k} - e\mathbf{A}) \\ \sigma.(\mathbf{k} - e\mathbf{A}) & -(E^* - M_b^*) \end{pmatrix} \begin{pmatrix} u_A \\ u_B \end{pmatrix} = 0. \quad (10)$$

To produce a constant magnetic field in the z direction we, choose: $A_2 = A_3 = 0$, $A_1 = -By$. The solution of this eigenvalue equation is well known in the literature [5, 8, 13]:

$$E^* = \sqrt{M_b^{*2} + k_z^2 + 2\nu|e|B},$$

$$E = \sqrt{M_b^{*2} + k_z^2 + 2\nu|e|B} + g_{v,b}\omega_0 + g_{\rho,b} I_{3b}\rho_0 = \mu, \quad (11)$$

where the discrete parameter ν is called Landau level (LL), and μ is the chemical potential. The first LL, $\nu = 0$, is nondegenerate and all the others are two-fold degenerate. For the leptons, since they do not feel the strong force,

$$E_l = \sqrt{m_l^2 + k_z^2 + 2\nu|e|B} = \mu_l. \quad (12)$$

The expected values for the mesons are as follows:

$$\omega_0 = \sum_{ub} \frac{g_{v,b}}{m_v^2} n^{ub} + \sum_{cb} \frac{g_{v,b}}{m_v^2} n^{bc}, \quad (13)$$

$$\sigma_0 = \sum_{ub} \frac{g_{s,b}}{m_s^2} n_s^{ub} + \sum_{cb} \frac{g_{s,b}}{m_s^2} n_s^{bc} - \frac{1}{2}\frac{\kappa}{m_s^2}\sigma_0^2 - \frac{1}{6}\frac{\lambda}{m_s^2}\sigma_0^3, \quad (14)$$

$$\rho_0 = \sum_{ub} \frac{g_{\rho,b}}{m_\rho^2} n^{ub} I_{3b} + \sum_{cb} \frac{g_{\rho,b}}{m_\rho^2} n^{cb} I_{3b}, \quad (15)$$

where n^{cb} and n^{ub} are the number density of the "charged baryons" and "uncharged baryons," respectively [14, 15], and n_s^{cb} and n_s^{ub} are called scalar density for the charged and uncharged baryons [8]. In $T = 0$[1] and they are given by

$$\mathrm{d}n^{ub} = \frac{8\pi k^2}{(2\pi)^3} \quad \rightarrow \quad n^{ub} = \int_0^{k_f} \frac{8\pi k^2}{(2\pi)^3} = \frac{k_f^3}{3\pi^2}, \quad (16)$$

$$\mathrm{d}n^{cb} = \frac{|e|B}{(2\pi)^2}\eta(\nu) dk_z,$$

$$n^{cb} = \frac{|e|B}{(2\pi)^2}\sum_\nu^{\nu_{\max}} \eta(\nu) \int_{-k_f}^{k_f} \mathrm{d}k_z = \frac{|e|B}{2\pi^2}\sum_\nu^{\nu_{\max}} \eta(\nu) k_f, \quad (17)$$

$$n_s^{ub} = \frac{1}{\pi^2}\int_0^{k_f} \frac{M_b^* k^2 \mathrm{d}k}{\sqrt{M_b^{*2} + k^2}}, \quad (18)$$

$$n_s^{cb} = \frac{|e|B}{2\pi^2}\sum_\nu^{\nu_{\max}} \eta(\nu) \int_0^{k_f} \frac{M_b^* \mathrm{d}k_z}{\sqrt{M_b^{*2} + k_z^2 + 2\nu|e|B}}. \quad (19)$$

The summation in ν in the above expressions ends at $\nu_{\max}$, the largest value of ν for which the square of Fermi momenta of the particle is still positive and which corresponds to the closest integer from below defined by the ratio:

$$\nu_{\max} \leq \frac{\mu^2 - M_b^{*2}}{2|e|B}, \quad \text{charged baryons} \quad (20)$$

$$\nu_{\max} \leq \frac{\mu_l^2 - m_l^2}{2|e|B}. \quad \text{leptons} \quad (21)$$

Now we couple the equations imposing β equilibrium and zero total net charge:

$$\mu_{b_i} = \mu_n - e_i\mu_e, \quad \mu_e = \mu_\mu, \quad \sum_b e_b n^b + \sum_l e_l n^l = 0, \quad (22)$$

where μ_{b_i} and e_i are the chemical potential and electric charge of the i-th baryon; and μ_n, μ_e, and μ_μ are the chemical potential of the neutron, electron, and muon respectively; and n^b is the number density of the baryons, and n^l is the number density of the leptons.

The energy density of the neutron star is

$$\epsilon = \sum_{ub} \epsilon_{ub} + \sum_{cb} \epsilon_{cb} + \sum_l \epsilon_l + \sum_m \epsilon_m + \frac{B^2}{8\pi}, \quad (23)$$

[1] This can be justified since the Fermi temperature of the neutron stars is very high compared to its own temperature [16].

where the energy densities for the uncharged baryons, charged baryons, leptons, and mesons have the following forms:

$$\epsilon_{ub} = \frac{1}{\pi^2}\int_0^{k_f}\sqrt{M_b^{*2}+k^2}k^2dk, \tag{24}$$

$$\epsilon_{cb} = \frac{|e|B}{2\pi^2}\sum_{\nu}^{\nu_{\max}}\eta(\nu)\int_0^{k_f}\sqrt{M_b^{*2}+k_z^2+2\nu|e|B}\mathrm{d}k_z, \tag{25}$$

$$\epsilon_{l} = \frac{|e|B}{2\pi^2}\sum_{\nu}^{\nu_{\max}}\eta(\nu)\int_0^{k_f}\sqrt{m_l^{2}+k_z^2+2\nu|e|B}\mathrm{d}k_z, \tag{26}$$

$$\epsilon_m = \frac{1}{2}m_s^2\sigma_0^2+\frac{1}{2}m_v^2\omega_0^2+\frac{1}{2}m_\rho^2\rho_0^2+\frac{1}{3!}\kappa\sigma_0^3+\frac{1}{4!}\lambda\sigma_0^4. \tag{27}$$

To find the pressure, we use the second law of thermodynamics, which gives an isotropic pressure:

$$p = \sum_i \mu_i n^i - \epsilon + \frac{B^2}{8\pi}, \tag{28}$$

where the sum runs over all fermions. Note that the contribution from electromagnetic fields should be taken into account in the calculation of the energy density and the pressure.

2.1 Tolman–Oppenheimer–Volkoff Equations and the Density-Dependent Magnetic Field

The magnetic field on the surface of the magnetars are of order of $10^{15}G$ but can reach more than $10^{18}G$ in their cores. To reproduce this behavior, we use a density-dependent magnetic field given by [5, 13, 17]:

$$B(n) = B^{suf} + B_0\left[1 - exp\left\{-\beta\left(\frac{n}{n_0}\right)^{\alpha}\right\}\right], \tag{29}$$

where B^{suf} is the magnetic field on the surface of the neutron stars, taken as $10^{15}G$; n is the total number density, $n = \sum n^b$, B_0 is the constant magnetic field. The two free parameters β and α are chosen to reproduce a weak magnetic field below the nuclear saturation density, and a quickly growing one when $n > n_0$, in such a way that $B(n) > 0.95B_0$ when $n = 6n_0$. To reproduce this behavior, we have set $\beta = -6.5\cdot 10^{-3}$ and $\alpha = 3.5$. Now B is replaced by $B(n)$ in the term $B^2/8\pi$ in our EoS.

To finish our analytical analysis, we write the Tolman–Oppenheimer–Volkoff (TOV) [18] equations:

$$\frac{\mathrm{d}M}{\mathrm{d}r} = 4\pi r^2\epsilon(r), \tag{30}$$

$$\frac{\mathrm{d}p}{\mathrm{d}r} = -\frac{G\epsilon(r)M(r)}{r^2}\left[1+\frac{p(r)}{\epsilon(r)}\right] \times\left[1+\frac{4\pi p(r)r^3}{M(r)}\right]\left[1-\frac{2GM(r)}{r}\right]^{-1}, \tag{31}$$

which are the differential equations for the structure of a static, spherically symmetric, relativistic star in hydrostatic equilibrium. The equation of states developed in this work are used as input for these equations. There are three minor problems with this approach. First, the pressure is not really isotropic. Anisotropies arise due to the preferential direction z of the magnetic field. Second, the energy of the magnetic field itself is a further source of gravitation that may induce a gravitational collapse. Third, the gradient of the pressure is a source of repulsion, which counter-balances gravity. So, if the magnetic field is strong enough, the neutron star may blow up. We discuss the validity of the symmetric TOV equations more carefully at the end of the paper.

3 Results and Discussion

We consider two families of neutron stars: one containing just protons, electrons, and neutrons, which we call "atomic stars" denoted by the letter A in the legends; and other containing protons, electrons, neutrons,

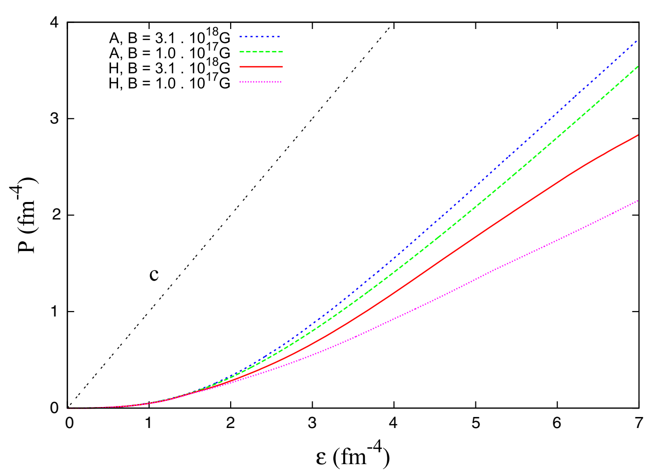

Fig. 1 (Color online) EoS for two atomic and two hyperonic stars obtained with different values of the magnetic field. The *straight line* corresponds to the causal limit, for which $\epsilon = p$

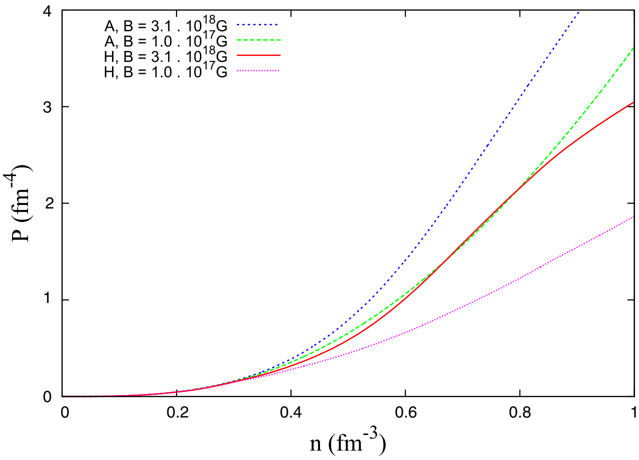

Fig. 2 (Color online) Pressure as function of number density for the four stars. The relative difference between *blue-green* (*red-pink lines*) for atomic (hyperonic) stars is the direct influence of the magnetic field

muons, and hyperons, which we call "hyperonic stars" denoted by the letter H in the legends. In the results, we also include the crust of neutron star through the BPS EoS [19] but always take into account the contribution of the magnetic field through the term $B(n)^2/8\pi$ in the EoS.

We choose two values for the magnetic field: $1.0 \cdot 10^{17}G$ and $3.1 \cdot 10^{18}G$ to produce a weak and a strong influence. We also include here some theoretical and observational constrains. First, all our EoS are causal and obey the Le Chantelier principle, i.e., the quantity $\mathrm{d}p/\mathrm{d}\epsilon$ lies between 0 and 1. We plot the numerical results of four EoS in Fig. 1.

As we can see in Fig. 1, the presence of hyperons softens the EoS more than the influence of the magnetic field can stiffen it. No matter how strong is the magnetic field in the interior of the magnetar, the EoS of an atomic star is always stiffer than a hyperonic one. We can also see that all our EoS are causal (the "c" line is the causality limit).

In Figs. 1 and 3, we note that both the EoS and the fraction of particles $Y_i = n_i/n$ are not affected significantly by a magnetic field about $10^{17}G$. The reason is that a field of this magnitude is too weak to contribute to the final EoS and to the fraction of particles. We also see that the magnetic field affects more the hyperonic stars than the atomic ones. As the hyperonic stars are softer than the atomic ones, they are therefore more sensitive to the presence of the magnetic field. Also the hyperonic stars are denser than the atomic stars, with a bigger central density n_c. Moreover, for a fixed value of density, the pressure of the hyperonic stars are smaller than the atomic ones. So, as the magnetic field couples to the number density through (29), the contribution of the magnetic field is always greater in hyperonic than in atomic stars. We show this result plotting the pressure p in function of n in Fig. 2.

Figures 3 and 4 show the fraction of particles. We can see that for the strong field, the appearance of charged particles is favored at low densities due to their dependence on the magnetic field, as expected from (17). The behavior of the particles in the presence of a strong magnetic field is also altered. For weak fields, the population of a kind of particle is always well behaved, while for the strong one, many kinks appear. The reason is that in the absence of a magnetic field, the number density of a determined kind of particle grows

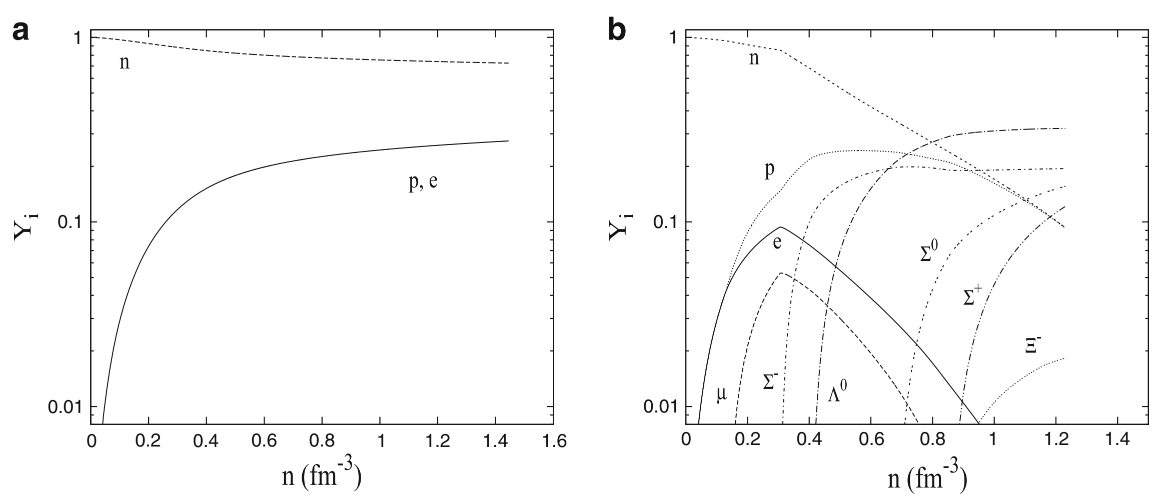

Fig. 3 **a** Atomic stars and **b** hyperonic stars for a magnetic field of $1.0 \cdot 10^{17}G$

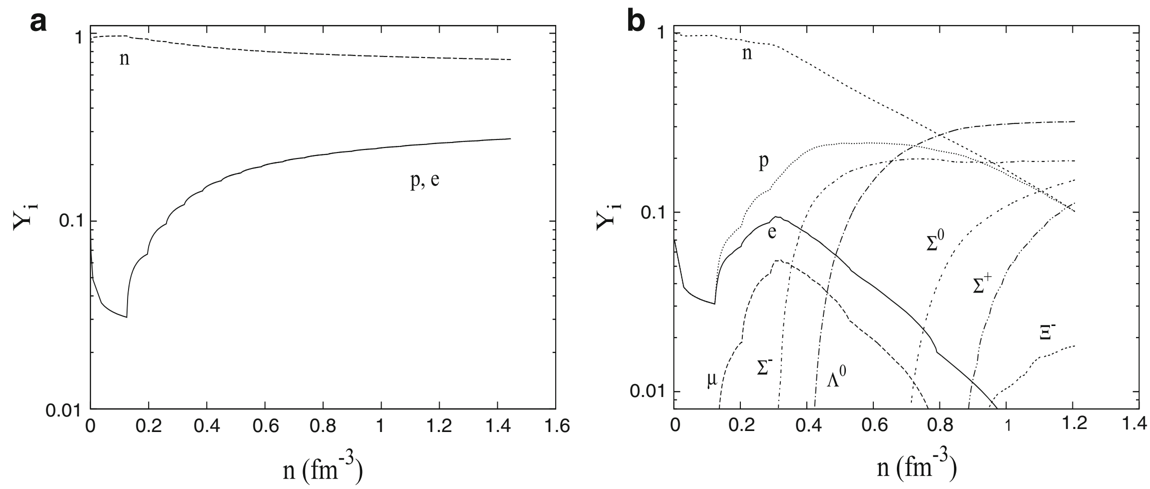

Fig. 4 **a** Atomic stars and **b** hyperonic stars for a magnetic field of $3.1 \cdot 10^{18} G$

smoothly with the momentum. When a magnetic field is present, there is also a dependence of the discrete LL. For a weak magnetic field, a lot of Landau levels are occupied; but for a strong magnetic field, just a few of them are filled. So the orbit normal to the z direction is tightly quantized. This effect is more evident in the hyperonic stars. Each nozzle in a slope of a determined particle indicates that the density is high enough to create another Landau level. For a high density, there are so many Landau levels available that the distribution approaches to the continuous, while for a weak magnetic field, even in a low density, there are several LL, so there is no significant difference with the case in the absence of the magnetic field. One can notice that for hyperonic stars, at densities on the order of $0.8\ fm^{-3}$, the neutron is no longer the most important constituent. From this point, the Λ^0 hyperon dominates in the region of high densities.

We can also ask how the magnetic field affects the strangeness fraction, defined as

$$f_s = \frac{1}{3} \frac{\sum_j |s_j| n_j}{n}, \tag{32}$$

where s_j is the strangeness of baryon j and n is the total number density. We plot the results in Fig. 5.

As we can see, although the magnetic field causes a difference in the fraction of a single particle, no difference is found in the total strangeness fraction. For a fixed density, if a new particle is created, another has to disappear due to the β equilibrium condition. Hence, although the Landau quantization affects a single particle, this effect is washed out when all of them are summed.

It is hard to compare quantitatively our results with others existing in the literature due to many different parametrizations. In the absence of hyperons, we see that our EoS is stiffer than those presented in [8, 9] since the authors of those references do not consider a density-dependent magnetic field. With relation to the fraction of particles Y_i, the fraction of protons in a low-density region is more favored in [8] than in our work. Now for a magnetic field of $1.0 \cdot 10^{17} G$ there is virtually no difference compared with the results with zero magnetic field presented in [11].[2]

When a strong magnetic field is applied, we see that our parametrization does not prevent the hyperon formation in a low-density region as showed in [20]. Also, in our work, the influence of Landau quantization

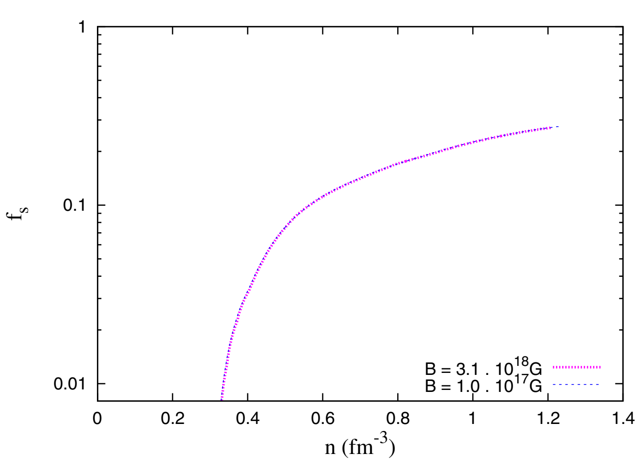

Fig. 5 Strangeness fraction for a weak and a strong magnetic field

[2]The same holds for the TOV solution showed.

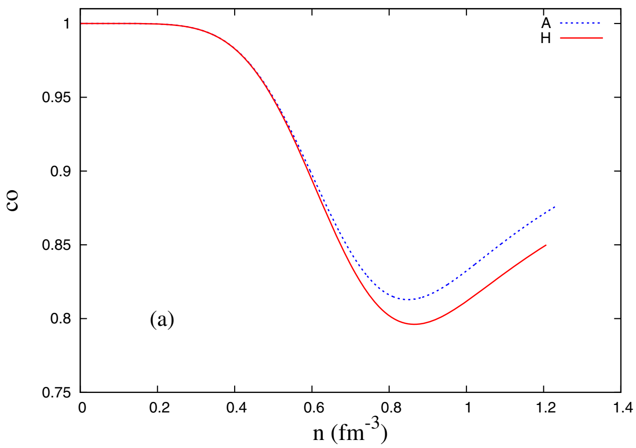

Fig. 6 (Color online) Quantity of co, showing that the energy density of the magnetic field is not dominant

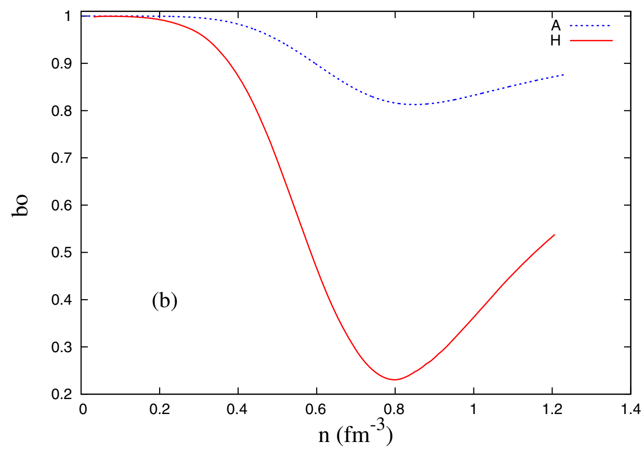

Fig. 7 (Color online) Quantity of bo, showing that the neutron star is always gravitationally bound

is much more evident even for a smaller value of the magnetic field.

In order to validate our EoS, we have to solve the TOV equations and check if the results agree with observational constraints. But, first, we have to check if the TOV equations are allowed to be used as a good approximation. The first point is that the pressure is not really isotropic. However, a recent work [21] showed that the anisotropy is not significant until the magnetic field reaches $B \simeq 3.2 \cdot 10^{18} G$. This is the reason we use the value $3.1 \cdot 10^{18} G$ as the strongest magnetic field. The second concerns the magnetic field as a source of gravitational energy. We expect that the magnetic field induces gravitational collapse. A simple way to avoid it is by requiring that the energy of the strongest magnetic field, $B_0 = 3.1 \cdot 10^{18} G$, to not be dominant. In other words, we require that $\epsilon_M > B(n)^2/8\pi$, where ϵ_M is the energy density of matter, given by (23) without the last term, which is the energy of the magnetic field itself. We can define the dimensionless quantity co

$$co = \left(1 - \frac{B(n)^2}{8\pi\epsilon_M}\right), \tag{33}$$

and require that co^3 is never negative. As we can see in Fig. 6, this imposition is fully filled.

The last point refers to the pressure of the magnetic field. If the magnetic field is too strong, the neutron star cannot be gravitationally bounded. In order to study the limits of the magnetic field such that it does not blow up the star, we use the approximation for the radial magnetic field presented in [22]. For the neutron star to be gravitationally bound, the term $M(r) + 4\pi r^3(p - B(n)^2/8\pi)$—where p here is the pressure of matter given by Eq. (28) without the term of the magnetic field itself—has always to be positive.[4] So, if the term $p - B(n)^2/8\pi$ is positive, of course the total sum also is. Now we define another dimensionless quantity bo:

$$bo = \left(1 - \frac{B(n)^2}{8\pi p}\right), \tag{34}$$

and require that bo is always positive. In Fig. 7, we see that this requirement is fulfilled, so we guarantee that the neutron star is gravitationally bound for all fields used in this work.

Now we can return to the TOV constraints.The star masses cannot exceed the maximum theoretical neutron star mass of 3.2 solar masses [23]. The EoS has to be able to predict the 1.97-solar mass neutron star [24] and to be in agreement with the redshift measurements (z) of two neutron stars. A redshift of $z = 0.35$ has been obtained from three different transitions of the spectra of the X-ray binary EXO0748-676 [25]. This redshift corresponds to M/R=0.15 $M_\odot$/km. Another constraint on the mass–radius ratio comes from the observation of two absorption features in the source spectrum of the 1E 1207.4-5209 neutron star, with redshift from $z = 0.12$ to $z = 0.23$, which gives M/R=0.069 $M_\odot$/km to M/R=0.115 $M_\odot$/km [26]. Besides these constrains, the stars with central density above that of the maximum mass stars are mechanically unstable [1]. Due to this fact, the Ξ^0 hyperon is not present in the neutron star interior (the density required to create it is too high[5]).

[3]We have named co and bo after their relation with the collapse and the bound state, respectively.

[4]We do not use this modified TOV equations since they imply the existence of a magnetic monopole.

[5]Indeed, the Ξ^0 hyperon appears in an insignificant quantity $Y_{\Xi^0} = 10^{-5}$ at $1.2\, fm^{-3}$.

Table 2 Neutron stars properties computed from the four EoS used as input to the TOV equation

Kind	M/$M_\odot$ (max.)	R (km)	n_c (fm^{-3})	B_0	B_c
A	2.48	12.21	0.741	31.0	25
A	2.39	12.10	0.840	1.0	1.0
H	2.22	11.80	0.824	31.0	28
H	2.01	11.86	0.952	1.0	1.0

From the central density n_c, we see that the hyperonic stars are denser than the atomic ones for a fixed magnetic field. B_0 and B_c are given in multiple of $10^{17}G$

Solving the TOV equations for the EoS, we obtain the results presented in Table 2.

The fact that the EoS of hyperonic stars are always softer than the atomic ones reflects in the maximum mass of these stars. Further, in Table 2, we can see that the hyperonic stars are denser than the atomic ones for a fixed magnetic field, as stated before. Moreover, denser stars can support stronger magnetic fields in their center (B_c) as a consequence of (29). A curious fact is that while the radius of the hyperonic star with the maximum mass decreases for strong magnetic fields, in the atomic stars, the radius grows with B. This result is a consequence of the fact that the radii of the neutron stars are not related to the stiffness of the EoS but with its symmetry energy slope [27]. We plot the TOV solutions in Fig. 8.

We can see that all our models are in agreement with the previously discussed constrains [23–26]. The inclined straight lines are the constrains of the measured redshift, while the horizontal one is the 1.97 $M_\odot$ pulsar. Every single dot in the curves is a possible neutron star. We can see also that a magnetar can reach a mass of 2.48$M_\odot$. However, as we neither know nor expect to discover any white dwarf with mass above 1.4$M_\odot$ due to the Chandrasekhar limit [2], the same holds to the neutron stars with masses above 2.0$M_\odot$ due to the Oppenheimer–Volkoff limit if we believe that hyperons and muons exist in their interior. The maximum possible mass is very close to the 1.97 $M_\odot$ known neutron star. The discovery of neutron stars with masses above 2.0$M_\odot$ will imply that either hyperons are absent or the star is a rare strongly magnetized neutron star, as shown in Fig. 8.

We have seen in Fig. 5 that the strangeness fraction f_s is not affected by the magnetic field for a fixed density. Let us see what happens inside different stars with different central densities, which can support different magnetic fields. We first calculate the strangeness fraction in the inner core by fixing the mass and check what happens with f_s when we vary the magnetic field. We then fix the maximum mass and look at its strangeness content. We write these results in the Table 3. We see

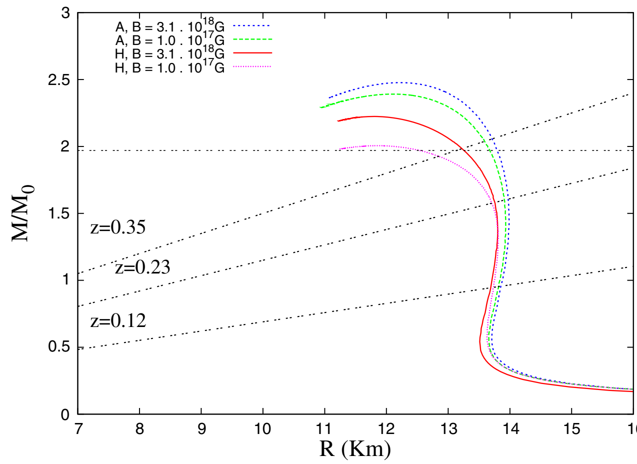

Fig. 8 (Color online) Mass–radius relation for two atomic and two hyperonic stars with different values of magnetic field. The *straight lines* are the observational constraints

that the strangeness fraction is not significant for lower mass stars. Indeed, the hyperons just become important for stars with masses above 1.6 $M_\odot$. The magnetic field is also not important for stars with low masses due to the low magnetic field strengths reached in their center. For masses above 1.6$M_\odot$, both the magnetic field and the hyperonic constituents become important. Since the magnetic field reduces the central density, it suppresses the hyperon formation. To conclude this discussion we note that the f_s is the value calculated at the center of the neutron stars, so the quantity of hyperons at lower densities is even more insignificant.

To conclude this paper, we discuss the cooling of the neutron stars due the direct Urca effect. As pointed out previously, Lattimer et al. [7] showed that the neutron stars can, too, cool much faster if their proton fractions reach values of $\simeq 11 - 15$ %. If it occurs, the direct Urca process enhances neutrino emission, cooling the neutron star much faster than any other process. We

Table 3 The influence of the magnetic field in the strangeness fraction for some standard mass

M ($M_\odot$)	f_s	n_c (fm^{-3})	B_0	B_c
1.2	0.00	0.29	31.0	1.8
1.2	0.00	0.29	1.0	0.07
1.4	0.01	0.32	31.0	2.6
1.4	0.01	0.34	1.0	0.12
1.6	0.03	0.38	31.0	4.5
1.6	0.04	0.40	1.0	0.22
1.8	0.05	0.44	31.0	7.2
1.8	0.08	0.52	1.0	0.40
2.22 (Max)	0.18	0.824	31.0	28
2.01 (Max)	0.23	0.952	1.0	1.0

The B's are given in multiple of $10^{17}G$

Table 4 Lower mass of the neutron star for a proton fraction of 13 %

M ($M_\odot$)	Kind	f_s	n_c (fm^{-3})	B_0	B_c
1.50	A	–	0.33	31.0	2.9
1.46	A	–	0.33	1.0	0.11
1.12	H	0.00	0.27	31.0	1.4
1.08	H	0.00	0.27	1.0	0.06

next show the strangeness fraction of the hyperonic stars when the direct Urca process is allowed by writing the mass of the stars when the proton fraction reaches 13 % for atomic (A) and hyperonic (H) stars in Table 4.

We see that the magnetic field causes almost no change in the lower mass that enables direct Urca process. On the other hand, the presence of muons (since hyperons are not present at low densities) has a great influence on the lower mass when the proton fraction reaches 13 %.

4 Conclusion

In this work, we consider a hadronic neutron star composed by hyperons submitted to a strong magnetic field. We see that while the presence of hyperons reduce the maximum mass by softening the EoS [1, 11, 28–31], the presence of a density-dependent magnetic field tends to increase the maximum mass stiffening the EoS [13, 17, 20]. Our study shows also that although the influence of a strong magnetic field is stiffer than the EoS, a hyperonic magnetar still has a softer EoS compared with a common atomic stars. We also see how the magnetic field can change the chemical composition of neutron stars due the Landau quantization and offer an explanation about the nonexistence of neutron stars with masses above the 1.97 $M_\odot$. We have shown that the TOV equations can be used as an approximation for neutron stars subject to strong magnetic fields up to a certain limit and that the effect of strong magnetic fields has to be taken into account in a description of massive magnetars, mainly if we believe that there are hyperons in their interior. In this case, the magnetic field increases the mass by more than 10 % although its effect is not significant for low mass neutron stars because the central magnetic field B_c is also low. According to the present model and parametrization, it is unlikely that hyperons are present in neutron stars with masses below 1.4$M_\odot$ even if the magnetic field is taken into account.

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