Braz J Phys (2012) 42:475–481
DOI 10.1007/s13538-012-0105-y

PARTICLES AND FIELDS

Dilaton Quantum Cosmology with a Schrödinger-like Equation

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Received: 24 August 2012 / Published online: 14 November 2012

Abstract A quantum cosmological model with radiation and a dilaton scalar field is analyzed. The Wheeler–DeWitt equation in the minisuperspace induces a Schrödinger equation, which can be solved. An explicit wavepacket is constructed for a particular choice of the ordering factor. A consistent solution is possible only when the scalar field is a phantom field. Moreover, although the wavepacket is time-dependent, a Bohmian analysis allows to extract a bouncing behavior for the scale factor.

Keywords Quantum cosmology · Gravitation · Quantum gravity

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1 Introduction

A scalar field is the simplest fundamental matter field that can be introduced in a cosmological model, since it adds essentially just one degree of freedom and it is naturally invariant under coordinate transformation. Even so, two different couplings of a scalar field can be quoted: the minimal coupling, leading to the so-called Einstein's frame, and the non-minimal coupling, corresponding to the Jordan's frame. While the minimal coupling is the simplest one, the non-minimal coupling is one of the predictions of string theory: the scalar field emerging from this theory, called dilaton field, appears non-minimally coupled to the gravity term [1]. Moreover, an interesting alternative to the general relativity theory, the Brans–Dicke theory [2], is based on the non-minimal coupling.

The Einstein and the Jordan frames may be connected by a conformal transformation. There is intensive discussion in the literature about the meaning of such connection, in the sense that it may be just a mathematical mapping of one system into another, or hide a deep physical meaning; see for example [3] and references therein. For certain observables, it is clear that the predictions in one frame are not equivalent to the predictions in the other frame. In quantum cosmology, this problem has been treated by Colistete et al. [4] and Fabris et al. [5] who showed that it is possible—even at the quantum level—to map the equations in one frame into those in another frame, although the resulting predictions for the evolution of the universe are different.

In Colistete et al. [4] and Fabris et al. [5], a system consisting of gravity and a scalar field, minimally or non-minimally coupled, has been considered. In this case, the analysis presents a major challenge, typical of quantum cosmology: how to obtain predictions for the evolution of the universe as a function of time. In fact, it is well known that the Wheeler–DeWitt equation resulting from quantizing the Einstein–Hilbert action has no time parameter due to the invariance of the gravitational system under time reparametrization. The introduction of a scalar field does not change this situation. In those papers, the WKB method has been used in order to obtain predictions out of the wavefunction for the evolution of the universe from the (timeless) Wheeler–DeWitt equation in the minisuperspace scenario. For discussions concerning this question, see [6] and references therein.

Another possibility to obtain the time evolution of a quantum cosmological system in the minisuperspace is to introduce matter in the form of a fluid, employing the Schutz description in terms of a potential that conveys the degrees of freedom of the fluid [9, 10]. It has been shown that such a description leads, always in the minisuperspace, to a Schrödinger-like equation, with the degrees of freedom of the matter playing the role of time. This proposal has been presented by Lapchinskii and Rubakov [11] and extensively analyzed by Alvarenga et al. [12].

Here, our aim is to address the problem of time evolution of the universe in the presence of a dilaton-like field from a quantum cosmological perspective. Since we are mainly interested in the primordial universe, a radiative fluid will be introduced. This allows us to recover the time variable through the Schutz formulation. Performing a conformal transformation (which does not affect the radiative fluid, since it is conformally invariant), we rewrite the dilaton–gravity system in Einstein's frame, from which the Wheeler–DeWitt equation in the minisuperspace is constructed, resulting in a Schrödinger-like equation. From this Schrödinger-like equation, an explicit solution is obtained by using a specific ordering factor. A wavepacket is determined, but its norm is time-dependent. Hence, it does not fit in the usual many-worlds interpretation of quantum mechanics [6, 7]. Nonetheless, it admits a Bohmian analysis [6, 8]. The Bohmian trajectories contain universes that are singularity-free.

The paper is organized as follows. In the next section, we quantize the dilaton–gravity–radiation system in the minisuperspace. In Section 3, a wavepacket is constructed and a (formal) many-world analysis is performed. In Section 4, the Bohmian trajectories are studied. In Section 5, we present our conclusions.

2 Dilaton–gravity System with Radiative Fluid

Let us consider the non-minimal coupling of gravity and a scalar field, represented by the Brans–Dicke theory:

$$\mathcal{L} = \sqrt{-\tilde{g}}\phi\left\{\tilde{R} - \tilde{\omega}\frac{\phi_{;\rho}\phi^{;\rho}}{\phi^2}\right\} + \mathcal{L}_\mathrm{m}, \qquad (1)$$

where L_m is the matter Lagragian supposed to be conformal invariant (a radiative fluid). In the strict string dilatonic case, we have $\tilde{\omega} = -1$. This defines the theory in Jordan's frame. Performing a conformal transformation such that $g_{\mu\nu} = \phi^{-1}\tilde{g}_{\mu\nu}$, we transpose the action (1) to the corresponding expression written in the Einstein's frame:

$$\mathcal{L} = \sqrt{-g}\left\{R - \omega\frac{\phi_{;\rho}\phi^{;\rho}}{\phi^2}\right\} + \mathcal{L}_\mathrm{m}, \qquad (2)$$

where $\omega = \tilde{\omega} + 3/2$.

Restricting ourselves to the FLRW metric, in an appropriate coordinate system, the line element can be written as

$$\mathrm{d}s^2 = N(t)^2\mathrm{d}t^2 - a(t)^2\gamma_{ij}\mathrm{d}x^i\mathrm{d}x^j \qquad (3)$$

where $N(t)$ is the lapse function, $a(t)$ is the scale factor, and γ_{ij} is the induced metric of the homogeneous and isotropic spatial hypersurfaces with curvature $k = 0, \pm 1$. From now on, we will fix $k = 0$. With this metric, the gravitational Lagrangian becomes

$$\mathcal{L}_G = \frac{V_0 a^3}{N}\left\{-6\left[\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 - \frac{\dot{a}}{a}\frac{\dot{N}}{N}\right] - \omega\frac{\dot{\phi}^2}{\phi^2}\right\} \quad , \qquad (4)$$

where V_0 is a constant and can be interpreted as the physical volume of the compact universe divided by a^3. Since we shall have an identical multiplicative constant in front of the matter Lagrangian, we can drop it from our analysis (this can also be understood as a normalization of the fields). Hence, discarding a surface term, the gravitational Lagrangian can be written in the following form:

$$\mathcal{L}_G = \frac{1}{N}\left\{6a\dot{a}^2 - \omega a^3\frac{\dot{\phi}^2}{\phi^2}\right\}. \qquad (5)$$

Defining,

$$\sigma = \sqrt{|\omega|}\ln\phi, \qquad (6)$$

we obtain,

$$\mathcal{L}_G = \frac{1}{N}\left\{6a\dot{a}^2 - \epsilon a^3\dot{\sigma}^2\right\}, \qquad (7)$$

where $\epsilon = \pm 1$ according ω is positive (upper sign) or negative (lower sign).

The canonical momenta associated with the scale factor and the scalar field are

$$p_a = 12\frac{a\dot{a}}{N} \quad , \quad p_\sigma = -2\epsilon\frac{a^3\dot{\sigma}}{N} \quad , \tag{8}$$

respectively.

This leads to the following expression in terms of the conjugate momentum:

$$\mathcal{L}_G = p_a\dot{a} + p_\sigma\dot{\sigma} - N\left\{\frac{1}{24}\frac{p_a^2}{a} - \epsilon\frac{p_\sigma^2}{4a^3}\right\} \quad . \tag{9}$$

Considering a radiative matter component (for the computation of the conjugate momentum associated with the fluid, see Refs. [11, 12]), we find the total Hamiltonian

$$H = N\left\{\frac{1}{24}\frac{p_a^2}{a} - \epsilon\frac{p_\sigma^2}{4a^3} - \frac{p_T}{a}\right\}. \tag{10}$$

The resulting Schrödinger equation is

$$-\frac{\partial^2\Psi}{\partial a^2} + \frac{\epsilon}{a^2}\frac{\partial^2\Psi}{\partial\sigma^2} = i\frac{\partial\Psi}{\partial T}, \tag{11}$$

where we introduced the redefinition $\frac{\sigma}{\sqrt{6}} \to \sigma$ and $\frac{T}{24} \to T$.

Equation (11) immediately raises two questions. First, what is the sign of ϵ? If it is positive, we have a hyperbolic Schrödinger equation. This implies that the "energy" E is not positive defined, as it can be seen by multiplying the Schrödinger equation by Ψ^* and integrating over σ and a. Moreover, in this case, the argument of the Bessel function can become imaginary, which may not be very serious, but may pose problems in the construction of the wavepacket, since the integration in E must be done along the entire real axis, and for $E < 0$, the Bessel function becomes the modified Bessel function. On the other hand, if the sign is -1, the positivity of E seems to be assured, and the construction of the wavepacket, not problematic, since the integration on E is carried out only in the positive semiaxis. For all these reasons, for the moment, we consider the case $\epsilon = -1$. It corresponds to a phantom scalar behavior induced to the dilaton field.

Second, we have not taken into account a possible factor ordering. If such a factor ordering is introduced, with $\epsilon = -1$, we have the following Schrödinger equation:

$$\frac{\partial^2\Psi}{\partial a^2} + \frac{p}{a}\frac{\partial\Psi}{\partial a} + \frac{1}{a^2}\frac{\partial^2\Psi}{\partial\sigma^2} = -i\frac{\partial\Psi}{\partial T} \quad . \tag{12}$$

Even though there is no unique way to choose the factor ordering, a possible, consistent, adequate choice is the covariant ordering, which is invariant through fields redefinitions. The minisuperspace for our model is two dimensional. Thus, using the covariant factor ordering, we can rewrite the Wheeler–DeWitt equation as a Klein–Gordon-like equation by defining the minisuperspace metric such that (12) reads

$$\frac{1}{\sqrt{g}}\partial_\mu\left(\sqrt{g}g^{\mu\nu}\partial_\nu\right)\Psi = -i\frac{\partial\Psi}{\partial T} \quad , \tag{13}$$

where μ, $\nu = 0, 1$, and the coordinates represent the fields, i.e., $x^0 = a$ and $x^1 = \sigma$. The only possible value for p in (12) allowing the covariant factor ordering is $p = 1$. In this case, we have

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 \\ 0 & a^2 \end{pmatrix} \quad \Rightarrow \quad \sqrt{g} = a \tag{14}$$

and the Wheeler–DeWitt equation reads

$$\left[\frac{1}{a}\frac{\partial}{\partial a}\left(a\frac{\partial}{\partial a}\right) + \frac{1}{a^2}\frac{\partial^2}{\partial\sigma^2}\right]\Psi = -i\frac{\partial\Psi}{\partial T} \quad . \tag{15}$$

This equation can be solved by the technique of separation of variables. Hence, through the ansatz

$$\Psi(a, \sigma, T) = \xi(a)e^{ik\sigma}e^{-iET}, \tag{16}$$

we obtain,

$$\xi'' + \frac{\xi'}{a} + \left\{E - \frac{k^2}{a^2}\right\}\xi = 0, \tag{17}$$

where the prime means derivative with respect to a. Noticing that this is just a Bessel's equation, its solution reads

$$\Psi = AJ_\nu(\sqrt{E}a)e^{ik\sigma}e^{-iET}, \quad \nu = k \quad , \tag{18}$$

with A being a normalization constant.

3 Wavepacket and Expectation Values

Let us construct a wavepacket by choosing conveniently the function $A = A(E, k)$. We have, for (18),

$$\Psi(a, \sigma, T) = \int_0^\infty \int_0^\infty r^{\nu+1}e^{-(\gamma+iT)r^2}e^{-(\alpha-i\sigma)k}J_k(ra)\mathrm{d}k\,\mathrm{d}r, \tag{19}$$

where $r = \sqrt{E}$ and α, γ are positive constants. Using the formula (6.631-4) from [13], the final result is

$$\Psi(a, \sigma, T) = C\frac{e^{-\frac{a^2}{4B(T)}}}{B(T)\,g_\alpha(a, B, \sigma)}, \tag{20}$$

where

$$B(T) = (\gamma + i\,T), \quad g_\alpha(a, \sigma, T) = -\alpha + \ln\left[\frac{a}{2B(T)}\right] \pm i\sigma, \tag{21}$$

and C is a normalization constant. Notice that, in order to give physical meaning to this wave packet, the condition $\Re\, g_\alpha(a, \sigma, T) < 0$ must be satisfied, assuring that the integral in the separation parameter k is convergent.

Let us calculate this normalization constant. Unitarity requires that

$$N = \int \Psi^* \Psi \mathrm{d}a\, \mathrm{d}\sigma = 1; \tag{22}$$

but,

$$\Psi^*\Psi = C^2 \frac{e^{\frac{-\gamma a^2}{2B^* B}}}{2B^* B} \frac{1}{g^* g}. \tag{23}$$

Notice that

$$B = \gamma + iT = \mathrm{D}e^{i\theta}, \tag{24}$$

where

$$D = \sqrt{\gamma^2 + T^2} = \sqrt{B^* B}, \tag{25}$$

$$\theta = \arctan\left(\frac{T}{\gamma}\right). \tag{26}$$

Hence,

$$g^* g = h^2 + (\sigma - \theta)^2, \quad h = -\alpha + \ln\left(\frac{a}{2(B^* B)^{\frac{1}{2}}}\right). \tag{27}$$

For the scalar field σ and for a, we must compute a double integral. In doing so, we must fix the range of σ. In principle, it is from $-\infty < \sigma < +\infty$, implying $0 \leq \phi < +\infty$, that is, a positive gravitational coupling. The norm of the wave function becomes

$$N = \frac{C^2}{(2B\,B^*)^{1/2}} \int_0^\infty \int_{-\infty}^{+\infty} \frac{e^{-\gamma \frac{a^2}{2BB^*}}}{(2B^* B)^{\frac{1}{2}}} \frac{\mathrm{d}\sigma}{[h^2 + (\sigma - \theta)^2]} \mathrm{d}a. \tag{28}$$

Under the substitutions,

$$u = \frac{a}{(2B^* B)^{\frac{1}{2}}}, \quad v = -\theta + \sigma, \tag{29}$$

the above integral becomes,

$$N = \frac{C^2}{(B\,B^*)^{1/2}} \int_0^\infty \int_{-\infty}^{+\infty} \frac{e^{-\gamma u^2}}{h^2 + v^2} \mathrm{d}u\, \mathrm{d}v. \tag{30}$$

But,

$$\int_{-\infty}^{+\infty} \frac{\mathrm{d}v}{h^2 + v^2} = \frac{\pi}{h}. \tag{31}$$

Hence,

$$N = \frac{C^2}{(B\,B^*)^{1/2}} \pi \int_0^\infty \frac{e^{-\gamma u^2}}{\alpha + \ln\left(\frac{u}{2}\right)} \mathrm{d}u = \frac{C^2}{(B\,B^*)^{1/2}} \pi\, I_1, \tag{32}$$

where I_1 is the definite integral,

$$I_1 = \int_0^\infty \frac{e^{-\gamma u^2}}{\alpha + \ln\left(\frac{u}{2}\right)} \mathrm{d}u. \tag{33}$$

The above-determined wavepacket is time-dependent and does not convey unitarity.

As a formal exercise, even in absence of unitarity, we evaluate the expectation value of the scalar field and of the scale factor for this wavepacket:

$$< \sigma >_T = \frac{\int_0^\infty \int_{-\infty}^{+\infty} \Psi^* \,\sigma\, \Psi \,\mathrm{d}a\, \mathrm{d}\sigma}{\int_0^\infty \int_{-\infty}^{+\infty} \Psi^* \Psi \,\mathrm{d}a\, \mathrm{d}\sigma}, \tag{34}$$

$$< a >_T = \frac{\int_0^\infty \int_{-\infty}^{+\infty} \Psi^* \,a\, \Psi \,\mathrm{d}a\, \mathrm{d}\sigma}{\int_0^\infty \int_{-\infty}^{+\infty} \Psi^* \Psi \,\mathrm{d}a\, \mathrm{d}\sigma}. \tag{35}$$

Let us first evaluate the expectation value for the scalar field. In the numerator of (34), we have that

$$\int_{-\infty}^{+\infty} \frac{\sigma \mathrm{d}\sigma}{h^2 + (\sigma - \theta)^2} = \int_{-\infty}^{+\infty} \frac{(v + \theta)\mathrm{d}v}{h^2 + v^2}. \tag{36}$$

The first integral is zero, and the second one leads to the equality

$$\int_{-\infty}^{+\infty} \frac{\sigma \mathrm{d}\sigma}{h^2 + (\sigma - \theta)^2} = \theta \int_{-\infty}^{+\infty} \frac{\mathrm{d}v}{h^2 + v^2} = \theta \frac{\pi}{h}. \tag{37}$$

Reinserting this result in the expression for the scale factor, we find that

$$< \sigma >_T = \arctan\left(\frac{T}{\gamma}\right). \tag{38}$$

For the scale factor, integration over σ yields

$$< a >_T = \pi \int_0^\infty \frac{a}{h} \frac{e^{-2\gamma \frac{a^2}{B^* B}}}{B^* B} \mathrm{d}a. \tag{39}$$

Performing, as before, the substitution

$$\frac{a}{\sqrt{B^* B}} = \frac{a}{\sqrt{\gamma^2 + T^2}} = u, \tag{40}$$

we then obtain the result

$$< a >_T = \sqrt{\gamma^2 + T^2} \int_0^\infty \frac{u}{h} e^{-2\gamma u^2} \mathrm{d}u = \frac{I_2}{I_1} \sqrt{\gamma^2 + T^2}, \tag{41}$$

where

$$I_2 = \int_0^\infty \frac{u}{h} e^{-2\gamma u^2} \mathrm{d}u. \quad (42)$$

Hence,

$$< a >_T = a_0(\gamma^2 + T^2)^{1/2}, \quad (43)$$

which is formally the same solution as when there is no scalar field [12]. The norm of the wavepacket is time-dependent and a unitary framework cannot be invoked.

The fact that the result for the scale factor is essentially the same obtained in the absence of the scalar field may be seen as contradictory. Notice, however, that the logarithmic derivative of the scale factor and the derivative of the scalar field appear in the Friedmann equation. Hence, the presence of the scalar field simply changes the value of a_0 with respect to the case in which it is absent.

4 Bohmian Trajectories

In order to extract some physical insight from the model developed until now, given the nature of the wavepacket employed in the previous section, we will use a Bohmian analysis [6]. Note that the computation of the Bohmian trajectories makes sense even in the absence of unitarity, a notion that constitutes one of the main features of the ontological interpretation of quantum mechanics.

The Bohm–de Broglie interpretation is most easily understood from the polar form of the wave function. Indeed, decomposing the wave function as $\Psi = Re^{iS}$, the Wheeler–DeWitt Eq. (15) splits in two real nonlinear equations for the two real function $R(a, \sigma, T)$ and $S(a, \sigma, T)$

$$\frac{\partial R^2}{\partial T} + \frac{1}{a}\frac{\partial}{\partial a}\left(R^2\, 2a\frac{\partial S}{\partial a}\right) + \frac{1}{a}\frac{\partial}{\partial \sigma}\left(R^2 \frac{2}{a}\frac{\partial S}{\partial \sigma}\right) = 0 \quad (44)$$

$$\frac{\partial S}{\partial T} + \left(\frac{\partial S}{\partial a}\right)^2 + \left(\frac{1}{a}\frac{\partial S}{\partial \sigma}\right)^2 + Q = 0 \quad (45)$$

with

$$Q \equiv -\frac{1}{R}\left[\frac{1}{a}\frac{\partial}{\partial a}\left(a\frac{\partial R}{\partial a}\right) + \frac{1}{a^2}\frac{\partial^2 R}{\partial \sigma^2}\right] \quad (46)$$

Equation (45) is a modified Hamilton–Jacobi equation with the presence of the quantum potential Q. Through this equation, we can identify the momenta as

$$P_a = \frac{\partial S}{\partial a} \quad , \quad P_\sigma = \frac{\partial S}{\partial \sigma} \quad . \quad (47)$$

The metric in the minisuperspace (14) allows us to define covariant derivatives [14]. Therefore, (44) represents a continuity equation written in that minisuperspace, with R^2 playing the rôle of a probability density, and we can directly read the associated velocities within each parenthesis, i.e.,

$$v^\mu = g^{\mu\nu}\frac{\partial S}{\partial x^\nu} \quad \Rightarrow \quad \dot{a} = 2\frac{\partial S}{\partial a} \quad , \quad \dot{\sigma} = \frac{2}{a^2}\frac{\partial S}{\partial \sigma}$$

The wavefunction can be written as

$$\Psi(a, \sigma, T) = A\frac{e^{-\frac{a^2}{4B}}}{g\, B} = A\frac{e^{-\frac{a^2 B^*}{4BB^*}}}{g^* g\, B^* B} g^*\, B^*, \\ = f(a, \sigma, T)e^{iS(a,\sigma,T)}, \quad (48)$$

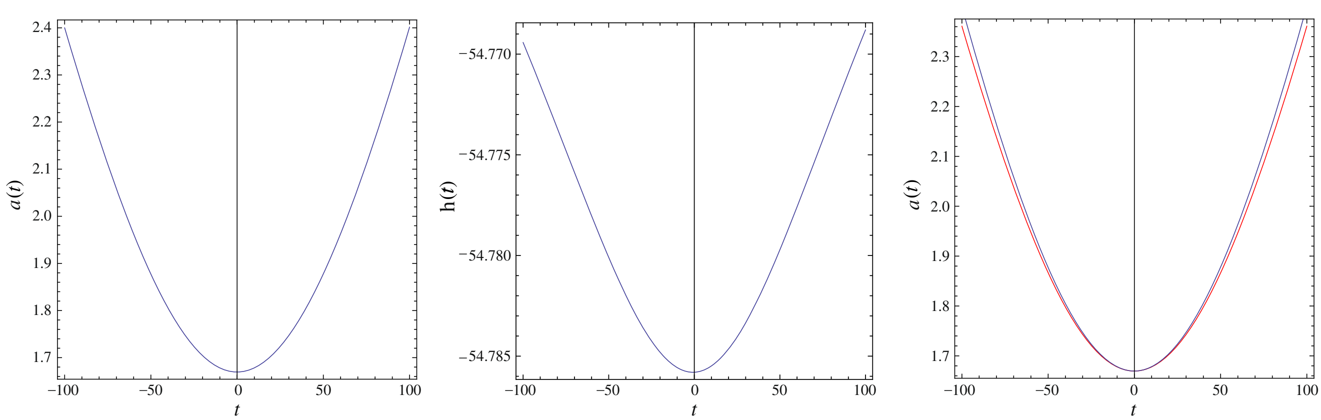

Fig. 1 (Color online) *Left panel*: behavior of the scale factor for $\gamma = 100$ and $\alpha = 50$. *Central panel*: the function h. *Right panel*: solution obtained using the Bohmian trajectories (*inner curve*, blue online), compared with the solution using the expectation value (*outer curve*, red online)

with

$$f(a,\sigma,T) = A\frac{e^{-\frac{a^2\gamma}{4BB^*}}}{\sqrt{g^*g\,B^*B}}, \tag{49}$$

$$S(a,\sigma,T) = \frac{a^2T}{4(\gamma^2+T^2)} - \arctan\left(\frac{T}{\gamma}\right) + \arctan\left(\frac{-\theta+\sigma}{h}\right). \tag{50}$$

Remembering that

$$h = -\alpha + \ln\left(\frac{a}{2\sqrt{\gamma^2+T^2}}\right) = \Re\, g_T(a,\sigma). \tag{51}$$

and that, for a radiative fluid $N = a$ in (8) [12], taking into account the time and field redefinitions, we have the following expressions for the Bohmian trajectories:

$$\dot{a} = \frac{aT}{(\gamma^2+T^2)} + \frac{2}{a}\frac{\theta-\sigma}{h^2+(\theta-\sigma)^2}, \tag{52}$$

$$\dot{\sigma} = \frac{2}{a^2}\frac{h}{h^2+(\theta-\sigma)^2}, \tag{53}$$

where θ and h are given by (26), (51).

These equations must be integrated numerically.[1]

The numerical solutions of (52) and (53) provide an interesting perspective depending on the initial conditions and on the free parameters of the model, namely γ and α. We can show that there is at least a class of non-singular solutions, satisfying the wavepacket conditions of consistency. Figure 1 displays a nonsingular solution. The function h is also plotted, showing that the condition for the convergence of the wavepacket is satisfied. In the same figure, we plot the results obtained using the expectation value of the previous section and the numerical solutions for the Bohmian trajectories. The Bohmian trajectories predict an asymmetric bounce, while the expectation value displays a perfectly symmetric bounce.

5 Conclusions

The possibility to make predictions and subsequently extract a family of solutions (trajectories in the minisuperspace) is a recurrent problem in quantum cosmology. Although the well-known WKB method, including decoherence features, has provided a vast range of particular results, there is an alternative framework by means of transforming the WdW equation into a Schrödinger equation, with the time variable induced by a matter component [9–11]. Quite interesting results have appeared in the literature [12] using this procedure.

In this paper, we investigated that alternative framework in search of predictions, using a system characterized by a dilaton field and radiation expressed using the Schutz variables. The dynamics of the radiative fluid implies a time variable.

We found that, in order to ensure that the Schrödinger equation be elliptic, leading to a positive energy spectrum, the dilaton field must have a phantom behavior. On the other hand, the construction of a quasi-Gaussian superposition, conjugated with certain convergence criteria, leads to a wavepacket with a time-dependent norm; hence, unitarity could not be invoked. Nevertheless, this still allows us to investigate the wavepacket under a Bohmian perspective. Interestingly, we found a bouncing behavior for the scale factor, i.e., the singularity is avoided. In this sense, we extend the results of Alvarenga et al. [15], where the imposition of unitarity led to the conclusion that the dilaton field should be constant. At same time, such approach opens new perspective concerning anisotropic quantum models [16].

There are still many open directions to explore in this program. In particular, we must (a) look for other wavepackets, presumably by using numerical methods, and (b) study the dynamics in the Jordan frame. We hope to address these problems in a future work.

Acknowledgements We thank the CNPq for partial financial support. JM and PVM were also supported by the grant CERN/FP/116373/2010. PVM is grateful to the CENTRA-IST for financial assistance. He also wants to thank the hospitality of the UFES, where this work was completed.

[1] It can already be stated that the solutions are not equivalent to the expectation values found before, a consequence of the absence of unitarity: when the wavepacket is unitary, the Bohmian trajectories reproduces the expectation values.

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