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STATISTICAL

Not Fully Developed Turbulence in the Dow Jones Index

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Abstract The shape of the curves relating the scaling exponents of the structure functions to the order of these functions is shown to distinguish the Dow Jones index from other stock market indices. We conclude from the shape differences that the information-loss rate for the Dow Jones index is reduced at smaller time scales, while it grows for other indices. This anomaly is due to the construction of the index, in particular to its dependence on a single market parameter: price. Prices are subject to turbulence bursts, which act against full development of turbulence.

Keywords Intermittency of the turbulence · Financial markets · Dow Jones index · Scaling exponents

1 Introduction

An analogy between physics and economics has been extensively developed in recent years [1–3]. Following this line of thought, we here ask whether fluid dynamics can help us predict the behavior of the stock market and, if so, whether the evolution of stock market indices can be identified with turbulent behavior. To answer these questions, we draw a parallel with the phenomenon known as turbulence intermittency. The analogy between fluid dynamics and stock markets has limitations, especially because market indices never reach stationary conditions. References [1] and [4] have identified analogies, such as intermittency and non-Gaussian statistics, but they have also found distinctions in the time evolution of the second moment and in the shape of the probability density functions (PDFs). Fluid turbulence must be analyzed in short periods, no more than a few minutes, a constraint aimed at separating the turbulent scale from the influence of motions in the mesoscale or synoptic scales, so that only non-stationary conditions are associated with the turbulence. In long series of financial data, by contrast, the lack of stationarity is due not only to market volatility, but also to trends or eventual economic cycles. Our analysis is not immune to this difficulty. Even though we remove trends and consider only subsets of the integral scale, the relatively large time span in our study may distort the results, since it admits contributions other than the turbulent component.

These shortcomings notwithstanding, the conclusions drawn from methods similar to those in fluid theory help explain certain behaviors of stock indices and how those indices are constructed within an economy [5]. Efforts in this direction are particularly important in view of the strongly complex fluctuating dynamics of financial time series [6]. In noteworthy examples, the anomalous diffusive properties and second-order correlations of price fluctuations [see 2] have been addressed through multivariate [7] and non-linear [8] models.

The dynamics of the turbulent regime is driven by a cascade linking larger scales to smaller ones. Scaling analysis, i.e., the quest for patterns or properties repeated at different scales, is the most appropriate approach to problems defined by that phenomenon. First applied in physics, scaling has become a standard tool in other disciplines, e.g., to assess the volatility of financial markets. Among the methods employed to characterize the scaling behavior of a data set are the Detrended Fluctuation Analysis [9], Average Wavelet Coefficient Method [10], ARFIMA estimation by exact maximum likelihood [11], multi-affine analysis [12], and generalized Hurst exponent method [13].

The scaling behavior of turbulence implies a fractal geometry. Many attempts have exploited the concept of multifractality

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to explain the phenomenon of intermittency. Models have been constructed from analyses of the evolution of PDFs or of the velocity-increment structure functions. The intermittency of turbulence in fluids is analogous to the variability of the financial market volatility in economics [14]. The volatility of financial time series is a stylized fact that has been documented in Refs. [15–17] or in the more recent Ref. [18]. The traditional Generalized Autoregressive Conditional Heteroskedasticity models of Engle [19] and Bollerslev [20] deserve special attention, for they are able to describe the recurrent volatility clusters. Nonetheless, the importance of the aforementioned models based on fractality should not be underestimated, as illustrated by the work of Mandelbrot, the founder of fractal geometry, whose fractal view of the world of finances has reevaluated the standard tools and models of modern financial theory [21].

In summary, scaling techniques have been frequently applied to the study of financial markets. More specifically, scaling analyses have led to conclusions concerning the development of the markets and provided thorough description of the data.

2 Intermittency/Variability of the Volatility

In general terms, the turbulence of a fluid is called intermittent [22] when a turbulent variable, such as the velocity, changes along the scales of the energy cascade [23]. The turbulent regime is characterized by a set of eddies with different sizes. In Richardson's depiction, the largest eddies, comparable in size to the characteristic scale of the main flow, interact with and extract energy from that flow. The large eddies are unstable and disappear after breaking up and releasing their energy to smaller eddies. This process is repeated at progressively smaller scales until sufficiently small eddies are reached, at which size viscosity becomes capitally important and dissipates energy into heat. Dissipation occurs only at the end of the cascade, in the so-called dissipation range.

At a certain scale in the cascade, the distribution of the random variable, which is associated with the change in the variable between two points separated by a distance at that scale, is measured by its PDF or by the entire set of its statistical moments. The evolutions of the PDF's or of the statistical moments along the cascade therefore provide estimates of the intermittency [24–28]. In the absence of intermittency, the PDF's would be scale invariant. In intermittency of the turbulence, by contrast, the shape of the PDF's is expected to evolve as the scale shrinks. To quantify the evolution, it is sufficient to monitor any parameter characteristic of the PDF shape, such as its flatness [29].

The statistical moments at a given scale are associated with the structure functions of the changes in the variable at that scale. The pth order structure function of the velocity v is defined by the expression

$$S_p(\tau) = \frac{\langle |v(t+\tau) - v(t)|^p \rangle}{\langle |v(t)|^p \rangle} \tag{1}$$

where τ is a time scale.

If S_p scales with a power law,

$$S_p \propto \tau^{\zeta_p} \tag{2}$$

then the statistical moments can be represented by the exponents ζ_p.

In the absence of intermittency and under homogeneity and isotropy conditions, on the basis of dimensional analysis Kolmogorov's theory requires that [30]

$$S_p = C_p (l\langle\varepsilon\rangle)^{p/3} \tag{3}$$

where $\langle\varepsilon\rangle$ is the global average of the energy dissipation rate ε, and C_p is a universal constant. If $\langle\varepsilon\rangle$ is constant, $S_p \propto l^{p/3}$, and the scaling exponent ζ_p is $p/3$. The scaling exponent is therefore linearly related to the order of the structure function.

Intermittency breaks the linearity. The greater the intermittency, the greater the curvature of the ζ_p vs. p plots [31, 32]. Reference [33] presents one interpretation of this curvature, drawn from Obukhov's procedure [34].

As already explained, intermittency can alternatively be regarded as multi-fractal, or as a multi-scaling process [13]. Multi-fractality can be characterized by the generalized Hurst exponent $H(p)$, defined by the following expression for the scaling exponent:

$$\zeta_p = pH(p) \tag{4}$$

Monofractality, which is expected in the absence of intermittency, implies constant $H(p)$, i.e., linear relation between ζ_p and p.

The scaling law (2) holds under homogeneity and isotropy conditions in the inertial range. Assuming Taylor's hypothesis [35], we also replace homogeneity by stationarity. Under these conditions, ζ_p vs. p plots must obey the so-called 4/5 law [22], according to which ζ_3 must be equal to 1, intermittency again implying deviation from the straight line $\zeta_p = p/3$. If the flow is non-stationary, a condition often found in fluid dynamics and even more frequent in economics, the 4/5 law may not hold, and as we will see, ζ_p may be far from unity. Intermittency is then flagged by deviation from a linear form—not necessarily from the expression $\zeta_p = p/3$.

On the other hand, irrespective of the conditions, the structure functions scale as power laws of the third-order structure function, i.e., follow the expression

$$S_p \propto S_3^{\overline{\zeta_p}} \tag{5}$$

where the $\overline{\zeta_p} = \zeta_p/\zeta_3$ are the relative scaling exponents, normalized by the exponent of order 3. The proportionality (5) defines the Extended Self Similarity (ESS) condition [36]. While their physical meaning remains the subject of debate, the relative scaling exponents serve as surrogates allowing analysis of intermittency even when the scaling law $S_p \propto l^{\zeta_p}$ fails [37]. In contrast with the ζ_p, the $\overline{\zeta_p}$ cover nearly all regimes. They are, however, more uniform and therefore less discriminatory than the absolute exponents [38].

The scaling $S_p \propto l^{\zeta_p}$ is sufficient to validate scaling exponent analysis. If the turbulence is fully developed, Hölder's inequality forces the plot of ζ_{2p} as a function of p to be concave [22]. Several models try to determine the dependence of ζ_p on p in the case of fully developed turbulence; examples are found in Refs. [39, 40].

Apart from situations represented by different intermittency models, other cases exist in which turbulence is not fully developed, so that the ζ_{2p} vs. p curves are not necessarily concave. If conditions in the atmospheric boundary layer favor stratification with very strong stability, for example, vertical motion will be inhibited and waves of high frequency will be trapped. The breaking up of these internal waves may then give rise to bursts of turbulence that block full development of turbulence over a certain time period. The bursts may redistribute energy throughout the scaling range. As a result, the energy dissipation rate may diminish at smaller scales and make the p dependence of ζ_{2p} convex, instead of rising at smaller scales as it usually does [41].

The analogy between fluid dynamics and stock markets rests upon the notion that the flow of energy down the turbulent cascade in fluids is equivalent to the information cascade in the economic domain [42–44]. The analogue of the velocity is the difference between two index values with a τ time elapse, which is equivalent to a distance and therefore sets the scale.

An alternative would be to define the velocity as the logarithm of the ratio between two index values, but we prefer not to take advantage of this alternative. The logarithm does offer certain advantages. In particular, the log returns are additive, so that addition of n single period log returns yields the n-period log returns, a property that simplifies the assessment of investments over extended periods. Additivity is, however, irrelevant in the study of intermittency, since it would be inadequate to regard an n-period as the addition of single periods—by definition, intermittency is the variability along that n-period. Log returns also simplify the computation of derivatives and other calculus operators; unfortunately, they are unwieldy to compute structure functions. Log returns, moreover, are less realistic than discrete returns, since markets tend to quote the latter. To describe the evolution of a group of assets over a fixed period, discrete returns are convenient because they are linearly additive; weighted averages can therefore be computed to estimate portfolio returns, and this provides the basis upon which stock market indices are built. For all these reasons and also because tighter correspondence with the expression for the kinematic velocity will more clearly bring out the analogy between fluids and stock markets, we prefer to analyze discrete returns.

A market index is the weighted average, at a given moment, of the prices, and in certain cases of the quantity, of shares for a fixed set of businesses. The flux of information creates a cascade that, each moment determines the degree of confidence and the risk investors are willing to face and provides clues for efficient allocation of resources [45, 46]. Just as energy is dissipated by the fluid cascade, information is lost in the stock market. The information, which conveyed by many sources, such as the market itself, national and international agencies, and economic agents, may be truncated, corrupted, or distorted by manipulative expedients. Perhaps even more important, investors at the receiving end can lose pieces of information due to the lack of attention or incorrect interpretation. In the same way that viscosity dissipates energy, human factors downgrade information.

3 Description of the Indices in the Study

We have used the daily closing data from the most important stock exchanges in the Euro zone, the United States, the United Kingdom, and Japan. More specifically, we have analyzed the evolution of the following indices between January 1, 1992 and August 31, 2009, a total of 4609 days: FTSE 100, CAC 40, DAX 30, IBEX 35, NASDAQ 100, NIKKEI 225, and DOW JONES.

Different indices are generated by different calculation methods. The DOW JONES and the NIKKEI indices are price-weighted whereas the others are value weighted, i.e., calculated from the monetary value of each constituent: the shares in issue multiplied by the stock price [47]. They depend on capitalization—on price and volume—and are more affected by the large stocks within their portfolios [48].

Price-weighting is based on a simple notion. When economic activity booms, industrial profits attract investors, and share prices rise. An increase in the DOW JONES or NIKKEI index is a symptom of high-profit expectations. When the stock of a company splits, however, the share prices decline and the weight of the company in the price-weighted index is reduced, even though it may be large and growing—as a matter of fact, growing stocks tend to split [49].

There are also important differences between the calculation of the DOW JONES and NIKKEI indices. The NIKKEI index includes 225 top-rated Japanese companies; its constituents are changed every year, but its support comes mainly from the mostly liquid high-tech industries. Moreover, there is no specific weighting of industries: stocks are given equal weights, based on a par value of 50 yen per share. Unlike the

NIKKEI, the DOW JONES includes the prices of 30 stocks from large, well known, mostly industrial, US companies, i.e., from mature blue-chip stocks, which are similar in terms of scale and firm returns, and therefore, of capitalizations [50]. In 2011, it only included 17.22 % of technological firms. The NASDAQ, regarded as the US tech-based index, was introduced to fill the gap of technological firms in the DOW JONES. To make the index more consistent, the prices in the DOW JONES are weighted by a divider that is adjusted whenever one of the component stocks undergoes a stock split or stock dividend. Therefore, the DOW JONES gives more weight to large businesses than the NIKKEI.

4 Results

To draw results from the data, we first eliminate the trend in the original data sets to reduce the usual unsteadiness of the market. Next, to detect intermittency of turbulence in the studied markets, we have to monitor the scale dependence of the shape of the probability density functions. To this end, we examine the flatness F of the PDF's. Figure 1 shows the ratio $F/3$, that is, F normalized by Gaussian flatness, for each index as a function of scale, ranging from one day to the integral scale. The dashed lines show that all plots but the DOW JONES are well described by logarithmic fits. We therefore leave the DOW JONES data aside for the moment and compare the other plots.

Among those, the NASDAQ is the most intermittent one. While the coefficients of the logarithmic terms in the remaining five curves range from −0.259 to −0.40, the coefficient for the NASDAQ plot is −0.9085. Rare events on a small scale are therefore more likely to occur in the latter index than in the former ones. The variability of change being therefore most frequent, the NASDAQ is the most intermittent index. This

Fig. 2 **a**, **c**, **e**, **g**, **i**, **k**, **m** First-, second-, and third-order structure functions vs. scale for the seven indices in our study; **b**, **d**, **f**, **h**, **j**, **l**, **n** First- and second-order structure functions vs. the third-order structure function for all indices. Each *straight line* represents linear regression applied to the data of matching color, with the indicated determination coefficient

result is not surprising. The more traditional firms constituting the other indices work with more widespread knowledge than the high-techs in the NASDAQ, which is moreover a basket of small- and medium-sized firms, more exposed to market movements [5].

The smallest coefficient of the logarithmic term is found in the poor fit to the DOW JONES data. Although the small correlation coefficient makes any estimate of intermittency drawn from this fit quantitatively unreliable, Fig. 1 shows that the flatness of the DOW JONES PDF's varies much less at small scales than those of the other indices.

Given the inadequacy of the flatness plots to quantify the intermittency of the DOW JONES index, we turn to the structure functions defined by Eq.(1). Figure 2a, c, e, g, i, k, and m show the structure functions S_p (p=1,2,3) for the indices in our study as functions of the scale, while Fig. 2b, d, f, h, j, l, and n depict S_p (p=1,2) as functions of S_3. Our analysis is restricted to $p \leq 3$ to respect the bound imposed by statistical uncertainty, mathematically expressed by the following expression [41]:

$$\mathrm{maxorder} \approx \log(n) - 1 \qquad (6)$$

where n is the number of available data points.

Had we limited our analysis to $p \leq 2$, the scaling laws would be more clearly recognized in Fig. 2a, c,…, m, but our conclusions would be unchanged, since the intermittency, i.e., the curvature of the ζ_p vs. p plots, is already defined at these lower orders. Inclusion of p=3 in our discussion, on the other hand, gives access to the plots in Fig. 2b, d, … , n,

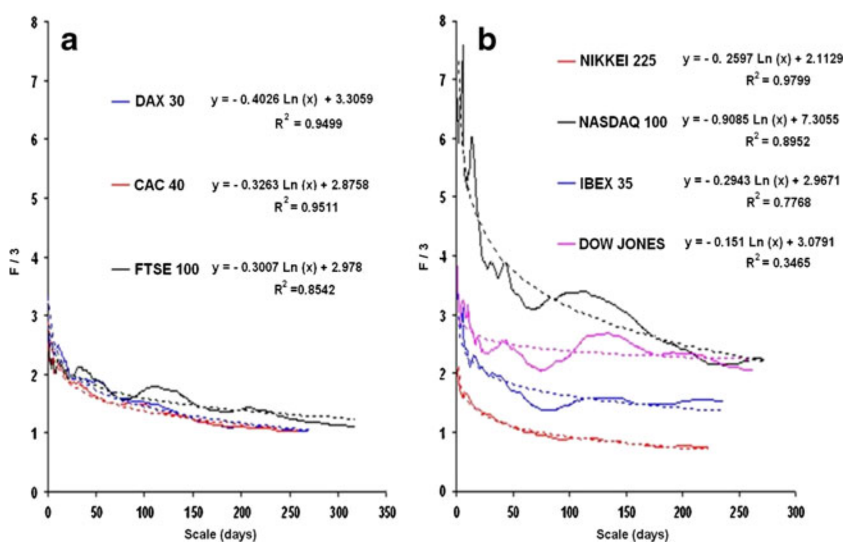
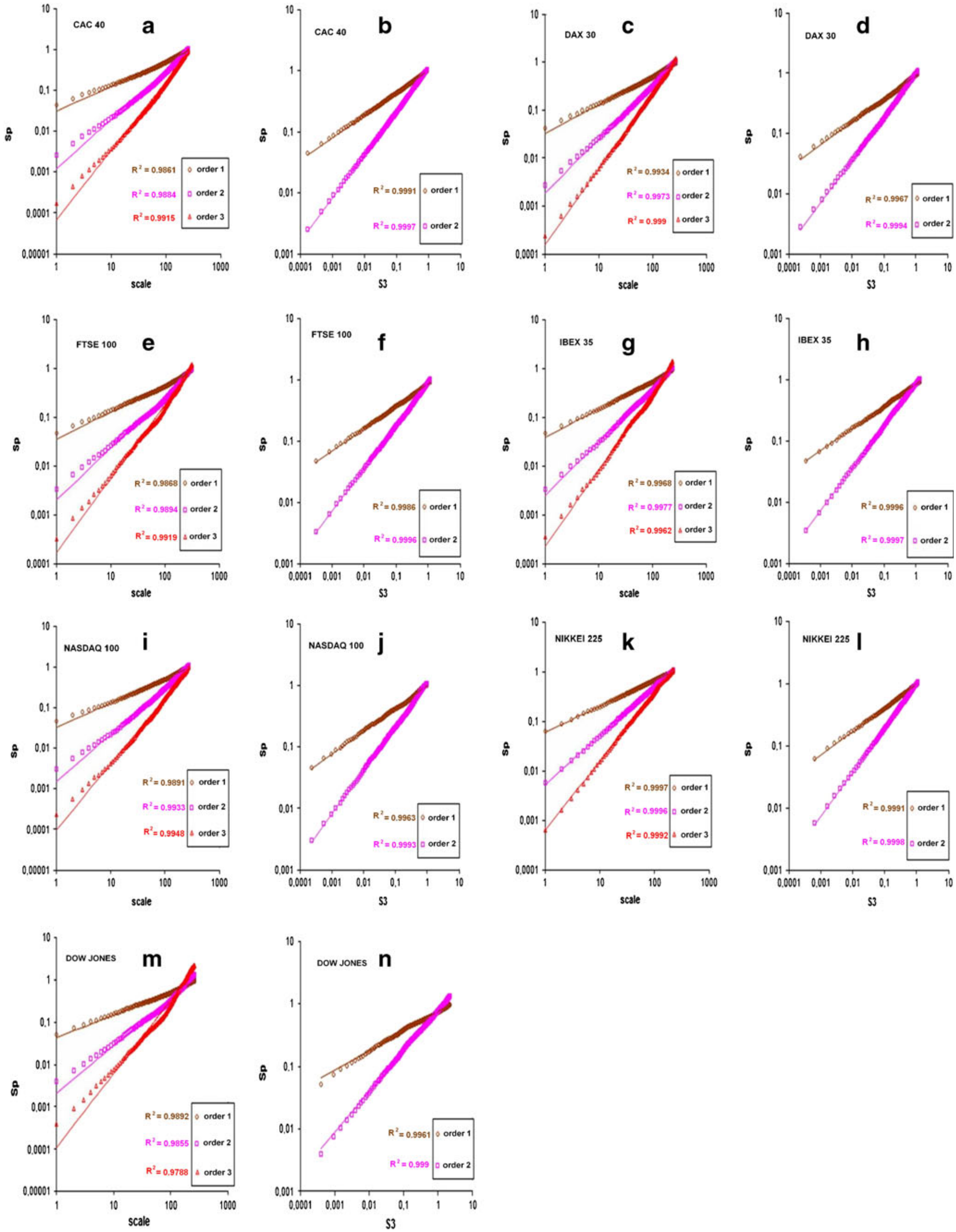


Fig. 1 **a** Evolution of the flatness ($F/3$) with scale for (**a**) the DAX 30, CAC 40, and the FTSE 100 indices, and **b** the NIKKEI, NASDAQ, IBEX 35 and the DOW JONES indices. The *dashed lines* display the indicated logarithmic fits to the *solid curve of matching color*


CAC 40
a
R² = 0.9861 order 1
R² = 0.9884 order 2
R² = 0.9915 order 3
Sp
scale
CAC 40
b
R² = 0.9991 order 1
R² = 0.9997 order 2
S3
DAX 30
c
R² = 0.9934 order 1
R² = 0.9973 order 2
R² = 0.999 order 3
DAX 30
d
R² = 0.9967 order 1
R² = 0.9994 order 2
FTSE 100
e
R² = 0.9868 order 1
R² = 0.9894 order 2
R² = 0.9919 order 3
FTSE 100
f
R² = 0.9986 order 1
R² = 0.9996 order 2
IBEX 35
g
R² = 0.9968 order 1
R² = 0.9977 order 2
R² = 0.9962 order 3
IBEX 35
h
R² = 0.9996 order 1
R² = 0.9997 order 2
NASDAQ 100
i
R² = 0.9891 order 1
R² = 0.9933 order 2
R² = 0.9948 order 3
NASDAQ 100
j
R² = 0.9963 order 1
R² = 0.9993 order 2
NIKKEI 225
k
R² = 0.9997 order 1
R² = 0.9996 order 2
R² = 0.9992 order 3
NIKKEI 225
l
R² = 0.9991 order 1
R² = 0.9998 order 2
DOW JONES
m
R² = 0.9892 order 1
R² = 0.9855 order 2
R² = 0.9788 order 3
DOW JONES
n
R² = 0.9961 order 1
R² = 0.999 order 2

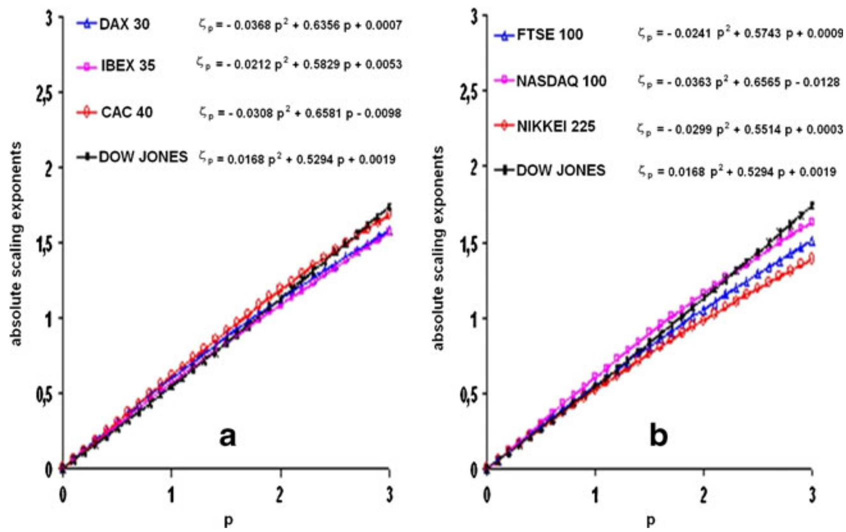


Fig. 3 Absolute scaling exponents as functions of the structure functions for **a** the DAX 30, IBEX 35, CAC 40 and the DOW JONES indices, and **b** the FTSE 100, NASDAQ 100, NIKKEI 225, and the DOW JONES indices. The mathematical expression of the optimal quadratic fit to each curve is shown, to identify its curvature

in which the (ESS) power laws relating S_p (p=1, 2) to S_3 can be more clearly discerned.

To explore the alternatives associated with Eq. (4), Fig. 3 plots ζ_p as a function of p, the seven indices being divided into two panels for clarity. All curves are concave, except the one representing the DOW JONES index [51]. The similar trends exhibited by all plots indicate that they have the same physical content. More clearly than Fig. 2, these curves show that the rate of information loss grows at shorter scales, in consistency with the notion of "bounded rationality", an expression that Simon [52] coined to describe the limited human ability to process information. The author pointed out that attention, not information is the scarce resource (for a model of information transmission, see [53], for instance). Indices affected by quantities, not by prices only, transmit more information in the long run, for the growth of quantities tends to take time [54].

The DOW JONES index stands out among the indices because the pertinent plot is convex, i.e., has positive curvature. For emphasis, Fig. 3 displays the quadratic form best fitting each form. Even more patent is the contrast between the DOW JONES and the other six indices in the plots of $\overline{\zeta_p}$ as functions of p in Fig. 4 [51], as in Fig. 3 dividing for clarity the seven indices into two panels. True, the plots of relative scaling exponents for the latter indices are less discriminatory than the plots of the absolute scaling exponents. In fact, they are nearly congruent, while the shape of the DOW JONES curve is clearly different. From a broad perspective, the resolution of the relative scaling exponents therefore seems insufficient to distinguish the intermittent behavior of different indices. Nonetheless, given the inadequacy of the flatness-evolution analysis to estimate the intermittency of the DOW JONES index, the relative exponents offer an important complementary view that will offer new insights into the behavior of this index.

In contrast with the rates of information loss for other indices, the rate of loss in the DOW JONES index declines at shorter temporal intervals, the distinction between short- and

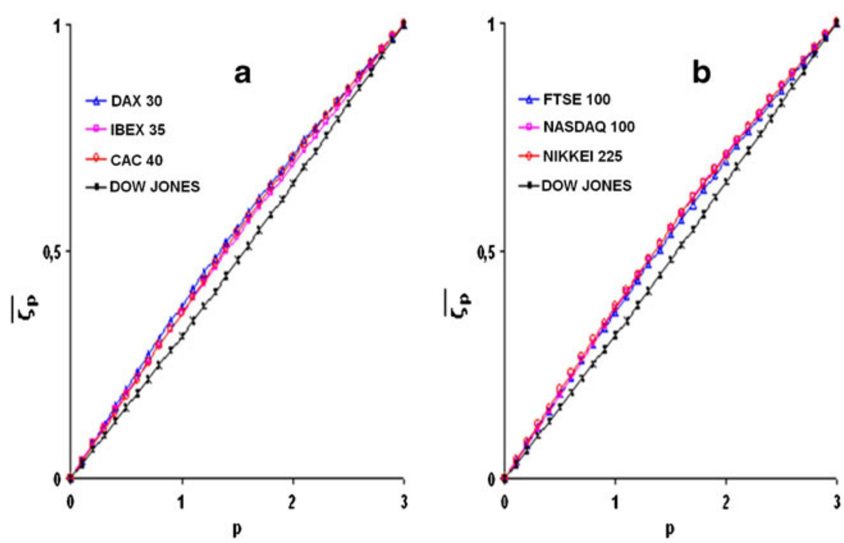


Fig. 4 Relative scaling exponents as functions of the order p of the structure function for **a** the DAX 30, IBEX 35, CAC 40 and the DOW JONES indices, and **b** the FTSE 100, NASDAQ 100, NIKKEI 225, and the DOW JONES indices

long-term transmissions being less pronounced. The large, mainly non-tech businesses with similar features in the DOW JONES index tends to cause less information loss in the short term because rare events are less frequent than in the other indices (the performance of big businesses in periods of crisis is very well documented; see for instance [55–60]).

The smaller frequency is due to bursts to which the DOW JONES index is prone. In the stock market characterized by inelastic output, prices rise promptly in response to an exogenous rise in the demand for stocks. This burst exhausts the exogenous change in demand. For that reason, the DOW JONES (and the NIKKEI) index, dependent only on prices, may undergo bursts of turbulence that oppose the development of full turbulence. Such turbulent price bursts may be due to animal spirits—emotions spontaneously urging action—, the reception of financial news, and the rapid transmission of information. A massive flow of information in a very short time period, i.e., small scale, implies that the rate of information loss at that scale is lesser than the rate at larger scales, i.e., in the long run or secular period. According to Marshall [54], the prices return to normal in the secular period: quantity is increased, but all acquired information on prices is lost [61]. When this happens, the curvature of the ζ_p vs. p plot will be opposite to the usual one—convex instead of concave—as in the DOW JONES curves in Figs. 3 and 4. The bursts therefore seem to be catharsis shocks making the index less intermittent, more relaxed.

More specifically, as shown in [62], macroeconomic announcements, such as the communications from the Federal Open Market Committee meetings, Employment Situation Report from the Bureau of Labor Statistics, CPI and PPI are the most relevant information bursts affecting the DOW JONES index. The restraint on the development of turbulence caused by the bursts is similar to the aforementioned freezing of one degree of freedom—the vertical component—in the atmospheric boundary layer under conditions of very strong stability.

The NIKKEI index is also quantity independent. However, its high-tech industry profile and its computational method give less weight to large businesses than the DOW JONES index does, counterbalance the price-weighting effect and lose more information in the short run. The sum of the two opposite tendencies makes it more volatile/intermittent than the DOW JONES index, but as shown by Ref. [5], less than the NASDAQ, which is the most volatile index in our study. The new, risky high-tech corporations suffer from information asymmetry that gives rise to speculation and gain differentials. Traders of their stock have to make decisions without the common knowledge that eases the transfer of information in transactions dealing with the stock of the large, well-established businesses.

5 Conclusions

Several studies have attempted to relate turbulence to market fluctuations. Analogies and distinctions have been found regarding, for example, the shape of the probability–density functions and the emergence of intermittency. This article contributes to the understanding of both facets of the problem. Our parallel is drawn not only from statistical concepts such as structure function analysis, but also from the definition and characteristics of the stock market indices and the physical aspects of turbulence.

PDF analysis proved to be unsuitable to describe the intermittency of the DOW JONES index; complementary analysis based on the scaling exponents of the structure functions therefore became necessary. From the latter analysis, we inferred from the behavior of the DOW JONES index that bursts of turbulence curb the full development of turbulence and that the rate of information transmission exceeds the rate of information loss at short time intervals.

The plot of ζ_{2p} as a function of p is concave in fully developed turbulences. All such plots in our work are concave, except the one for the DOW JONES. For the latter, we have shown that the structure functions follow power scaling laws and satisfy the ESS condition. The deviations of the ζ_p vs. p and $\overline{\zeta_p}$ vs. p curves for the DOW JONES data from the corresponding curves for the other indices signals the absence of fully developed turbulence in the former index. The dependence of the DOW JONES index on a single parameter—prices—freezes out one degree of freedom. This result is similar to the phenomenon found in the planetary boundary layer under strong stability conditions: the constraints of atmospheric stability make the p dependence of the exponents convex.

The NIKKEI index, by contrast, evolves under the competition of two opposite forces. While only price dependent, it is constituted by high-tech industries and its definition underweights large companies. The resulting scaling exponents show the ζ_{2p} vs. p concavity characteristic of fully developed turbulence, as do the exponents derived from the data for the five indices in our study that depend on price and quantity.

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