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Noncommutative Dp -Brane in General Background Fields

Maryam Zoghi · Davoud Kamani

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Abstract We investigate the noncommutativity of open strings, attached to a Dp -brane, in the presence of the linear dilaton, tachyon, $U(1)$ gauge field as well as constant antisymmetric B -field backgrounds. The noncommutativity parameter, open string metric, and some special cases will be studied. The mode-dependent noncommutativity, inspired by the tachyon field, will be discussed in detail.

Keywords D-brane · Noncommutativity · Dilaton · Tachyon

1 Introduction

D-branes, as inevitable objects in string theory [1], have been studied from various points of view. On the other hand, we have noncommutative geometry which has been used as a framework for many subjects in physics and chiefly in high energy physics. The advent of noncommutative string theory in the B -field background and appearance of nontrivial commutators in spatial directions of the D-brane [2–5], revived investigation of noncommutativity in the string theory with different background fields like dilaton field [6, 7], tachyon field [8–10], as well as moving D-branes.

Unstable D-branes in background fields are mainly studied in world-volume field theory and boundary state descriptions [11–13]. In this paper, we consider a Dp -brane of the bosonic string theory in the presence of a $U(1)$ gauge field, linear dilaton, tachyon, and Kalb–Ramond B -field,

simultaneously. This is a system with generalized setup. For this system, we obtain the boundary conditions and propagator of an open string, attached to the Dp -brane. This propagator enables us to extract a set of open string metrics and noncommutativity parameters. We observe that these variables depend on the open string modes. On these variables, some scaling limits and approximations will be investigated. Finally, we apply our results for the D1-brane and D2-brane as special cases.

2 Extended Boundary Conditions of Open String

We consider a Dp -brane in a general background, which contains the antisymmetric field $B_{\mu\nu}$, the dilaton field Φ , the open string tachyon field $T(x)$, and a $U(1)$ gauge field A_α which lives on the Dp -brane world-volume. We shall use the indices $\alpha \in \{0, 1, \dots, p\}$ and $i \in \{p+1, \dots, d-1\}$ for directions along the world-volume of the Dp -brane and perpendicular to it, respectively. Therefore, the string sigma model action in these background fields has the following form:

$$S = \frac{1}{4\pi\alpha'} \int_{\Sigma} d^2\sigma \left(\sqrt{-h} h^{ab} g_{\mu\nu} \partial_a X^\mu \partial_b X^\nu - 2\pi i \alpha' \epsilon^{ab} B_{\mu\nu} \partial_a X^\mu \partial_b X^\nu + \alpha' \sqrt{-h} \Phi R^{(2)} \right) + \frac{1}{2\pi\alpha'} \int_{\partial\Sigma} d\tau \{ -2\pi i \alpha' A_\alpha \partial_\tau X^\alpha + T(X) \}, \quad (1)$$

where Σ indicates the string worldsheet which has the boundary $\partial\Sigma$ and the metric h_{ab} with $h = -\det h_{ab}$. The scalar curvature $R^{(2)}$ is constructed from the metric h_{ab} . The space-time metric also is $g_{\mu\nu}$. Note that the indexes of worldsheet and space-time take their values from the sets $a, b \in \{\sigma, \tau\}$ and $\mu, \nu \in \{0, 1, \dots, d-1\}$, respectively.

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The variety of the background fields enables us to restrict them to obtain a solvable model. For this, we take the background fields $g_{\mu\nu}$ and $B_{\mu\nu}$ to be constant with $g_{\alpha i} = 0$ and only $B_{\alpha\beta} \neq 0$. In addition, for the $U(1)$ gauge field, we elect the gauge $A_\alpha = -\frac{1}{2}F_{\alpha\beta}X^\beta$, where the field strength $F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha$ is constant. Moreover, we suppose that the dilaton field has a linear form along the brane world-volume, i.e., $\Phi = a_\alpha X^\alpha$ where the parameters $\{a_\alpha|\alpha = 0, 1, \dots, p\}$ are constant. In the conventional case, the dilaton is an arbitrary function, which causes the appearance of third-order derivative in the action and, hence, is put away [14, 15]. In the action Eq. (1), the scalar curvature $R^{(2)}$ contains a second-order derivative. This implies that by considering a linear dilaton, only the usual second-order derivative is obtained.

Remember that the exponential of the dilaton field gives the strength of the string coupling. So the linear dilaton background describes a world in which strings are weakly coupled for large negative x and strongly coupled for large positive x . Thus, one could worry about the reliability of the formalism in such setup. However, adding a tachyon background term of the form $T_0 \exp(u \cdot X)$ suppresses the contribution of the strongly coupled region, and this keeps things under control [16]. The conventional inhomogeneous tachyon profile is a linear form which appears as a squared term in the boundary of the string action [17, 18]. This motivates us to consider tachyon profile as $T(X) = \frac{1}{2}U_{\alpha\beta}X^\alpha X^\beta$ with the constant symmetric matrix $U_{\alpha\beta}$, which originates from the expansion of exponential form $T_0 \exp(u_\alpha X^\alpha)$ for small parameters $\{u_\alpha|\alpha = 0, 1, \dots, p\}$. This form of the tachyon field accompanied by the linear dilaton gives a Gaussian theory and, hence, is exactly solvable [19–22].

The reparametrization invariance of the bulk part of the action Eq. (1) enables us to choose the conformal gauge for the worldsheet metric, i.e., $h_{ab}(\sigma, \tau) = e^{\rho(\sigma, \tau)}\eta_{ab}$. Note that due to the presence of the dilaton, the action does not have the Weyl symmetry. Thus, $\rho(\sigma, \tau)$ is a nonzero worldsheet field.

The vanishing of the variation of the action leads to the equations of motion for the worldsheet fields X^μ and ρ as in the following:

$$\partial^2 X_\mu + \frac{1}{2}a_\mu \partial^2 \rho = 0, \quad (2)$$

$$a_\alpha \partial^2 X^\alpha = 0, \quad (3)$$

where $\partial^2 = \eta^{ab}\partial_a\partial_b$. Now, we consider a noncritical string theory, i.e., $a^2 = a_\alpha a^\alpha \propto d - 26 \neq 0$. Then Eqs. (2) and (3) split into $\partial^2 X^\mu = \partial^2 \rho = 0$. In addition, the vanishing of this variation defines the boundary conditions of the string.

For example, for the open string end at $\sigma = 0$, we have the boundary equations:

$$\begin{aligned} (g_{\alpha\beta}\partial_\sigma X^\beta + 2\pi i\alpha' \mathcal{F}_{\alpha\beta}\partial_\tau X^\beta + U_{\alpha\beta}X^\beta)_{\sigma=0} &= 0, \\ (\delta X^i)_{\sigma=0} &= 0, \\ (a_\alpha\partial_\sigma X^\alpha)_{\sigma=0} &= 0. \end{aligned} \quad (4)$$

Here, $\mathcal{F}_{\alpha\beta} = B_{\alpha\beta} - F_{\alpha\beta}$ is total field strength.

Removing $\partial_\sigma X^0$ from the first boundary condition, via the third one, leads us to define the variable $\tilde{g}_{\alpha\bar{\beta}}$ as follows:

$$\tilde{g}_{\alpha\bar{\beta}} \equiv g_{\alpha\bar{\beta}} - g_{\alpha 0} \frac{a_{\bar{\beta}}}{a_0}, \quad (5)$$

where $\bar{\beta} \in \{1, 2, \dots, p\}$. To obtain desirable boundary conditions, we demand $\tilde{g}_{0\bar{\beta}}$ to be zero which gives $a_{\bar{\beta}} = \frac{a_0}{g_{00}}g_{0\bar{\beta}}$. In addition, we impose the extra conditions $\mathcal{F}_{\alpha 0} = U_{0\beta} = 0$. Therefore, the boundary condition of open string along the brane directions takes the following form:

$$(\tilde{g}_{\bar{\alpha}\bar{\beta}}\partial_\sigma X^{\bar{\beta}} + 2\pi i\alpha' \mathcal{F}_{\bar{\alpha}\bar{\beta}}\partial_\tau X^{\bar{\beta}} + U_{\bar{\alpha}\bar{\beta}}X^{\bar{\beta}})_{\sigma=0} = 0. \quad (6)$$

The symmetric matrix $\tilde{g}_{\bar{\alpha}\bar{\beta}} = g_{\bar{\alpha}\bar{\beta}} - \frac{g_{00}}{a_0^2}a_{\bar{\alpha}}a_{\bar{\beta}}$ effectively possesses the treatment of a metric in the brane volume. By having the coordinates $\{X^{\bar{\alpha}}\}$, we are able to specify X^0 via its equation $\partial^2 X^0 = 0$ and the boundary condition: $\partial_\sigma X^0|_{\sigma=0} = -\frac{a_{\bar{\alpha}}}{a_0}\partial_\sigma X^{\bar{\alpha}}|_{\sigma=0}$.

3 Noncommutativity Variables

The open string propagator $\mathcal{G}^{\bar{\alpha}\bar{\beta}}$ can be calculated via the following equation:

$$\partial\bar{\partial}\mathcal{G}^{\bar{\alpha}\bar{\beta}}(z, z') = -2\pi\tilde{g}^{\bar{\alpha}\bar{\beta}}\delta^{(2)}(z - z'), \quad (7)$$

and the boundary condition

$$[(\tilde{g} + 2\pi\alpha'\mathcal{F})\partial\mathcal{G} - (\tilde{g} - 2\pi\alpha'\mathcal{F})\bar{\partial}\mathcal{G} - iU\mathcal{G}]^{\bar{\alpha}\bar{\beta}}|_{\sigma=0} = 0, \quad (8)$$

where the complex variable is $z = \tau + i\sigma$. The solution of these equations is given by

$$\begin{aligned} \mathcal{G}^{\bar{\alpha}\bar{\beta}}(z, z') &= -\alpha'\tilde{g}^{\bar{\alpha}\bar{\beta}}\ln|z - z'| \\ &+ \frac{\alpha'}{2}\sum_{n=1}^{\infty}\left(\frac{\tilde{g} - 2\pi\alpha'\mathcal{F} - \frac{\alpha'}{2n}U}{\tilde{g} + 2\pi\alpha'\mathcal{F} + \frac{\alpha'}{2n}U}\right)^{(\bar{\alpha}\bar{\beta})} \\ &\quad \times \frac{(z\bar{z}')^n + (\bar{z}z')^n}{n} \\ &+ \frac{\alpha'}{2}\sum_{n=1}^{\infty}\left(\frac{\tilde{g} - 2\pi\alpha'\mathcal{F} - \frac{\alpha'}{2n}U}{\tilde{g} + 2\pi\alpha'\mathcal{F} + \frac{\alpha'}{2n}U}\right)^{[\bar{\alpha}\bar{\beta}]} \\ &\quad \times \frac{(z\bar{z}')^n - (\bar{z}z')^n}{in}. \end{aligned} \quad (9)$$

According to the prototype initiated in the ref. [3], the above propagator defines an open string metric and a non-commutativity parameter, for each string mode, as in the following:

$$G_n^{\bar{\alpha}\bar{\beta}} = \left(\frac{1}{\tilde{g} + 2\pi\alpha'\mathcal{F} + \frac{\alpha'}{2n}U} \right)^{(\bar{\alpha}\bar{\beta})} \\ = \left(\frac{1}{\tilde{g} + 2\pi\alpha'\mathcal{F} + \frac{\alpha'}{2n}U} \right. \\ \left. \times \left(\tilde{g} + \frac{\alpha'}{2n}U \right) \frac{1}{\tilde{g} - 2\pi\alpha'\mathcal{F} + \frac{\alpha'}{2n}U} \right)^{\bar{\alpha}\bar{\beta}}, \quad (10)$$

$$\theta_n^{\bar{\alpha}\bar{\beta}} = 2\pi\alpha' \left(\frac{1}{\tilde{g} + 2\pi\alpha'\mathcal{F} + \frac{\alpha'}{2n}U} \right)^{[\bar{\alpha}\bar{\beta}]} \\ = -(2\pi\alpha')^2 \left(\frac{1}{\tilde{g} + 2\pi\alpha'\mathcal{F} + \frac{\alpha'}{2n}U} \right. \\ \left. \times \mathcal{F} \frac{1}{\tilde{g} - 2\pi\alpha'\mathcal{F} + \frac{\alpha'}{2n}U} \right)^{\bar{\alpha}\bar{\beta}}, \quad (11)$$

where $n \in \{1, 2, 3, \dots\}$. Note that these variables depend on the positive string mode numbers. Analogous results can be seen for noncommutative D -branes in the pp -wave background [23]. The peculiar result that we obtained is a consequence of the tachyon field in the volume of the brane. In the absence of the tachyon, all modes of the open string probe the same value for each of these variables. However, the tachyon splits this degeneracy. By adjusting the parameters $\tilde{g}_{\bar{\alpha}\bar{\beta}}$, $\mathcal{F}_{\bar{\alpha}\bar{\beta}}$ and $U_{\bar{\alpha}\bar{\beta}}$, one can receive expedient values of the noncommutativity variables.

Now, we introduce the averaged values of the noncommutativity variables:

$$\bar{G}^{\bar{\alpha}\bar{\beta}} = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=1}^N G_n^{\bar{\alpha}\bar{\beta}}, \\ \bar{\theta}^{\bar{\alpha}\bar{\beta}} = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=1}^N \theta_n^{\bar{\alpha}\bar{\beta}}. \quad (12)$$

By using the Eqs. (10) and (11), we find that

$$\bar{G}^{\bar{\alpha}\bar{\beta}} = \left(\frac{1}{\tilde{g} + 2\pi\alpha'\mathcal{F}} \tilde{g} \frac{1}{\tilde{g} - 2\pi\alpha'\mathcal{F}} \right)^{\bar{\alpha}\bar{\beta}}, \\ \bar{\theta}^{\bar{\alpha}\bar{\beta}} = -(2\pi\alpha')^2 \left(\frac{1}{\tilde{g} + 2\pi\alpha'\mathcal{F}} \mathcal{F} \frac{1}{\tilde{g} - 2\pi\alpha'\mathcal{F}} \right)^{\bar{\alpha}\bar{\beta}}. \quad (13)$$

These are independent of the tachyon matrix. Therefore, they are counterparts of the well-known standard noncommutativity variables, as expected. In fact, due to the dilaton parameters $\{a_{\bar{\alpha}}|\bar{\alpha} = 1, \dots, p\}$ in the matrix $\tilde{g}_{\bar{\alpha}\bar{\beta}}$, these are generalized version of the ordinary case.

3.1 Noncommutativity After Tachyon Condensation

The ground state of the bosonic open string has negative mass squared and hence is tachyonic. In fact, the tachyon field describes the dynamics of an unstable Dp -brane. That is, after tachyon condensation (i.e., when at least one of the nonzero elements of the matrix $U_{\bar{\alpha}\bar{\beta}}$ goes to infinity), the brane becomes unstable. Therefore, a Dp -brane in the presence of a tachyon field decays to the lower dimensional branes or decays to the closed string vacuum.

According to the Eqs. (10) and (11), since each mode feels its own noncommutativity, tachyon condensation depends on the mode number. Let tachyon condensation take place in the x^p -direction of the brane. For the finite mode numbers, this implies the limit $\frac{1}{n}U_{pp} \rightarrow \infty$. Thus, the last columns and the lowest rows of the matrices G_n and θ_n vanish. The nonzero $(p-1) \times (p-1)$ matrices inside the matrices G_n and θ_n elaborate the open string metric and noncommutativity parameter of a $D(p-1)$ -brane. In other words, the Dp -brane loses its extension along the direction x^p , as expected. The features of the new noncommutativity variables are similar to the previous case, i.e., the Eqs. (10) and (11), with $\bar{\alpha}, \bar{\beta} \in \{1, 2, \dots, p-1\}$. For very large mode numbers, if the quantity $\frac{1}{n}U_{pp}$ tends to infinity again, we have the above discussion. If the infinite values of U_{pp} and n lead to a finite value for $\frac{1}{n}U_{pp}$, then tachyon condensation does not occur.

We observe that the averaged values of the noncommutativity variables are independent of the tachyon, and hence they are not affected by condensation of the tachyon. In Section 5, the tachyon condensation again will be illustrated.

4 Scaling Limits

4.1 The Zero Slope Limit

Now, we impose the zero slope limit ($\alpha' \rightarrow 0$) on the open string variables. This is useful for studying the low energy behavior of the open string. Since open strings are sensitive to G and θ , we should take the limit in the manner that these variables are to be fixed rather than the closed string parameters [3]. Therefore, we apply the limits $\alpha' \sim \epsilon^{\frac{1}{2}} \rightarrow 0$, $g_{\bar{\alpha}\bar{\beta}} \sim \epsilon \rightarrow 0$ and $U_{\bar{\alpha}\bar{\beta}} \sim \epsilon^{\frac{1}{2}}$, while \mathcal{F} is fixed. The Eq. (5) gives the scaling $\tilde{g}_{\bar{\alpha}\bar{\beta}} \sim \epsilon \rightarrow 0$. In these limits, we find the following results:

$$G_n^{\bar{\alpha}\bar{\beta}} = -\frac{1}{(2\pi\alpha')^2} \left(\mathcal{F}^{-1} \left(\tilde{g} + \frac{\alpha'}{2n}U \right) \mathcal{F}^{-1} \right)^{\bar{\alpha}\bar{\beta}}, \\ \theta_n^{\bar{\alpha}\bar{\beta}} = \left(\mathcal{F}^{-1} \right)^{\bar{\alpha}\bar{\beta}}, \quad (14)$$

$$\begin{aligned}\bar{G}^{\bar{\alpha}\bar{\beta}} &= -\frac{1}{(2\pi\alpha')^2} \left(\mathcal{F}^{-1} \tilde{g} \mathcal{F}^{-1} \right)^{\bar{\alpha}\bar{\beta}}, \\ \bar{\theta}^{\bar{\alpha}\bar{\beta}} &= \left(\mathcal{F}^{-1} \right)^{\bar{\alpha}\bar{\beta}}.\end{aligned}\quad (15)$$

Since $\tilde{g}_{\bar{\alpha}\bar{\beta}}$, $\alpha'U$ and α'^2 have the same limiting behavior so the above variables are finite.

4.2 Small \mathcal{F} and U Limit

A small tachyon and total gauge field limit can be obtained by using the scaling $g_{\bar{\alpha}\bar{\beta}} \sim \epsilon$, $U_{\bar{\alpha}\bar{\beta}} \sim \epsilon$ and $\mathcal{F}_{\bar{\alpha}\bar{\beta}} \sim \epsilon^{\frac{1}{2}}$. In this limit, the corresponding noncommutativity variables reduce to

$$\begin{aligned}G_n^{\bar{\alpha}\bar{\beta}} &= -\frac{1}{(2\pi\alpha')^2} \left(\mathcal{F}^{-1} \left(\tilde{g} + \frac{\alpha'}{2n} U \right) \mathcal{F}^{-1} \right)^{\bar{\alpha}\bar{\beta}}, \\ \theta_n^{\bar{\alpha}\bar{\beta}} &= \left(\mathcal{F}^{-1} \right)^{\bar{\alpha}\bar{\beta}},\end{aligned}\quad (16)$$

$$\begin{aligned}\bar{G}^{\bar{\alpha}\bar{\beta}} &= -\frac{1}{(2\pi\alpha')^2} \left(\mathcal{F}^{-1} \tilde{g} \mathcal{F}^{-1} \right)^{\bar{\alpha}\bar{\beta}}, \\ \bar{\theta}^{\bar{\alpha}\bar{\beta}} &= \left(\mathcal{F}^{-1} \right)^{\bar{\alpha}\bar{\beta}}.\end{aligned}\quad (17)$$

Though the features of these variables and their counterparts in the zero slope limit are the same, they have different values. For example, the noncommutativity parameters in Eqs. (14) and (15) are finite, while in the Eqs. (16) and (17), they are very large. However, the closed string metrics in both cases are finite.

4.3 Large \mathcal{F} and U Limit

Now, we examine large values for the matrices \mathcal{F} and U :

$$\alpha' \sim \epsilon^{\frac{1}{2}}, \quad \mathcal{F}_{\bar{\alpha}\bar{\beta}} \sim \epsilon^{-\frac{1}{2}}, \quad U_{\bar{\alpha}\bar{\beta}} \sim \epsilon^{-\frac{1}{2}}, \quad g_{\bar{\alpha}\bar{\beta}} \sim \epsilon \rightarrow 0. \quad (18)$$

This scaling leads to the following equations:

$$\begin{aligned}G_n^{\bar{\alpha}\bar{\beta}} &= \frac{2n}{\alpha'} \left(\frac{1}{U + 4\pi n \mathcal{F}} U \frac{1}{U - 4\pi n \mathcal{F}} \right)^{\bar{\alpha}\bar{\beta}}, \\ \theta_n^{\bar{\alpha}\bar{\beta}} &= -(4\pi n)^2 \left(\frac{1}{U + 4\pi n \mathcal{F}} \mathcal{F} \frac{1}{U - 4\pi n \mathcal{F}} \right)^{\bar{\alpha}\bar{\beta}}.\end{aligned}\quad (19)$$

$$\begin{aligned}\bar{G}^{\bar{\alpha}\bar{\beta}} &= -\frac{1}{(2\pi\alpha')^2} \left(\mathcal{F}^{-1} \tilde{g} \mathcal{F}^{-1} \right)^{\bar{\alpha}\bar{\beta}}, \\ \bar{\theta}^{\bar{\alpha}\bar{\beta}} &= \left(\mathcal{F}^{-1} \right)^{\bar{\alpha}\bar{\beta}}.\end{aligned}\quad (20)$$

From the open string metrics, only G_n is finite, while \bar{G} behaves like ϵ . The noncommutativity parameters θ_n and $\bar{\theta}$ also go to zero in analogy with to $\epsilon^{\frac{1}{2}}$.

4.4 Small Tachyon Approximation

An interesting approximation is given by the small tachyon matrix elements. In this case, the noncommutativity variables are truncated to the following versions:

$$\begin{aligned}\lim_{U \rightarrow 0} G_n^{\bar{\alpha}\bar{\beta}} &= \bar{G}^{\bar{\alpha}\bar{\beta}} + \frac{\alpha'}{2n} \left(\frac{1}{\tilde{g} + 2\pi\alpha'\mathcal{F}} U \frac{1}{\tilde{g} - 2\pi\alpha'\mathcal{F}} \right. \\ &\quad \left. - U \frac{1}{\tilde{g} + 2\pi\alpha'\mathcal{F}} \bar{G} - \bar{G} \frac{1}{\tilde{g} - 2\pi\alpha'\mathcal{F}} U \right)^{\bar{\alpha}\bar{\beta}} \\ &\quad + \mathcal{O}(U^2)^{\bar{\alpha}\bar{\beta}},\end{aligned}\quad (21)$$

$$\begin{aligned}\lim_{U \rightarrow 0} \theta_n^{\bar{\alpha}\bar{\beta}} &= \bar{\theta}^{\bar{\alpha}\bar{\beta}} - \frac{\alpha'}{2n} \left(\bar{\theta} \frac{1}{\tilde{g} - 2\pi\alpha'\mathcal{F}} U + U \frac{1}{\tilde{g} + 2\pi\alpha'\mathcal{F}} \bar{\theta} \right)^{\bar{\alpha}\bar{\beta}} \\ &\quad + \mathcal{O}(U^2)^{\bar{\alpha}\bar{\beta}}.\end{aligned}\quad (22)$$

We observe that the first terms of these limits are the averaged noncommutativity variables, as expected. The first-order corrections also slow down by the factor $1/n$.

5 Examples

To provide a more intuitive physical view of the above general setup, we describe the noncommutativity variables for two simple examples. These are D1-brane and D2-brane.

5.1 D1-Brane

Consider a D1-brane lying in the X^1 -direction. The general forms of the closed string metric and the matrix U can be written as follows:

$$g_{\alpha\beta} = \begin{pmatrix} -g_0 & g_1 \\ g_1 & g_2 \end{pmatrix}, \quad U_{\alpha\beta} = \begin{pmatrix} 0 & 0 \\ 0 & u_2 \end{pmatrix}. \quad (23)$$

Note that in our formulation, there are $\mathcal{F}_{\bar{\alpha}0} = U_{0\beta} = 0$. According to $\tilde{g}_{11} = g_2 + \frac{g_1^2}{g_0}$, the open string metrics reduce to

$$\begin{aligned}G_{(n)11} &= g_2 + \frac{g_1^2}{g_0} + \frac{\alpha'}{2n} u_2, \\ \bar{G}_{11} &= g_2 + \frac{g_1^2}{g_0}.\end{aligned}\quad (24)$$

Since there is only one spatial direction, the noncommutativity parameters vanish.

5.2 D2-Brane

For a D2-brane in the $X^1 X^2$ -plane, there are

$$g_{\alpha\beta} = \begin{pmatrix} -g_0 & g_1 & g_3 \\ g_1 & g_2 & g_4 \\ g_3 & g_4 & g_5 \end{pmatrix}, \quad \mathcal{F}_{\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & b \\ 0 & -b & 0 \end{pmatrix},$$

$$U_{\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & u_2 & u_4 \\ 0 & u_4 & u_5 \end{pmatrix}. \quad (25)$$

The closed string metric defines the matrix \tilde{g} as in the following:

$$\tilde{g}_{\tilde{\alpha}\tilde{\beta}} = \begin{pmatrix} g_2 + \frac{g_1^2}{g_0} & g_4 + \frac{g_1 g_3}{g_0} \\ g_4 + \frac{g_1 g_3}{g_0} & g_5 + \frac{g_3^2}{g_0} \end{pmatrix}, \quad \tilde{\alpha}, \tilde{\beta} \in \{1, 2\}. \quad (26)$$

Therefore, the noncommutativity variables take the following forms:

$$G_n^{\tilde{\alpha}\tilde{\beta}} = \begin{pmatrix} \frac{\frac{g_3^2}{g_0} + g_5 + \frac{g_1^2}{2n} u_5}{P_n} & -\frac{2n(2ng_1 g_3 + 2ng_0 g_4 + \alpha' g_0 u_4)}{Q_n} \\ -\frac{2n(2ng_1 g_3 + 2ng_0 g_4 + \alpha' g_0 u_4)}{Q_n} & \frac{\frac{g_1^2}{g_0} + g_2 + \frac{g_1^2}{2n} u_2}{P_n} \end{pmatrix},$$

$$\theta_n^{\tilde{\alpha}\tilde{\beta}} = \begin{pmatrix} 0 & -\frac{8\pi\alpha' n^2 b g_0}{Q_n} \\ \frac{8\pi\alpha' n^2 b g_0}{Q_n} & 0 \end{pmatrix}, \quad (27)$$

where the variables P_n and Q_n have the following definitions:

$$P_n = \left(\frac{g_1 g_3}{g_0} + g_4 - 2\pi\alpha' b + \frac{\alpha'}{2n} u_4 \right) \times \left(\frac{g_1 g_3}{g_0} + g_4 + 2\pi\alpha' b + \frac{\alpha'}{2n} u_4 \right) + \left(\frac{g_1^2}{g_0} + g_2 + \frac{\alpha'}{2n} u_2 \right) \left(\frac{g_3^2}{g_0} + g_5 + \frac{\alpha'}{2n} u_5 \right),$$

$$Q_n = 4n^2 g_0 g_4^2 + 2n\alpha' g_3^2 u_2 + 2n\alpha' g_0 g_5 u_2 - 4n\alpha' g_0 g_4 u_4 + 16\pi^2 n^2 \alpha'^2 b^2 g_0 - \alpha'^2 g_0 u_4^2 + \alpha'^2 g_0 u_2 u_5 - 4ng_1 g_3 (2ng_4 + \alpha' u_4) + 2ng_1^2 (2ng_5 + \alpha' u_5) + 2ng_2 (2ng_3^2 + 2ng_0 g_5 + \alpha' g_0 u_5). \quad (28)$$

Regarding to the tachyon condensation, consider the limit $(U_{22} = u_5) \rightarrow \infty$. The Eq. (27) implies that the noncommutativity matrix vanishes, and the only nonzero element of the open string metric reduces to $G_n^{11} = 1/(g_2 + \frac{g_1^2}{g_0} + \frac{\alpha'}{2n} u_2)$. This is exactly the inverse of the first Eq. (24). That is, the D2-brane has deformed to a D1-brane, as expected.

6 Conclusions and Summary

The noncommutativity of open strings, which are attached to a Dp -brane in the presence of the massless field—Kalb–Ramond, a $U(1)$ gauge field, dilaton, and a tachyon field which is massive—was investigated. The presence of the linear dilaton field effectively deforms the closed string metric and, hence, the noncommutativity variables. The appearance of various parameters due to the background fields, i.e., $\{B_{\tilde{\alpha}\tilde{\beta}}, F_{\tilde{\alpha}\tilde{\beta}}, a_{\tilde{\alpha}}, U_{\tilde{\alpha}\tilde{\beta}}\}$ enables us to adjust these variables to desirable values.

The noncommutativity variables are influenced by all background fields, but the tachyon field has more control over them. More precisely, the tachyon field decomposes a noncommutativity which has been originated by the Kalb–Ramond and the $U(1)$ gauge fields into infinite number of noncommutativities. That is, each open string mode feels its own noncommutativity. The average values of the noncommutativity variables are independent of the tachyon. More accurately, they are equal to the noncommutativity of infinite massive states of open string, or equivalently, they are the ordinary noncommutativity variables, which appear in the systems without tachyon field. As expected, tachyon condensation deforms the noncommutativity variables of a Dp -brane to that of a $D(p-1)$ -brane. However, this reduction for light string modes always takes place, while for very large mode numbers, its occurrence depends on the orders of the infinities of “ n ” and “ U_{pp} ”.

Finally, various scaling limits were studied. The tachyon matrix enabled us to introduce new appealing scaling limits.

Note that for a moving Dp -brane in our setup, with a velocity perpendicular to the brane directions, the noncommutativity structure is the same as we studied. By contrast, a moving Dp -brane along itself generates a new noncommutativity structure which call for additional investigation.

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