

Brazilian Journal of Physics ISSN: 0103-9733 luizno.bjp@gmail.com Sociedade Brasileira de Física Brasil

de Lima Bernardo, Bertúlio
Time Correlation and Decoherence in a Two-Particle Interferometer
Brazilian Journal of Physics, vol. 44, núm. 2-3, -, 2014, pp. 202-207
Sociedade Brasileira de Física
Sâo Paulo, Brasil

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ATOMIC PHYSICS



Time Correlation and Decoherence in a Two-Particle Interferometer

Bertúlio de Lima Bernardo

Received: 16 June 2013 / Published online: 11 March 2014 © Sociedade Brasileira de Física 2014

Abstract A two-particle interferometer is theoretically analyzed to show how decoherence induced by interactions with the environment affects time correlations, a process we call time correlation decoherence. Specifically, on the basis of simple mathematical analysis, we show how the interaction between a bipartite entangled system and a photon bath representing the environment can efface the oscillations in the coincidence detection rate of the interferometer. We discuss the dependence of this kind of decoherence on the photon energy and density.

Keywords Decoherence · Interference · Entanglement

1 Introduction

As famously stated by Feynman [1], Young's double-slit experiment has in it the *heart of quantum mechanics* and *contains the only mystery of the theory*. From the quantum mechanical point of view, this experiment consists of a group of particles, such as electrons, approaching a screen with two slits. After traversing the slits, the particles impinge on a distant detector screen, which registers permanently their positions. If no information is available concerning the passage of the particles through the slits, the particle density on the detection screen displays an interference pattern described by the expression $\rho(x) = \frac{1}{2} |\psi_1(x) + \psi_2(x)|^2$, where $\psi_1(x)$ and $\psi_2(x)$ are the partial wave functions associated with the passage through slits 1 and 2, respectively. On the other hand, if the experimental

B. de Lima Bernardo (☒)
Departamento de Física, CCEN, Universidade Federal da Paraíba,
Caixa Postal 5008, 58059-900, João Pessoa, PB, Brazil
e-mail: bertulio@fisica.ufpb.br



procedure determines the slit traversed by each particle, the interference pattern disappears and the detector exhibits the classical addition of two patterns, one due to the particles that have traversed slit 1 and the other due to those that have traversed slit 2, i.e., $\rho(x) = \frac{1}{2} |\psi_1(x)|^2 + \frac{1}{2} |\psi_2(x)|^2$. This experiment leads to the conclusion that quantum interference is incompatible with which-path information.

Let us now consider the behavior of a classical macroscopic object immersed in a large environment of gaseous molecules, light, thermal photons, etc. At any moment, a huge number of environmental particles collide with the object, in such a way that they will carry some information about the object, on its position and orientation in space, for instance. In this case, the information is associated with the scattering positions and deflection angles. We see that every object interacts with its environment, as a result of which, information about the physical properties of the former is inevitably encoded in the latter.

Interactions between quantum objects and their environments are significantly weaker because quantum systems are several orders of magnitude smaller than classical ones. Nonetheless, system-environment interactions are ubiquitous in quantum physics and can transfer which-path (or which-state) information to the environment by the aforementioned mechanism. In other words, an interacting environment suppresses interference (wave-like behavior) in atomic systems and consequently bars quantum manifestations at the macroscopic scale. System-environment interactions explain how the classical behavior of the macroscopic world emerges from the quantum properties of its building blocks [2–4].

A quantum superposition depends on the relative phases between its components. System-environment interactions transfer which-path (or which-state) information to the environment at the expense of the coherence among those relative phases. This inevitable monitoring of the system by the environment therefore amounts to the so-called *decoherence* process. In the two-slit interferometer, one can always regard any mechanism offering information on the particle path as a form of system-environment interaction responsible for a specific kind of decoherence, i.e., the destruction of the interference pattern on the detection screen [4, 5]. This remarkably simple, evident form of decoherence is the object of our analysis.

We are particularly interested in a class of interference devices first developed in the 1980s, the two-particle interferometer [7–9]. Certain experiments have shown that when two entangled particles separately go through a singleparticle interferometer, such as the Young interferometer, an interference pattern results when the rate of coincident arrival is measured, while no such pattern appears when only one particle is observed [10]. Entangled particles, in this context, are particles simultaneously created by the same source in such a way that, due to momentum conservation, one only has to determine the position of one particle to predict the position of the other. This kind of interferometer, as we shall see, is particularly sensitive to entanglement correlation. Nonetheless, the literature contains no detailed description of decoherence in two- and many-particle systems of this kind.

The purpose of this paper is to establish a simple, direct connection between decoherence and two-particle interferometry. To this end, we discuss a *gedanken* experiment originally devised by Horne and Zeilinger [11], which is convenient because simple calculations suffice to describe the system after interaction with the bath of monochromatic photons that here represents the environment. As already mentioned, in the two-particle interferometer under study, interference is only observed in time correlation measurements. For this reason, we will refer to the environmental disturbance as *time correlation decoherence* (TCD), to distinguish it from the well-known spatial decoherence that is commonplace in the single-particle systems.

2 Two-Particle Interferometry

The *gedanken* experiment, which Gottfried has also explored very well [12], analyzes the particles produced by the decay process $A \rightarrow a + b$, each daughter particle going through a double-slit apparatus, as shown in Fig. 1. If A is approximately at rest, momentum conservation forces a and b to travel in approximately opposite directions. Therefore, if a passes through one of the slits on the right, b must pass through the diametrically opposite slit on the left.

Let $|R_1\rangle$ and $|R_2\rangle$ denote the quantum states of particle *a* corresponding to passage through slits 1 and 2 on the

right, and $|L_1\rangle$ and $|L_2\rangle$ denote the quantum states of particle b corresponding to the passage through slits 1 and 2 on the left, respectively. We can then write the two-particle quantum state in the form

$$|\psi\rangle = \frac{1}{\sqrt{2}} (|R_1\rangle|L_2\rangle + |R_2\rangle|L_1\rangle). \tag{1}$$

On the right-hand side of Eq. (1), we recognize a state that is entangled, in the above-defined sense, since $|\psi\rangle$ cannot be factorized into a simple product of a and b states, i.e., no two states $|R_i\rangle$ and $|L_j\rangle$ can be found such that $|\psi\rangle = |R_i\rangle|L_j\rangle$. Therefore, the state of particle cannot be specified without reference to the other particle.

The concepts of density matrix and reduced density matrix have capital importance in decoherence theory. We therefore adopt those concepts from the outset, to familiarize the reader with them. We shall make only simple use of these tools. To compare our formalism with the quantum state formalism, we recommend Gottfried's analysis of the same *gedanken* experiment [12]. A clear introduction to the density matrix and reduced density matrix in the context of decoherence can be found in Ref. [4].

To start, we write the density matrix $\rho = |\psi\rangle\langle\psi|$ for this system in the following form

$$\rho = \frac{1}{2} \sum_{\substack{ij=1\\i\neq j}}^{2} |R_i\rangle |L_j\rangle \langle L_j| \langle R_i| + \frac{1}{2} \sum_{\substack{ij=1\\i\neq j}}^{2} |R_i\rangle |L_j\rangle \langle L_i| \langle R_j|.$$
(2)

To describe the behavior of one of the particles, the reduced density matrix associated with that particle is convenient. If we are interested in particle a, for instance, to compute the reduced density matrix ρ_a , we trace Eq. (2) over the states of particle b in the following way:

$$\rho_a = \text{Tr}_{\mathsf{b}} |\psi\rangle\langle\psi| = \frac{1}{2} \sum_{i=1}^{2} \langle L_i |\psi\rangle\langle\psi| L_i\rangle. \tag{3}$$

Since
$$\langle L_1|L_2\rangle=0$$
, we have that $\rho_a=1/2\sum_{i=1}^2|R_i\rangle\langle R_i|$.

This density matrix corresponds to a particle density $\rho(y_a)$ on the detecting screen on the right-hand side of Fig. 1 given by the expression

$$\rho(y_a) \equiv \langle y_a | \rho_a | y_a \rangle = \frac{1}{2} |\psi_{a1}(y_a)|^2 + \frac{1}{2} |\psi_{a2}(y_a)|^2, \tag{4}$$

where
$$\psi_{ai}(y_a) = \langle y_a | R_i \rangle$$
 $(i = 1, 2)$.

As we can see, the distribution of a particles on the detection screen exhibits no interference pattern. Given the symmetry of the apparatus, we see that mutatis mutandis the same result describes the distribution of b particles on the left-hand detection screen. Physically speaking, the absence of interference patterns stems from assuming particle A to



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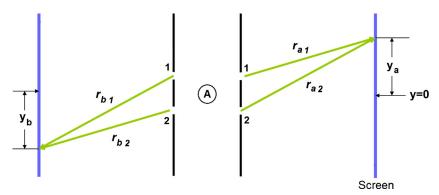


Fig. 1 (Colored online) Schematic representation of the gedanken experiment discussed in the text. Particle A, with approximately zero momentum, decays into two particles a and b. To conserve momentum, the daughter particles must travel in approximately diametrically

opposite paths. Each particle traverses a double-slit apparatus before being detected by a screen, at y_a or y_b . The lengths r_{a1} , r_{a2} , r_{b1} , and r_{b2} are the distances from the slits to the detection points

be approximately at rest initially. According to the uncertainty principle, we have almost no information on the initial position of *A*. Therefore, particle *A* is equivalent to a large source of daughter particles. Consequently, single-particle interference cannot occur.

Let us now analyze the system as a whole. The probability density of simultaneously detecting particle a at y_a and particle b at y_b is given by the expression

$$\rho(y_a, y_b) \equiv \langle y_b | \langle y_a | \rho | y_a \rangle | y_b \rangle. \tag{5}$$

Substitution of Eq. (2) into Eq. (5) yields the result

$$\rho(y_a, y_b) = \frac{1}{2} \sum_{\substack{ij=1\\i\neq j}}^{2} \langle y_a | R_i \rangle \langle y_b | L_j \rangle \langle L_j | y_b \rangle \langle R_i | y_a \rangle$$

$$+ \frac{1}{2} \sum_{\substack{ij=1\\i\neq j}}^{2} \langle y_a | R_i \rangle \langle y_b | L_j \rangle \langle L_i | y_b \rangle \langle R_j | y_a \rangle. \tag{6}$$

Let us assume that, after passing through one of the slits, the wave functions of the particles are spherical waves, i.e., given by the expressions

$$\psi_{aj}(r_{aj}) = \langle r_{aj} | R_j \rangle = \frac{e^{ikr_{aj}}}{r_{aj}} \tag{7}$$

and

$$\psi_{bj}(r_{bj}) = \langle r_{bj} | L_j \rangle = \frac{e^{ikr_{bj}}}{r_{bj}} \qquad (j = 1, 2), \tag{8}$$

where the $r_{(a,b)j}$ denote the distances from the slits to the detection points, and k is the wave number.

If we let the distance between the slits and the detection screen be much larger than the separation between the two slits so that we are in the Fraunhofer diffraction limit, we have that $r_{a(1,2)} \approx L \mp \theta y_a$ and $r_{b(1,2)} \approx L \mp \theta y_b$ [13], with the coordinates y defined in Fig. 1, and the angle θ and

distance L defined in Fig. 2. Equations (7) and (8) then yield the approximate equalities

$$\langle y_a | R_{1,2} \rangle \approx \frac{e^{ik(L \mp \theta y_a)}}{L \mp \theta y_a}$$
 (9)

and

$$\langle y_b | L_{1,2} \rangle \approx \frac{e^{ik(L \mp \theta y_b)}}{L \mp \theta y_b}.$$
 (10)

If we substitute Eqs. (9) and (10) into Eq. (6), noticing that the denominators can all be absorbed into an irrelevant overall factor, for small diffraction angles we can write the following expression for the joint probability of detecting particle a at y_a and particle b at y_b :

$$\rho(y_a, y_b) \doteq \cos^2(k\theta(y_a - y_b)), \tag{11}$$

where the symbol \doteq stands for equality up to a constant factor. This equation shows that the coincident arrival rate is a periodic function of the relative position $y_a - y_b$, a functional form characteristic of interference. It is not difficult to identify the source of this curious behavior: as Eq. (1) shows, the daughter particles emitted in opposite directions by the decay of particle A can reach the screens in two alternative ways. The interference between these two paths is responsible for the oscillatory coincidence rate.

The contrast between Eqs. (4) and (11) constitutes a particular instance of a relation between the single- and two-particle interferences first identified by Jaeger et al. [6] in their analysis of the system depicted in Fig. 1. Ref. [6] showed that if the initial state is maximally entangled, the coincidence rate is affected by interference, as we have shown, while single-particle detection patterns are not. By contrast, when the particles are created in a separable state, only single-particle interference arises. More specifically, the more entangled the initial state is, the stronger the interference in the coincidence detection rate and the weaker the single-particle interference. This complementarity has been verified in a number of experiments [7–9].



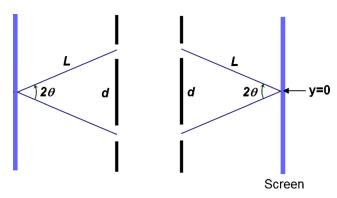


Fig. 2 (Colored online) Distance L and the angle θ associated with arrangement in Fig. 1

Although capturing essential features of experimental systems, Fig. 1 is schematic. One of its most important limitations is the absence of interaction between the particles and the environment. In the next section we show, for the first time, how this interaction undercuts the interference responsible for the oscillatory behavior in Eq. (11), i.e., how the system-environment interaction gives rise to decoherence.

3 Time Correlation Decoherence

We now turn to the open system, i.e., to particles that interact with the environment. As we shall see, decoherence will arise, i.e., time correlations will lose coherence.

First, we place a light source beyond the two slits on the right-hand side of Fig. 1, to play the role of the environment. In this arrangement, if we place a detector behind each slit, a photon that happens to be scattered by particle a near one of the slits will be recorded by the nearest detector. The collision between particle a and the photon represents the system-environment interaction, which, as we shall show, entangles them and causes decoherence. Next, we restrict our analysis to photons with a wavelength so short that diffraction can be ruled out and we can be sure that a photon scattered at slit 1 cannot reach the detector at slit 2 or vice versa. The purpose of the detectors is to detect interactions between particle a and the environment. In no way do they affect decoherence.

In analogy with the discussion in section 2, we describe the system-environment interaction by the expressions

$$|R_1\rangle|L_2\rangle|\epsilon_0\rangle \to |R_1\rangle|L_2\rangle|\epsilon_1\rangle$$
 (12)

and

$$|R_2\rangle|L_1\rangle|\epsilon_0\rangle \to |R_2\rangle|L_1\rangle|\epsilon_2\rangle,$$
 (13)

where the environmental states $|\epsilon_0\rangle$, $|\epsilon_1\rangle$, and $|\epsilon_2\rangle$ are the initial photonic quantum state, the state into which the arrival of a photon scattered near slit 1 triggers detector 1,

and the state into which the arrival of a photon scattered near slit 2 triggers detector 2, respectively. The initial environmental state $|\epsilon_0\rangle$ evolves into $|\epsilon_1\rangle$ or $|\epsilon_2\rangle$, depending on the system state. Equations (12) and (13) are valid only if particle a scatters a photon right after passing through one of the slits, without any other collision. Otherwise, it would be incorrect to write $|R_1\rangle$ or $|R_2\rangle$ (which, according to Eq. (9), represent spherical waves emerging from slits 1 and 2) on the right-hand sides of expressions (12) and (13). The linearity of the Schrödinger equation implies the von Neumann measurement scheme [2, 3]

$$\frac{1}{\sqrt{2}}(|R_1\rangle|L_2\rangle + |R_2\rangle|L_1\rangle)|\epsilon_0\rangle \rightarrow \\
|\phi\rangle = \frac{1}{\sqrt{2}}(|R_1\rangle|L_2\rangle|\epsilon_1\rangle + |R_2\rangle|L_1\rangle|\epsilon_2\rangle). \tag{14}$$

We see that the system states have become entangled with the environmental states, which encode information on the particle paths. The initial coherence between the system states $|R_2\rangle|L_1\rangle$ and $|R_1\rangle|L_2\rangle$ is now shared with the environment, i.e., is now a property of the system-environment state.

Let us analyze the behavior of this system in more detail. If we determine the reduced density matrix $\rho_a = \mathrm{Tr_{bE}}|\phi\rangle\langle\phi|$ for particle a, where $\mathrm{Tr_{bE}}$ stands for the trace over the states of particle b and the environment, and proceed to calculating $\langle y_a|\rho_a|y_a\rangle$, it results that the probability density $\rho(y_a)$ of finding particle a at position y_a on the screen is still given by Eq. (4). Not surprisingly, the entanglement between system and environment has no effect upon the already incoherent single-particle probability density.

On the other hand, if we calculate the reduced density matrix $\rho_{ab}=\mathrm{Tr_E}|\phi\rangle\langle\phi|$ for the two particles, where $\mathrm{Tr_E}$ stands for the trace over the environmental states only, we find the equality

$$\rho_{ab} = \frac{1}{2} \sum_{k=1}^{2} \langle \epsilon_k | O_{ij} + Q_{ij} | \epsilon_k \rangle, \tag{15}$$

where

$$O_{ij} = \sum_{\substack{ij=1,\\i\neq j}}^{2} |R_i\rangle |L_j\rangle |\epsilon_i\rangle \langle \epsilon_i| \langle L_j| \langle R_i|$$
(16)

and

$$Q_{ij} = \sum_{\substack{ij=1,\\i\neq j}}^{2} |R_i\rangle |L_j\rangle |\epsilon_i\rangle \langle \epsilon_j| \langle L_i| \langle R_j|.$$
(17)

If a given photon is recorded by detector 1, the same photon cannot be recorded by detector 2. Mathematically, this



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self-evident notion corresponds to the equality $\langle \epsilon_i | \epsilon_j \rangle = 0$ for $i \neq j$. Equation (15) therefore reduces to the equation

$$\rho_{ab} = \frac{1}{2} \sum_{\substack{ij=1,\\i\neq j}}^{2} |R_i\rangle |L_j\rangle \langle L_j|\langle R_i|,$$
(18)

and Eqs. (9) and (10) yield the following expression for the probability density of simultaneously detecting particle a at y_a and particle b at y_b :

$$\rho(y_a, y_b) \equiv \langle y_a | \langle y_b | \rho_{ab} | y_b \rangle | y_a \rangle = \text{const.}$$
 (19)

The probability distribution in Eq. (19) is position independent. We therefore see that the system-environment interaction has destroyed the coincidence-rate interference expressed by Eq. (11), i.e., the interference prevalent in the isolated, photon-free system. Since time correlation interference is lost, we call this phenomenon time correlation decoherence (TCD).

Interesting issues emerge when we examine the environmental properties. In particular, we are interested on the dependence of TCD upon the photon energy or, equivalently, upon the wavelength of the light. Up to this point, we have only considered the small wavelength limit, i.e., a wavelength λ that is dwarfed by the slit separation d. With larger wavelengths, diffraction allows photons scattered near slit 1 (2) to reach detector 2 (1). The light is now unable to resolve the separation between the slits and the environment and cannot encode a significant amount of information on the particle paths.

In order to account for these new possibilities, we now write the state of the system in the form

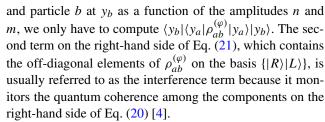
$$|\varphi\rangle = n|R_1\rangle|L_2\rangle|\epsilon_1\rangle + m|R_1\rangle|L_2\rangle|\epsilon_2\rangle + n|R_2\rangle|L_1\rangle|\epsilon_2\rangle + m|R_2\rangle|L_1\rangle|\epsilon_1\rangle, \tag{20}$$

where n and m are the probability amplitudes for a photon scattered near a given slit to be recorded by the detectors that are closer and farther from that slit, respectively. The right-hand side of Eq. (20) remains invariant under the change $1 \leftrightarrow 2$ because we are working with identical slits and symmetrically positioned detectors.

When we calculate the reduced density matrix, $\rho_{ab}^{(\varphi)} = \text{Tr}_{E}|\varphi\rangle\langle\varphi|$, under the condition $\langle\epsilon_{1}|\epsilon_{2}\rangle = 0$, the following result emerges:

$$\rho_{ab}^{(\varphi)} = (|n|^2 + |m|^2) \sum_{\substack{ij=1,\\i\neq j}}^{2} |R_i\rangle |L_j\rangle \langle L_j| \langle R_i|
+ (nm^* + n^*m) \sum_{\substack{ij=1,\\i\neq j}}^{2} |R_i\rangle |L_j\rangle \langle L_i| \langle R_j|.$$
(21)

Equation (21) is our central result. To find the probability density of simultaneously detecting particle a at y_a



In the small wavelength limit, we have $n=1/\sqrt{2}$ and m=0. In this case, as expected, Eq. (21) reduces to Eq. (18). TCD is maximum and the coincidence arrival rate displays no vestige of interference. In the large wavelength limit, on the other hand, n=m=1/2 since the amplitude of a photon reaching a detector is independent of the slit at which it was scattered. Under these conditions, Eq. (21) reduces to Eq. (2), TCD is minimum, and the coincidence arrival rate shows the interference features identified in our discussion of Eq. (11). No information on particle paths is conveyed to the environnment.

In the intermediate case, in which the detectors receive only partial which-path information, we have that $n > m \neq 0$. The coincidence-rate interference is weaker than in the large wavelength limit and the probability distribution combines a term reminiscent of Eq. (11) with a constant contribution, analogous to Eq. (19):

$$\rho(y_a, y_b) \equiv \langle y_a | \langle y_b | \rho_{ab} | y_b \rangle | y_a \rangle$$

$$\doteq |n|^2 + |m|^2 + (nm^* + mn^*) \cos(2k\theta(y_a - y_b)). (22)$$

Another important parameter is the intensity of the light source, i.e., the photon density in the region beyond the slits on the right-hand side of Fig. 1. So far, we have implicitly assumed the intensity to be sufficiently high to insure scattering, with 100 % certainty. This constraint relaxed, the properties of the system are described by a mixed density matrix [4] of the form $\rho = w_1 |\phi\rangle\langle\phi| + w_2 |\alpha\rangle\langle\alpha|$. Here $|\phi\rangle$ is defined as in Eq. (14), $|\alpha\rangle = |\psi\rangle|\epsilon_0\rangle$ is a separable (non-entangled) system-environment state associated with the absence of collisions, and w_1 and $w_2 = 1 - w_1$ are the classical probabilities of particle a scattering or not scattering a photon after passing through the slits, respectively. We calculate the reduced density matrix, $\rho_{ab} = \text{Tr}_{E}(\rho)$ and from ρ_{ab} , with the wave functions in Eqs. (9) and (10), we find the following expression for the probability density $\rho(y_a, y_b)$ to detect the two particles in coincidence:

$$\rho(y_a, y_b) \equiv \langle y_a | \langle y_b | \rho_{ab} | y_b \rangle | y_a \rangle$$

$$\doteq w_1 + 2w_2 \cos^2(k\theta(y_a - y_b)), \qquad (23)$$

which, as expected, combines features found in Eqs. (11) and (19).

For completeness, we cursorily discuss an alternative arrangement, with an additional light source and two other detectors beyond the slitted screen on the left-hand side of in Fig. 1. Qualitatively, the new arrangement is equivalent to the setup we have analyzed. As long as one of the particles



or both of them scatter photons after passing through the slits, the environmental state changes as it acquire information on the paths followed by the particles. Clearly, with two collision alternatives, the odds in favor of acquiring whichpath information are higher, and interference is weakened. Equations (19), (20), and (21) are still applicable, but given that each particle can now collide with a photon, the collision probability parameter w_1 in Eq. (23) is larger. For example, if the two light sources are identical, w_1 is twice larger than in the previous case.

4 Conclusion

In conclusion, we have quantitatively studied a class of quantum mechanical decoherence processes, to show how the system-environment interactions suppress coincidence-rate interference in a two-particle interferometer. The environment was modeled by a photon bath. Given the loss in particle time correlation coherence, we have called this process time correlation decoherence (TCD). In addition, we have brought to light the decisive importance of the photon energy and density in TCD.

Acknowledgments The author gratefully acknowledges Eric J. Heller for invaluable comments and discussions. This work was supported by Coordenação de Aperfeiçoamento de Pessoal de Nível Superior (CAPES) and by Conselho Nacional de Desenvolvimento Científico e Tecnolgico (CNPq).

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