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STATISTICAL

News and Views: About Complexity and Why to Care

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Abstract What is nowadays ubiquitously referred to as *complexity* emerges in a wide variety of natural, artificial, and social systems. This very rich concept is nontrivial to understand and is therefore hard to operationally define. Consequently, along the years, many intertwined characterizations have been proposed in the literature. Among those, a powerful and practical one consists in focusing on the entropic and statistical mechanical aspects of the system. We attempt here to put this active line of research into a contemporary perspective.

Keywords Complex systems · Nonadditive entropies · Nonextensive statistical mechanics

Statistical mechanics and its profound connection to thermodynamics constitute, together with classical, quantum, and relativistic mechanics and electromagnetism, one of the pillars of contemporary physics. It is primarily due to the genius of L. Boltzmann and J.W. Gibbs. Consistently, the Boltzmann–Gibbs entropy expression $S_{BG} = k \ln W$ belongs today—together with $F = ma$, $E = mc^2$, Maxwell, and Schroedinger equations—to the mathematical hardcore that no educated physicist and chemist, among others, can ignore. Statistical mechanics is based on mechanics and electromagnetism, but it adds to its very foundations a nontrivial ingredient, namely theory of probabilities. It takes into account the fact that there is, in science, a virtually infinite amount of information that we cannot know. Not only we cannot measure nor calculate it, but even more, we do not want to explicitly handle it in most practical occasions. It is obvious that such situation does not happen only in physics: it also occurs in computational sciences, engineering, biology, linguistics, economics, and in fact in all branches of human knowledge. This advances the ubiquitous relevance of the concept of *entropy*, which naturally emerges in all kinds of problems concerning generation, communication, and capture of information, far above its specific and unavoidable use in statistical mechanics and thermodynamics.

Having made these remarks, let us focus now on *complexity* and *complex systems*. And to start with, what is it that we call "complexity"? S. Lloyd discussed 31 definitions of complexity (quoted by Horgan in his 1995 paper *From Complexity to Perplexity*); B. Edmonds advances in 1999 over 40 definitions of complexity. It is unnecessary therefore to insist on the point that complexity, like beauty and a plethora of other crucial human concepts that have defied philosophers of all kinds and all times, is very hard to define. A slightly simpler problem is what is it that we call "complex systems"? In a kind of similar way to which we can recognize a beautiful piece of art, or a beautiful child, even if we do not know how to universally define beauty; we do recognize a complex system, even if we do not know how to neatly

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define complexity. Among its many characterizations, there is one which, in one way or another, is always there, namely that a complex system is usually constituted by many interacting elements, each of which might be thought (and represented) as considerably simpler than the whole system.

Consistently, *emergence* of collective phenomena frequently occurs, which are very hard to predict (sometimes nearly unthinkable a priori) in terms of the elements of the system. Motion of galaxies (made by stars), motion of granular matter (made by grains), nature of hadronic jets (produced by interactions between quarks and gluons after high-energy collisions of protons, for example), behavior of crowds (made by individuals), intellectual performances of the brain (made by neurons), behavior of economical crisis (triggered by relatively simple operations between traders, banks, customers), evolution of languages, of species, of cultures, of the Earth's biosphere and weather, processing of signals and images—the list is endless.

For one century and a half, statistical mechanics has proved to be a very successful theoretical approach to the connections between sensibly different (space-time) scales of a great variety of systems (condensed matter physics, theory of phase transitions and critical phenomena, renormalization group, stochastic equations, to mention but a few). It is therefore natural to try to adapt the concepts and methods of standard statistical mechanics in order to efficiently deal (improving in particular predictability, which is of course one of the aims of science, side by side with understanding) with an enormous variety of important natural, artificial, and social phenomena that we observe around us. In this regard, the concept of entropy naturally appears as the central ingredient to tackle with.

In 1865, Clausius introduced in thermodynamics and named the concept *entropy* [1]. It was introduced in completely macroscopic terms, with no reference at all to the microscopic world, whose existence was under strong debate at his time, and still so even several decades later. One of the central properties of this concept was to be thermodynamically *extensive*, i.e., to be proportional to the size of the system (characterized by its total d-dimensional hypervolume). In the 1870s, Boltzmann [2–4] made the genius connection of the thermodynamical entropy to the microcosmos. This connection was refined by Gibbs a few years later [5–7]. From this viewpoint, the thermodynamic entropic extensivity became the nowadays well-known property that the total entropy of a system should be proportional to N, the total number of its microscopic elements (or, equivalently, proportional to the total number of microscopic degrees of freedom). More precisely, in the $N \to \infty$ limit, it should asymptotically be $S(N) \propto N$. The entropic function introduced by Boltzmann and Gibbs (and later on adapted to quantum and information-theoretic scenarios by von Neumann and Shannon, respectively) is given (for systems described through discrete random variables) by

$$S_{BG}(N) = -k \sum_{i=1}^{W(N)} p_i \ln p_i \quad \left(\sum_{i=1}^{W(N)} p_i = 1 \right), \tag{1}$$

where k is a conventional positive constant (usually taken to be the Boltzmann constant k_{B} in physics, and $k = 1$ in other contexts) and i runs over all nonvanishing-probability microscopic configurations of the N-sized system, $\{p_i\}$ being the corresponding probabilities. In the particular case of equal probabilities (i.e., $p_i = 1/W(N)$, $\forall i$), we recover the celebrated Boltzmann formula $S_{BG}(N) = k \ln W(N)$. It is clear that *if the microscopic random variables are probabilistically (strictly or nearly) independent*, we have $W(N) \propto \mu^N$ $(\mu > 1;\ N \to \infty)$; hence, $S_{BG}(N) \propto N$, thus satisfying the (Clausius) thermodynamic expectation of extensivity.

But *if the random variables are strongly correlated*, $W(N)$ behaves quite differently. For instance, if $W(N) \propto N^{\rho}$ $(\rho > 0;\ N \to \infty)$, then $S_{BG}(N) \propto \ln N$, which makes S_{BG} thermodynamically inadmissible. Let us consider, instead of Eq. (1), the following (nonadditive, as we shall see hereafter) entropy:

$$S_q(N) = k \frac{1 - \sum_{i=1}^{W(N)} p_i^q}{q-1} \times \left(\sum_{i=1}^{W(N)} p_i = 1;\ q \in \mathcal{R};\ S_1 = S_{BG} \right). \tag{2}$$

Hence, for the particular case of equal probabilities, we have $S_q(N) = k\frac{[W(N)]^{1-q}-1}{1-q} \equiv k \ln_q W(N)$ (with $\ln_1 W(N) = \ln W(N)$). Then, we straightforwardly verify that, for $q = 1 - \frac{1}{\rho}$, we have $S_q \propto N$, which is thermodynamically admissible.

If we consider two *probabilistically independent* systems A and B $\left(i.e.,\ p_{ij}^{A+B} = p_i^A p_j^B\right)$, we easily verify that $\frac{S_q(A+B)}{k} = \frac{S_q(A)}{k} + \frac{S_q(B)}{k} + (1-q)\frac{S_q(A)}{k}\frac{S_q(B)}{k}$, which means that, unless $q = 1$ (i.e., the BG entropy), S_q is *nonadditive*. These few remarks imply a kind of change of paradigm, namely extensivity and additivity are sensibly different properties, which makes the functional connection between the thermodynamic entropy and the microscopic world *nonuniversal*. It is thanks to its nonadditivity that S_q can be thermodynamically extensive for a vast class of complex systems, where specific strong correlations are present between the elements of the system. This yields to a possible manner of defining what complexity is: *A system might be said to be complex if correlations between its elements are such that, in order to guarantee entropic extensivity, a nonadditive entropy is needed* (see [8–11]). It follows, as a natural consequence, that BG entropy and statistical mechanics are sufficient but not necessary for thermodynamics to be valid for large-enough systems.

This simple observation has, amazingly enough, far-reaching consequences. Indeed, over 4,000 articles substantially related to this standpoint have been published worldwide by over 6,000 scientists (see the regularly updated bibliography, as well as the selected theoretical, experimental, observational, and computational articles, in http://tsallis.cat.cbpf.br/biblio.htm). Through analytical, experimental, observational, and computational results, these articles contain a plethora of predictions, verifications, and applications in many natural, artificial, and social systems (see, for instance, recent reviews in [12, 13]).

Let us stress that we have briefly presented here only a few, though central, among the many scientific, technological, and epistemological issues of complexity. To the study of this area of knowledge—whose importance can be illustrated by Stephen Hawking's January 2000 declaration "I think the next century will be the century of complexity".—several centers have been created all over the world. We may mention the Santa Fe Institute in New Mexico and the Max Planck Institut fuer Physik komplexer Systeme in Dresden, among many others. In Brazil, substantial activity is being dedicated to the subject, in particular by the Instituto Nacional de Ciencia e Tecnologia de Sistemas Complexos, which presently aggregates 37 scientists from 18 Brazilian institutions.

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