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GENERAL AND APPLIED PHYSICS

Investigating Bulk Waves in Orthotropic Rectangular Nanoplates Based on Three Dimensional Elasticity Theory and Nonlocal Elasticity Theory

Mohammad Rahim Nami · Maziar Janghorban

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Abstract The propagation of bulk waves in rectangular nanoplates is studied on the basis of nonlocal three-dimensional elasticity theory. The nonlocal theory applies to both thin and thick rectangular orthotropic nanoplates. The dispersion relation for the waves is derived analytically. Our results are checked against data for macroplates. The influence of nonlocality and other parameters on the wave frequency and phase velocity is discussed.

Keywords Bulk waves · Three-dimensional elasticity theory · Nonlocal elasticity theory · Rectangular nanoplates · Orthotropic materials

1 Introduction

Waves propagate in distinct ways. Relative to the direction of propagation, the direction of oscillation can either be longitudinal or transversal [1, 2]. Various studies have been dedicated to wave propagation in different structures. Kapania and Raciti [3] summarised recent analyses of laminated beams and plates with emphasis on vibrations and wave propagation. Chow [4] derived equations for the dynamics of orthotropic laminated plates on the basis of Timoshenko's beam theory to include the effects of transverse shear and rotational inertia, discussed the propagation of flexural waves and studied the transient response of a rectangular plate to normal impact. Lowe and Diligent [5] analysed the reflection of the fundamental Lamb mode from surface-breaking rectangular notches on isotropic plates. Their results were extracted from finite-element time-domain simulations complemented by experiments. Mukdadi et al. [6] discussed guided waves in a layered elastic plate of rectangular cross-section, with finite width and thickness, the deformation of the cross-section being modelled by a two-dimensional finite-element analytical method. Lee and Staszewski [7] developed a local interaction simulation to describe wave propagation in metallic structures, with damage-detection applications in mind. Their study included a bond model for piezoceramic actuator/sensor diffusion. Mead and Yaman [8] presented an exact analytical method to describe the vibrational response of a finite, three-layered, rectangular sandwich plate with a viscoelastic core subject to a harmonic linear force. Their work benefited from previous knowledge on the flexural wave motion in an infinite parallel unstiffened plate under a single harmonic linear force or moment. Wang et al. [9] examined flexural wave propagation in doubled-layered nanoplates in the framework of the nonlocal continuum theory to derive an equation for wave motion. They computed the frequency, phase velocity, group velocity, and the ratio of the two velocities with different scale coefficients and wave numbers. Wang and Lu [10], whose work combined continuum mechanics and molecular dynamics simulation based on the Terroff-Brenner potential, described flexural wave propagation in a carbon nanotube. The study focused on the wave dispersion due to rotational inertia and shear deformation in the model of a Timoshenko beam. Toshiaki et al.'s continuum mechanics study [11] on wave propagation in single- and double-walled carbon nanotubes in a conveying fluid proposed a simplified set of Flügge shell equations governing the motion of the carbon nanotubes. Arash et al. [12] developed a nonlocal elastic plate model accounting for scale effects in the propagation of waves in graphene sheets and developed a finite element model from the weak form of the elastic plate model to present a comprehensive study of the waves in the sheets. Several other articles have also studied the wave propagation in nanoplates [13, 14]. Recently, we have reported a study of wave propagation on nanoplates on the basis of strain gradient elasticity

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Table 1 Orthotropic nano-plates in our study. For each nanoplate, the dimensions and nonlocal parameter are shown, along with the material properties [24]

Nanoplate type	*a* (nm)	*b* (nm)	*h* (nm)	$E_1(GPa)$	$E_2(GPa)$	$G_{12}(GPa)$	v_{12}	$\rho(kg/m^3)$	$\sqrt{\mu}(nm)$
Armchair sheet I	9.519	4.844	0.129	2,434	2,473	1,039	0.197	6,316	0.67
Armchair sheet II	6.995	4.847	0.143	2,154	2168	923	0.202	5,727	0.47
Armchair sheet III	4.888	4.855	0.156	1,949	1962	846	0.201	5,295	0.27
Zigzag sheet IV	9.496	4.877	0.145	2,145	2097	938	0.223	5,624	0.47
Zigzag sheet V	7.065	4.887	0.149	2,067	2054	913	0.204	5,482	0.32
Zigzag sheet VI	4.855	4.888	0.154	1,987	1974	857	0.205	5,363	0.22

[15]. Here, on the basis of three-dimensional nonlocal elasticity theory, we turn to bulk waves in rectangular nanoplates. Wave motion in the bulk is independent of boundary conditions and hence easier to describe than guided waves, even though the governing equations are the same [16]. Our study adopts an analytical method to study the effects of different parameters on the frequencies, phase velocity, and group velocities. We hope that our methodology will motivate future improvements in the modelling of wave propagation in nanostructures.

2 Nonlocal Elasticity Theory

The response of materials at the nanoscale is different from the response at the macroscopic scale. The stress at a reference

Table 2 Wave frequency (ω=$\omega\times 10^{13}$) in orthotropic nanoplates for various nonlocal parameters and wave numbers

	$\mu(nm^2)$					
	0.0 [27]	0.0	1.0	1.5	2.0	2.5
$k_z=1\times 10^9$						
Armchair sheet I	1.2826	1.2826	0.9069	0.8112	0.7405	0.6856
Armchair sheet II	1.2695	1.2695	0.8977	0.8029	0.7330	0.6786
Armchair sheet III	1.2640	1.2640	0.8938	0.7994	0.7298	0.6756
Zigzag sheet IV	1.2915	1.2915	0.9132	0.8168	0.7456	0.6903
Zigzag sheet V	1.2905	1.2905	0.9125	0.8162	0.7451	0.6898
Zigzag sheet VI	1.2641	1.2641	0.8939	0.7995	0.7298	0.6757
$k_z=5\times 10^9$						
Armchair sheet I	6.4129	6.4129	1.2577	1.0335	0.8980	0.8048
Armchair sheet II	6.3476	6.3476	1.2449	1.0230	0.8888	0.7966
Armchair sheet III	6.3201	6.3201	1.2395	1.0186	0.8850	0.7931
Zigzag sheet IV	6.4573	6.4573	1.2664	1.0407	0.9042	0.8103
Zigzag sheet V	6.4526	6.4526	1.2655	1.0399	0.9035	0.8097
Zigzag sheet VI	6.3206	6.3206	1.2396	1.0187	0.8851	0.7932
$k_z=7\times 10^9$						
Armchair sheet I	8.9781	8.9781	1.2697	1.0402	0.9023	0.8079
Armchair sheet II	8.8866	8.8866	1.2568	1.0296	0.8931	0.7997
Armchair sheet III	8.8481	8.8481	1.2513	1.0251	0.8893	0.7962
Zigzag sheet IV	9.0402	9.0402	1.2785	1.0474	0.9086	0.8135
Zigzag sheet V	9.0337	9.0337	1.2776	1.0466	0.9079	0.8129
Zigzag sheet VI	8.8488	8.8488	1.2514	1.0252	0.8893	0.7963
$k_z=9\times 10^9$						
Armchair sheet I	11.5433	11.5433	1.2747	1.0429	0.9041	0.8092
Armchair sheet II	11.4256	11.4256	1.2617	1.0323	0.8949	0.8009
Armchair sheet III	11.3761	11.3761	1.2563	1.0278	0.8910	0.7975
Zigzag sheet IV	11.6231	11.6231	1.2836	1.0502	0.9104	0.8148
Zigzag sheet V	11.6147	11.6147	1.2826	1.0494	0.9097	0.8142
Zigzag sheet VI	11.3770	11.3770	1.2564	1.0279	0.8911	0.7975

Table 3 The effects of nonlocal parameter and wave number on the phase velocities ($\frac{d\omega}{dk}=\frac{d\omega}{dk}\times 10^3$) of orthotropic nanoplates

	$\mu(\mathrm{nm}^2)$					
	0.0	0.5	1.0	1.5	2.0	2.5
$k_z=1\times10^9$						
Armchair sheet I	12.8259	10.4723	9.0693	8.1118	7.4050	6.8557
Armchair sheet II	12.6951	10.3655	8.9768	8.0291	7.3295	6.7858
Armchair sheet III	12.6401	10.3206	8.9379	7.9943	7.2978	6.7564
Zigzag sheet IV	12.9145	10.5447	9.1320	8.1679	7.4562	6.9031
Zigzag sheet V	12.9052	10.5371	9.1254	8.1620	7.4508	6.8981
Zigzag sheet VI	12.6411	10.3215	8.9386	7.9950	7.2984	6.7570
$k_z=5\times10^9$						
Armchair sheet I	12.8259	3.4908	2.5154	2.0671	1.7960	1.6095
Armchair sheet II	12.6951	3.4552	2.4897	2.0460	1.7777	1.5931
Armchair sheet III	12.6401	3.4402	2.4789	2.0371	1.7700	1.5862
Zigzag sheet IV	12.9145	3.5149	2.5327	2.0814	1.8084	1.6207
Zigzag sheet V	12.9052	3.5124	2.5309	2.0799	1.8071	1.6195
Zigzag sheet VI	12.6411	3.4405	2.4791	2.0373	1.7701	1.5864
$k_z=7\times10^9$						
Armchair sheet I	12.8259	2.5399	1.8139	1.4860	1.2890	1.1541
Armchair sheet II	12.6951	2.5140	1.7954	1.4708	1.2759	1.1424
Armchair sheet III	12.6401	2.5031	1.7876	1.4644	1.2704	1.1374
Zigzag sheet IV	12.9145	2.5575	1.8264	1.4962	1.2980	1.1621
Zigzag sheet V	12.9052	2.5556	1.8251	1.4952	1.2970	1.1613
Zigzag sheet VI	12.6411	2.5033	1.7877	1.4646	1.2705	1.1375
$k_z=9\times10^9$						
Armchair sheet I	12.8259	1.9910	1.4164	1.1588	1.0046	0.8991
Armchair sheet II	12.6951	1.9707	1.4019	1.1470	0.9944	0.8899
Armchair sheet III	12.6401	1.9621	1.3959	1.1420	0.9901	0.8861
Zigzag sheet IV	12.9145	2.0047	1.4262	1.1668	1.0115	0.9053
Zigzag sheet V	12.9052	2.0033	1.4251	1.1660	1.0108	0.9047
Zigzag sheet VI	12.6411	1.9623	1.3960	1.1421	0.9901	0.8861

point is now a functional of the strain field at every point of the continuum [17]. Nonlocal elasticity theory yields the following differential equations to represent the constitutive relations for a graphene sheet (http://silver.neep.wisc.edu/~lakes/Nonlocal.html) [18]:

$$\left(1-\mu\nabla^2\right)\sigma = t \tag{1}$$

where the pre-factor μ modulating the Laplacian is the *nonlocal parameter*, t is the macroscopic stress tensor at the position under study, and σ is the nonlocal stress tensor. Different authors have reported different values for the nonlocal parameter. For example, Duan and Wang [19] have used μ's ranging from 0 to 4 nm in their analysis of bending in circular micro/nanoplates. Wang and Wang [20] have pointed out µshould be less than 4 nm [21]. The same nonlocal parameters have been adopted by Nami and Janghorban [22]. A few other nonlocal parameters are listed in Table 1.

Equation (1) can be spelled out as follows:

$$\left\{\begin{array}{l}
\sigma_x-\mu\left(\frac{\partial^2\sigma_x}{\partial x^2}+\frac{\partial^2\sigma_x}{\partial y^2}+\frac{\partial^2\sigma_x}{\partial z^2}\right)\\
\sigma_y-\mu\left(\frac{\partial^2\sigma_y}{\partial x^2}+\frac{\partial^2\sigma_y}{\partial y^2}+\frac{\partial^2\sigma_y}{\partial z^2}\right)\\
\sigma_z-\mu\left(\frac{\partial^2\sigma_z}{\partial x^2}+\frac{\partial^2\sigma_z}{\partial y^2}+\frac{\partial^2\sigma_z}{\partial z^2}\right)\\
\tau_{yz}-\mu\left(\frac{\partial^2\tau_{yz}}{\partial x^2}+\frac{\partial^2\tau_{yz}}{\partial y^2}+\frac{\partial^2\tau_{yz}}{\partial z^2}\right)\\
\tau_{xz}-\mu\left(\frac{\partial^2\tau_{xz}}{\partial x^2}+\frac{\partial^2\tau_{xz}}{\partial y^2}+\frac{\partial^2\tau_{xz}}{\partial z^2}\right)\\
\tau_{xy}-\mu\left(\frac{\partial^2\tau_{xy}}{\partial x^2}+\frac{\partial^2\tau_{xy}}{\partial y^2}+\frac{\partial^2\tau_{xy}}{\partial z^2}\right)
\end{array}\right\}
=\left\{\begin{array}{cccccc}
C_{11} & C_{12} & C_{13} & 0 & 0 & 0\\
C_{12} & C_{22} & C_{23} & 0 & 0 & 0\\
C_{13} & C_{23} & C_{33} & 0 & 0 & 0\\
0 & 0 & 0 & C_{44} & 0 & 0\\
0 & 0 & 0 & 0 & C_{55} & 0\\
0 & 0 & 0 & 0 & 0 & C_{66}
\end{array}\right\}
\left\{\begin{array}{c}
\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\\ \gamma_{yz}\\ \gamma_{xz}\\ \gamma_{xy}
\end{array}\right\} \tag{2}$$

Table 4 The effects of nonlocal parameter and wave number on the group velocities of orthotropic nanoplates

	μ(nm^2)					
	0.5	0.75	1.0	1.25	1.5	1.75
k_z=3×10^9						
Armchair sheet I	38.5451	13.7769	6.4129	3.4886	2.1035	1.3646
Armchair sheet II	38.1522	13.6365	6.3476	3.4530	2.0821	1.3507
Armchair sheet III	37.9869	13.5774	6.3201	3.4381	2.0731	1.3449
Zigzag sheet IV	38.8115	13.8721	6.4573	3.5127	2.1181	1.3741
Zigzag sheet V	38.7836	13.8622	6.4526	3.5102	2.1166	1.3731
Zigzag sheet VI	37.9899	13.5785	6.3206	3.4383	2.0733	1.3450
k_z=5×10^9						
Armchair sheet I	2.6065	0.8324	0.3649	0.1912	0.1124	0.0716
Armchair sheet II	2.5799	0.8240	0.3611	0.1892	0.1112	0.0708
Armchair sheet III	2.5687	0.8204	0.3596	0.1884	0.1107	0.0705
Zigzag sheet IV	2.6245	0.8382	0.3674	0.1925	0.1132	0.0721
Zigzag sheet V	2.6226	0.8376	0.3671	0.1924	0.1131	0.0720
Zigzag sheet VI	2.5689	0.8205	0.3596	0.1884	0.1108	0.0705
k_z=7×10^9						
Armchair sheet I	0.3868	0.1192	0.0513	0.0266	0.0155	0.0098
Armchair sheet II	0.3828	0.1180	0.0508	0.0263	0.0154	0.0097
Armchair sheet III	0.3812	0.1175	0.0506	0.0262	0.0153	0.0097
Zigzag sheet IV	0.3894	0.1200	0.0517	0.0268	0.0156	0.0099
Zigzag sheet V	0.3891	0.1199	0.0516	0.0267	0.0156	0.0099
Zigzag sheet VI	0.3812	0.1175	0.0506	0.0262	0.0153	0.0097
k_z=9×10^9						
Armchair sheet I	0.0897	0.0272	0.0116	0.0060	0.0035	0.0022
Armchair sheet II	0.0888	0.0270	0.0115	0.0059	0.0035	0.0022
Armchair sheet III	0.0884	0.0268	0.0115	0.0059	0.0034	0.0022
Zigzag sheet IV	0.0903	0.0274	0.0117	0.0060	0.0035	0.0022
Zigzag sheet V	0.0903	0.0274	0.0117	0.0060	0.0035	0.0022
Zigzag sheet VI	0.0884	0.0268	0.0115	0.0059	0.0034	0.0022

3 Governing Equations

In this section, we derive the three-dimensional elasticity theory from the nonlocal elasticity theory. For rectangular nanoplates, the displacements are defined by the equalities

$$\begin{aligned} u &= u(x,y,z,t) \\ v &= v(x,y,z,t) \\ w &= w(x,y,z,t) \end{aligned} \tag{3}$$

where u, v, and w are the displacements in the x, y, and z directions, respectively. Also important for three-dimensional elasticity theory are the strain-displacement relations, expressed by the equalities

$$\varepsilon_x = \frac{\partial u}{\partial x}, \varepsilon_y = \frac{\partial v}{\partial y}, \varepsilon_z = \frac{\partial w}{\partial z}, \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}, \gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}, \gamma_{yz} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \tag{4}$$

The stress fields must satisfy the equilibrium condition, which yields the following equilibrium equations for a rectangular nanoplate [23]:

$$\begin{aligned} \frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xz}}{\partial z} + \frac{\partial \tau_{xy}}{\partial y} &= \rho \frac{\partial^2 u}{\partial t^2} \\ \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} + \frac{\partial \tau_{yx}}{\partial x} &= \rho \frac{\partial^2 v}{\partial t^2} \\ \frac{\partial \sigma_z}{\partial z} + \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} &= \rho \frac{\partial^2 w}{\partial t^2} \end{aligned} \tag{5}$$

We now combine the constitutive equations of the nonlocal theory, equilibrium equations, and strain-displacement relations to obtain a set of equations governing bulk-wave motion in rectangular nanoplates:

Table 5 The effects of nonlocal parameter and wave number on the frequency ($\omega=\omega\times10^{13}$) of orthotropic nanoplates

	μ(nm^2)					
	0.5	0.75	1.0	1.25	1.5	1.75
$k_x=k_z=3\times10^9$						
Armchair sheet I	0.9390	0.7798	0.6812	0.6125	0.5612	0.5209
Armchair sheet II	0.9276	0.7703	0.6730	0.6051	0.5544	0.5146
Armchair sheet III	0.9186	0.7629	0.6664	0.5992	0.5490	0.5096
Zigzag sheet IV	0.9340	0.7756	0.6776	0.6092	0.5581	0.5181
Zigzag sheet V	0.9310	0.7732	0.6754	0.6073	0.5564	0.5164
Zigzag sheet VI	0.9213	0.7651	0.6684	0.6010	0.5506	0.5110
$k_x=k_z=5\times10^9$						
Armchair sheet I	0.9706	0.7976	0.6930	0.6211	0.5677	0.5261
Armchair sheet II	0.9588	0.7879	0.6846	0.6135	0.5608	0.5197
Armchair sheet III	0.9495	0.7803	0.6779	0.6076	0.5554	0.5146
Zigzag sheet IV	0.9654	0.7933	0.6893	0.6177	0.5646	0.5232
Zigzag sheet V	0.9623	0.7908	0.6871	0.6158	0.5629	0.5216
Zigzag sheet VI	0.9522	0.7825	0.6799	0.6093	0.5570	0.5161
$\boldsymbol{k_x}=\boldsymbol{k_z}=7\times10^9$						
Armchair sheet I	0.9799	0.8027	0.6964	0.6235	0.5695	0.5275
Armchair sheet II	0.9680	0.7930	0.6879	0.6159	0.5626	0.5211
Armchair sheet III	0.9586	0.7853	0.6812	0.6099	0.5572	0.5161
Zigzag sheet IV	0.9746	0.7984	0.6926	0.6201	0.5665	0.5247
Zigzag sheet V	0.9715	0.7959	0.6904	0.6182	0.5647	0.5231
Zigzag sheet VI	0.9613	0.7876	0.6832	0.6117	0.5588	0.5176
$\boldsymbol{k_x}=\boldsymbol{k_z}=9\times10^9$						
Armchair sheet I	0.9837	0.8049	0.6977	0.6245	0.5703	0.5281
Armchair sheet II	0.9718	0.7951	0.6893	0.6169	0.5634	0.5217
Armchair sheet III	0.9624	0.7874	0.6826	0.6109	0.5579	0.5167
Zigzag sheet IV	0.9785	0.8005	0.6940	0.6211	0.5672	0.5253
Zigzag sheet V	0.9754	0.7980	0.6918	0.6192	0.5654	0.5237
Zigzag sheet VI	0.9652	0.7897	0.6846	0.6127	0.5595	0.5182

$$C_{11}\frac{\partial^2 u}{\partial x^2}+C_{12}\frac{\partial^2 v}{\partial x\partial y}+C_{13}\frac{\partial^2 w}{\partial x\partial z}+C_{66}\left(\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 v}{\partial x\partial y}\right)+C_{55}\left(\frac{\partial^2 u}{\partial z^2}+\frac{\partial^2 w}{\partial x\partial z}\right)=\rho\frac{\partial^2 u}{\partial t^2}-\mu\nabla^2\left(\rho\frac{\partial^2 u}{\partial t^2}\right) \tag{6}$$

$$C_{12}\frac{\partial^2 u}{\partial x\partial y}+C_{22}\frac{\partial^2 v}{\partial y^2}+C_{23}\frac{\partial^2 w}{\partial y\partial z}+C_{66}\left(\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial^2 v}{\partial x^2}\right)+C_{44}\left(\frac{\partial^2 v}{\partial z^2}+\frac{\partial^2 w}{\partial y\partial z}\right)=\rho\frac{\partial^2 v}{\partial t^2}-\mu\nabla^2\left(\rho\frac{\partial^2 v}{\partial t^2}\right) \tag{7}$$

$$C_{13}\frac{\partial^2 u}{\partial x\partial z}+C_{23}\frac{\partial^2 v}{\partial y\partial z}+C_{33}\frac{\partial^2 w}{\partial z^2}+C_{55}\left(\frac{\partial^2 u}{\partial x\partial z}+\frac{\partial^2 w}{\partial x^2}\right)+C_{44}\left(\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 v}{\partial y\partial z}\right)=\rho\frac{\partial^2 w}{\partial t^2}-\mu\nabla^2\left(\rho\frac{\partial^2 w}{\partial t^2}\right) \tag{8}$$

For bulk waves in the xz plane, v and the derivatives with respect to y are assumed to vanish. To study the propagation of plane waves in that plane, we write the remaining displacements in the forms

$$\begin{aligned} u &= u_1 e^{i(k_x x+k_z z-\omega t)} \\ w &= w_1 e^{i(k_x x+k_z z-\omega t)} \end{aligned} \tag{9}$$

Insertion of Eq. (9) in the governing Eqs. (6–8) then yields the following results:

$$C_{11}u_1\mathrm{k}_x^2+(C_{13}+C_{55})w_1k_xk_z+C_{55}u_1\mathrm{k}_z^2=\rho u_1\omega^2+\mu\rho\omega^2\left(u_1\mathrm{k}_x^2+u_1\mathrm{k}_z^2\right) \tag{10}$$

$$C_{33}w_1\mathrm{k}_z^2+(C_{13}+C_{55})u_1k_xk_z+C_{55}w_1\mathrm{k}_x^2=\rho w_1\omega^2+\mu\rho\omega^2\left(w_1\mathrm{k}_x^2+w_1\mathrm{k}_z^2\right) \tag{11}$$

We next eliminate u_1 andw_1 from Eqs. (10) and (11) to obtain the following expression, which relates the frequency to the wave numbers in the x and z directions:

Table 6 The effects of nonlocal parameter and wave number on the phase velocities $\left(\frac{d\omega}{dk} = \frac{d\omega}{dk} \times 10^3\right)$ of nanoplates

	μ(nm^2)					
	0.5	0.75	1.0	1.25	1.5	1.75
k_x=k_z=3 × 10^9						
Armchair sheet I	2.2133	1.8380	1.6057	1.4438	1.3227	1.2277
Armchair sheet II	2.1864	1.8157	1.5862	1.4263	1.3066	1.2128
Armchair sheet III	2.1652	1.7981	1.5708	1.4124	1.2940	1.2010
Zigzag sheet IV	2.2013	1.8281	1.5970	1.4360	1.3156	1.2211
Zigzag sheet V	2.1945	1.8224	1.5920	1.4315	1.3114	1.2173
Zigzag sheet VI	2.1714	1.8033	1.5753	1.4165	1.2977	1.2045
k_x=k_z=5×10^9						
Armchair sheet I	1.3726	1.1280	0.9801	0.8783	0.8028	0.7440
Armchair sheet II	1.3560	1.1143	0.9682	0.8677	0.7931	0.7350
Armchair sheet III	1.3428	1.1035	0.9588	0.8592	0.7854	0.7278
Zigzag sheet IV	1.3652	1.1219	0.9748	0.8736	0.7985	0.7400
Zigzag sheet V	1.3609	1.1184	0.9717	0.8708	0.7960	0.7377
Zigzag sheet VI	1.3467	1.1067	0.9615	0.8617	0.7877	0.7299
k_x=k_z=7×10^9						
Armchair sheet I	0.9898	0.8109	0.7034	0.6298	0.5753	0.5329
Armchair sheet II	0.9778	0.8010	0.6949	0.6222	0.5683	0.5264
Armchair sheet III	0.9683	0.7933	0.6881	0.6161	0.5628	0.5213
Zigzag sheet IV	0.9845	0.8065	0.6996	0.6264	0.5722	0.5300
Zigzag sheet V	0.9814	0.8040	0.6974	0.6244	0.5704	0.5284
Zigzag sheet VI	0.9711	0.7956	0.6901	0.6179	0.5644	0.5228
k_x=k_z=9 × 10^9						
Armchair sheet I	0.7729	0.6324	0.5482	0.4906	0.4481	0.4149
Armchair sheet II	0.7635	0.6247	0.5416	0.4847	0.4426	0.4099
Armchair sheet III	0.7561	0.6186	0.5363	0.4800	0.4383	0.4059
Zigzag sheet IV	0.7687	0.6290	0.5452	0.4880	0.4457	0.4127
Zigzag sheet V	0.7663	0.6270	0.5435	0.4865	0.4443	0.4114
Zigzag sheet VI	0.7583	0.6204	0.5378	0.4814	0.4396	0.4071

$$(C_{13}+C_{55})\mathrm{k}_x^2\mathrm{k}_z^2 = \left(\rho\omega^2+\mu\rho\omega^2\left(\mathrm{k}_x^2+\mathrm{k}_z^2\right)-C_{11}\mathrm{k}_x^2-C_{55}\mathrm{k}_z^2\right)\left(\rho\omega^2+\mu\rho\omega^2\left(\mathrm{k}_x^2+\mathrm{k}_z^2\right)-C_{55}\mathrm{k}_x^2-C_{33}\mathrm{k}_z^2\right) \quad (12)$$

From Eqs. (10) and (11) we can also find the following, equivalent expressions for the ratio $X=\frac{w_1}{u_1}$,

$$X=\frac{\rho\omega^2+\mu\rho\omega^2\left(\mathrm{k}_x^2+\mathrm{k}_z^2\right)-C_{11}\mathrm{k}_x^2-C_{55}\mathrm{k}_z^2}{(C_{13}+C_{55})k_x\mathrm{k}_z} \quad (13)$$

$$X=\frac{(C_{13}+C_{55})k_x\mathrm{k}_z}{\rho\omega^2+\mu\rho\omega^2\left(\mathrm{k}_x^2+\mathrm{k}_z^2\right)-C_{55}\mathrm{k}_x^2-C_{33}\mathrm{k}_z^2} \quad (14)$$

From Eq. (12), one can find that for k_x=0, $\mathrm{k}_z^2=\frac{\rho\omega^2}{C_{55}-\mu\rho\omega^2}$ and $\mathrm{k}_z^2=\frac{\rho\omega^2}{C_{33}-\mu\rho\omega^2}$. For a given frequency and k_z=0, the values of k_x can be find as $\mathrm{k}_x^2=\frac{\rho\omega^2}{C_{11}-\mu\rho\omega^2}$ and $\mathrm{k}_x^2=\frac{\rho\omega^2}{C_{55}-\mu\rho\omega^2}$. Similarly, dispersion relations can be found any other direction of propagation. The following section will present numerical results resulting from these expressions for different orthotropic nanoplates.

4 Numerical Results

We now study the effects of wave numbers and nonlocal parameters on the frequencies, phase velocities, and group velocities. As usual, the phase and group velocities are defined as ω/k and $d\omega/dk$, respectively. For a discussion of the importance of the phase and group velocities, the reader is referred to the bibliography at the end of this article. We compare the numerical results for the six distinct orthotropic nanoplates defined by the material properties in Table 1 [24], where the dimensions of the rectangular nanoplates can also be found. Table 2 displays results sampling the dependence of the frequency on the nonlocal parameter and z-direction wave number. The frequency decreases as the nonlocal parameter grows, but rises for all the orthotropic nanoplates as the wave number grows. Our results for macroplates, i.e., for $\mu=0$, agree with the results of Habeger et al. [25]. The Zigzag sheet IV has the highest frequency, independently of the nonlocal parameter and wave number.

Table 3 contains information on the effect of the nonlocal parameters and wave numbers upon the phase velocities. Again, six types of orthotropic nanoplates are used. The Zigzag sheet IV has the highest, and the Armchair sheet III, the lowest phase velocity. The phase velocity decreases as the nonlocal parameter grows. Comparison between Tables 3 and 2 shows that the nonlocal parameter affects more the phase velocity than the frequency. The phase velocity also decreases as the wave number grows. In the absence of small-scale effects, i.e., for $\mu=0$, the phase velocity becomes independent of the wave number, a result also found in studies of macroplates [26].

Table 4 shows the group velocities of rectangular nanoplates for various wave numbers and sizes. The phase velocity decreases as the nonlocal parameter and the wave number grows. It can be seen that increasing both the nonlocal parameters and the wave numbers will cause decreasing the group velocities. The phase velocity becomes nearly independent of μ for $k_z=9\times10^9$. The group velocity is maximum (minimum) for the Zigzag (Armchair) sheet IV (III); for the highest wave numbers, however, the group velocities for the Zigzag sheet IV and Armchair sheet III are approximately equal.

Table 5 discusses the dependences of the circular frequencies on the nonlocal parameter and x- and z-direction wave numbers, both of which are considered. Comparison between the results in Tables 2 and 5 shows that the frequencies are smaller when both wave numbers are considered. The Armchair sheet I in Table 5 has the highest frequency among the orthotropic nanoplates, a result that is independent of the nonlocal parameter and wave number. By contrast, the Armchair sheet III has the lowest frequency. As in Table 2, the frequency decreases as the nonlocal parameter grows, for all orthotropic nanoplates.

The results on Table 6 show how the phase velocities depend on the nonlocal parameter and x- and z-direction wave numbers. For all nanoplates, the phase velocities decrease as the wave numbers in the x- and z-directions grow. Comparison between Tables 5 and 6 shows that the nonlocal parameter affects the phase velocities in the same way that it affects the circular frequencies. Clearly, size effects cannot be ignored in studies of wave propagation on nanoplates. It would be interesting to extend the results in this paper to cylindrical coordinates, to describe wave propagation in nanotubes and nanowires [27–29].

5 Conclusion

We have discussed the propagation of bulk waves in orthotropic rectangular nanoplates on the basis of three-dimensional nonlocal elasticity theory. Our results show how the wave frequency, phase velocity, and group velocity depend on various parameters. The frequency was shown to diminish as the nonlocal parameter rises. Likewise, the phase velocity decreases as the wave number grows. Finally, we have shown that the group velocity decreases as the nonlocal parameter or the wave number grows.

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