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CONDENSED MATTER

Cylindrical and Spherical Ion-Acoustic Shock Waves in a Relativistic Degenerate Multi-Ion Plasma

M. R. Hossen · L. Nahar · A. A. Mamun

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Abstract A rigorous theoretical investigation has been made to study the existence and basic features of the ion-acoustic (IA) shock structures in an unmagnetized, collisionless multi-ion plasma system (containing degenerate electron fluids, inertial positively as well as negatively charged ions, and arbitrarily charged static heavy ions). This investigation is valid for both non-relativistic and ultra-relativistic limits. The reductive perturbation technique has been employed to derive the modified Burgers equation. The solution of this equation has been numerically examined to study the basic properties of shock structures. The basic features (speed, amplitude, width, etc.) of these electrostatic shock structures have been briefly discussed. The basic properties of the IA shock waves are found to be significantly modified by the effects of arbitrarily charged static heavy ions and the plasma particle number densities. The implications of our results in space and interstellar compact objects like white dwarfs, neutron stars, black holes, and so on have been briefly discussed.

Keywords Shock waves · Degenerate pressure · Nonplanar geometry · Relativity · Compact objects

1 Introduction

Plasma is found almost everywhere throughout the universe. Its appearance varies from diffuse interstellar clouds through stellar coronas to dense interiors of stars. They all have one thing in common: the matter is (partially) ionized. Generally, multi-ion plasma system is a system containing more than one types of ions and has a great importance to various fields of plasmas science and technology. The presence of the negative ions in the Earth's ionosphere [1] and coma of comet Halley [2] is well known. In different situations, (viz. plasma processing reactors, [3] neutral beam sources, [4] low-temperature laboratory experiments, [5] etc.) the existence of positive–negative ion plasmas has also been found.

Nowadays, relativistic degeneracy of plasmas has received a great attention because of their vital role in different astrophysical environments [6–8], where particle velocities become comparable to the speed of light. Astrophysical compact objects such as white dwarfs, neutron stars, quasars, black holes, pulsars, and so on are examples where relativistic degenerate plasmas are dominant and interesting new phenomena are investigated by several nonlinear effects in such plasmas. In case of such a compact object, the degenerate electron number density is very high (in white dwarfs it can be of the order of 10^{30} cm^{-3}, even more [6, 9]). For such interstellar compact objects, the equation of state for degenerate ions and electrons is mathematically explained by Chandrasekhar [10] for two limits, named as non-relativistic and ultra-relativistic limits. Chandrasekhar [10, 11] presented a general expression for the relativistic ion and electron pressures in his classical papers. The pressure for ion fluid can be given by the following equation

$$P_i = K_i n_i^{\alpha}, \tag{1}$$

where

$$\alpha = \frac{5}{3} \quad K_i = \frac{3}{5}\left(\frac{\pi}{3}\right)^{\frac{1}{3}} \frac{\pi \hbar^2}{m} \simeq \frac{3}{5}\Lambda_c \hbar c, \tag{2}$$

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for the non-relativistic limit (where $\Lambda_c = \pi\hbar/mc = 1.2 \times 10^{-10}$cm, and $\hbar$ is the Planck constant divided by 2π). While for the electron fluid,

$$P_e = K_e n_e^{\gamma}, \tag{3}$$

where

$$\gamma = \alpha;\ K_e = K_i \tag{4}$$

for non − relativistic limit and

$$\gamma = \frac{4}{3};\ K_e = \frac{3}{4}\left(\frac{\pi^2}{9}\right)^{\frac{1}{3}} \hbar c \simeq \frac{3}{4}\hbar c, \tag{5}$$

in the ultra-relativistic limit [10–15].

In present days, a large number of authors [14–27] have used the pressure laws (1) and (3) to investigate the linear and nonlinear properties of electrostatic and electromagnetic waves, by using the non-relativistic quantum hydrodynamic (QHD) [16] and quantum-magnetohydrodynamic (Q-MHD) [19] models and by assuming either immobile ions or non-degenerate uncorrelated mobile ions. Mamun and Shukla [28] considered a quantum plasma with non-relativistic and ultra-relativistic degenerate electron fluids and strongly coupled degenerate ion fluids and rigorously investigated the salient features of shock waves. Eliasson and Shukla [29] also considered a super-dense quantum plasma composed of relativistically degenerate electrons and fully ionized ions and studied the formation of electrostatic shock structures. Since, the dense astrophysical quantum plasmas can be confined by stationary heavy ions. Therefore, the effect of the heavy ions has to be taken into account, especially for astrophysical observations (such as white dwarfs, neutron stars, black holes, etc) where the degenerate plasma pressure, nonplanar geometry, and arbitrarily charged heavy ions play an important role in the formation and stability of the existing waves. More recently, Zobaer et al. [7, 30–32] considered an unmagnetized dusty plasma consisting of degenerate electron and ion fluids, and negatively charged static dust grains, and studied the basic features of shock waves. However, most of these works are limited to one-dimensional (planar) geometry, and they did not consider the effects of arbitrarily charged heavy ions, which may not be a realistic situation in space and laboratory devices, since the waves observed in space (laboratory devices) are certainly not infinite (unbound) in one dimension, and there is the possibility of having arbitrarily (positive/negative) charged static heavy ions. There are numerous cases of practical importance where planar geometry does not work and one would have to consider a nonplanar geometry. In this regard, capsule implosion (spherical geometry), shock tube (cylindrical geometry), white dwarfs, neutron star, and black hole are some well known examples where nonplanar geometry plays a vital role. Still now, no theoretical investigation has been made for both non-relativistic limit and ultra relativistic limit, and arbitrarily charged heavy ions in a cylindrical and spherical geometry. Therefore, in our present work, we attempt to study the basic features of nonplanar IA shock waves by deriving the modified Burgers equation in a multi-ion plasma system containing degenerate electron and ion fluids, and arbitrarily charged static heavy ions.

2 Governing Equations

We consider the propagation of an electrostatic perturbation mode in a degenerate dense multi-ion plasma system containing both non-relativistic and ultra-relativistic degenerate electrons, non-relativistic degenerate inertial ions having both positive and negative ions, and arbitrarily charged static heavy ions. Thus, at equilibrium, we have $Z_p n_{p0} + jZ_h n_{h0} = n_{e0} + Z_n n_{n0}$. We also consider the number density of positive and negative ions is equal at equilibrium, i.e., $n_{p0} = n_{n0}$. The dynamics of the electrostatic waves propagating in such plasma system are governed by the following normalized equations

$$\frac{\partial n_s}{\partial t} + \frac{1}{r^\nu}\frac{\partial}{\partial r}(r^\nu n_s u_s) = 0, \tag{6}$$

$$\frac{\partial u_p}{\partial t} + u_p\frac{\partial u_p}{\partial r} + \frac{\partial \phi}{\partial r} + \frac{K_1}{n_p}\frac{\partial {n_p}^{\alpha}}{\partial r} - \frac{\eta}{r^\nu}\frac{\partial}{\partial r}\left(r^\nu \frac{\partial u_p}{\partial r}\right) = 0, \tag{7}$$

$$\frac{\partial u_n}{\partial t} + u_n\frac{\partial u_n}{\partial r} - \beta\frac{\partial \phi}{\partial r} + \frac{K_1}{n_n}\frac{\partial {n_n}^{\alpha}}{\partial r} - \frac{\eta}{r^\nu}\frac{\partial}{\partial r}\left(r^\nu \frac{\partial u_n}{\partial r}\right) = 0, \tag{8}$$

$$n_e\frac{\partial \phi}{\partial r} - K_2\frac{\partial n_e^{\gamma}}{\partial r} = 0, \tag{9}$$

$$\frac{1}{r^\nu}\frac{\partial}{\partial r}\left(r^\nu \frac{\partial \phi}{\partial r}\right) = -\rho, \tag{10}$$

$$\rho = n_p - (1 + j\mu - \alpha_n)n_e - (1 + j\mu - \alpha_e)n_n, \tag{11}$$

where $\nu = 0$ for one dimensional planar geometry, and $\nu = 1$ (2) for nonplanar cylindrical (spherical) geometry; n_s ($s = p, n$) is the plasma species number density normalized by its equilibrium value n_{s0}, u_s is the plasma species ion fluid speed normalized by $C_{pm} = \left(m_e c^2/Z_p m_p\right)^{1/2}$ with m_e $\left(m_p\right)$ being the electron (plasma ion species) rest mass and c being the speed of light in vacuum, ϕ is the electrostatic wave potential normalized by $m_e c^2/e$ with e being the magnitude of the charge of an electron, the time variable (t) is normalized by $\omega_{pm} = \left(4\pi e^2 n_{p0}/m_p\right)^{1/2}$, and the space variable (r) is normalized by $\lambda_m = \left(m_e c^2/4\pi e^2 n_{p0}\right)^{1/2}$, $\beta \left(= Z_n m_p/Z_p m_n\right)$ is the ratio of negative and positive ion masses multiplied by their charge per ion, Z_s (where

$s = p, n$), $\alpha_e \left(= n_{e0}/Z_p n_{p0}\right)$ is the ratio of the number density of electron and positive ion multiplied by charge per positive ion Z_p, $\alpha_n \left(= Z_n n_{n0}/Z_p n_{p0}\right)$ is the ratio of the number density of negative and positive ions multiplied by their charge per ion, Z_s, and $\mu \left(= Z_h n_{h0}/Z_p n_{p0}\right)$ is the ratio of the number density of heavy ions and positive ions multiplied by their charge per ion $\left(Z_h/Z_p\right)$. When $j = +1(-1)$, the heavy ions act as positively (negatively) charged in this degenerate plasma system. We have defined $K_1 = n_{p0}^{\alpha-1} K_i/m_e c^2$ and $K_2 = n_{e0}^{\gamma-1} K_e/m_e c^2$.

3 Derivation of Modified Burgers Equation

Now, we derive modified Burgers equation by employing the reductive perturbation technique in order to examine the characteristics of the electrostatic shock waves propagating in a dense plasma system. We introduce the stretched coordinates [33] as follows:

$$\xi = -\epsilon \left(r + V_p t\right), \tag{12}$$

$$\tau = \epsilon^2 t, \tag{13}$$

where V_p is the wave phase speed (ω/k with ω being angular frequency and k being the wave number of the perturbation mode), and ϵ is a smallness parameter measuring the weakness of the dissipation ($0 < \epsilon < 1$). We then expand n_s, n_e, u_s, and ϕ, in power series of ϵ:

$$n_s = 1 + \epsilon n_s^{(1)} + \epsilon^2 n_s^{(2)} + \cdots, \tag{14}$$

$$n_e = 1 + \epsilon n_e^{(1)} + \epsilon^2 n_e^{(2)} + \cdots, \tag{15}$$

$$u_s = \epsilon u_s^{(1)} + \epsilon^2 u_s^{(2)} + \cdots, \tag{16}$$

$$\phi = \epsilon \phi^{(1)} + \epsilon^2 \phi^{(2)} + \cdots, \tag{17}$$

$$\rho = \epsilon \rho^{(1)} + \epsilon^2 \rho^{(2)} + \cdots, \tag{18}$$

and develop equations in various powers of ϵ. To the lowest order in ϵ, (6–11) give $u_s^{(1)} = -V_p n_s^{(1)}$, $n_p^{(1)} = \phi^{(1)}/\left(V_p^2 - K_1'\right)$, $n_n^{(1)} = -\beta\phi^{(1)}/\left(V_p^2 - K_1'\right)$, $n_e^{(1)} = \phi^{(1)}/K_2'$, and $V_p = \sqrt{\frac{K_2'\{1+\beta(1+j\mu-\alpha_e)\}}{1+j\mu-\alpha_n} + K_1'}$ where $K_1' = \alpha K_1$ and $K_2' = \gamma K_2$. The relation $V_p = \sqrt{\frac{K_2'\{1+\beta(1+j\mu-\alpha_e)\}}{1+j\mu-\alpha_n} + K_1'}$ represents the dispersion relation for the ion-acoustic type electrostatic waves in the degenerate multi-ion plasma under consideration.

We are interested in studying the nonlinear propagation of these dissipative ion-acoustic type electrostatic waves in a degenerate multi-ion plasma. To the next higher order in ϵ, we obtain a set of equations

$$\frac{\partial n_s^{(1)}}{\partial \tau} - V_p \frac{\partial n_s^{(2)}}{\partial \xi} - \frac{\partial}{\partial \xi}\left[u_s^{(2)} + n_s^{(1)} u_s^{(1)}\right] - \frac{\nu u_s^{(1)}}{V_p \tau} = 0,$$

$$\frac{\partial u_p^{(1)}}{\partial \tau} - V_p \frac{\partial u_p^{(2)}}{\partial \xi} - u_p^{(1)} \frac{\partial u_p^{(1)}}{\partial \xi} - \frac{\partial \phi^{(2)}}{\partial \xi} \tag{19}$$

$$-K_1' \frac{\partial}{\partial \xi}\left[n_p^{(2)} + \frac{(\alpha-2)}{2}\left(n_p^{(1)}\right)^2\right] - \eta \frac{\partial^2 u_p^{(1)}}{\partial \xi^2} = 0,$$

$$\frac{\partial u_n^{(1)}}{\partial \tau} - V_p \frac{\partial u_n^{(2)}}{\partial \xi} - u_n^{(1)} \frac{\partial u_i^{(1)}}{\partial \xi} + \beta \frac{\partial \phi^{(2)}}{\partial \xi} \tag{20}$$

$$-K_1' \frac{\partial}{\partial \xi}\left[n_n^{(2)} + \frac{(\alpha-2)}{2}\left(n_n^{(1)}\right)^2\right] - \eta \frac{\partial^2 u_n^{(1)}}{\partial \xi^2} = 0, \tag{21}$$

$$\frac{\partial \phi^{(2)}}{\partial \xi} - K_2' \frac{\partial}{\partial \xi}\left[n_e^{(2)} + \frac{(\gamma-2)}{2}\left(n_e^{(1)}\right)^2\right] = 0, \tag{22}$$

$$(1 + j\mu - \alpha_n)\, n_e^{(2)} - n_p^{(2)} + (1 + j\mu - \alpha_e)\, n_n^{(2)} = 0. \tag{23}$$

Now, combining (19–23), we deduce a modified Burgers equation

$$\frac{\partial \phi^{(1)}}{\partial \tau} + A\phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \xi} + \frac{\nu \phi^{(1)}}{2\tau} = B \frac{\partial^2 \phi^{(1)}}{\partial \xi^2}, \tag{24}$$

where

$$A = \frac{\left(V_p^2 - K_1'\right)^2}{2V_p\{1 + (1 + j\mu - \alpha_e)\}\beta}\left[\frac{3V_p^2 + K_1'(\alpha-2)}{\left(V_p^2 - K_1'\right)^3} + \frac{(\gamma-2)(1+j\mu-\alpha_n)\beta^2}{K_2'^2} - \frac{(1+j\mu-\alpha_e)\left\{3V_p^2\beta^2 + K_1'(\alpha-2)\beta^2\right\}}{\left(V_p^2 - K_1'\right)^3}\right] \tag{25}$$

$$B = \frac{\eta}{2}. \tag{26}$$

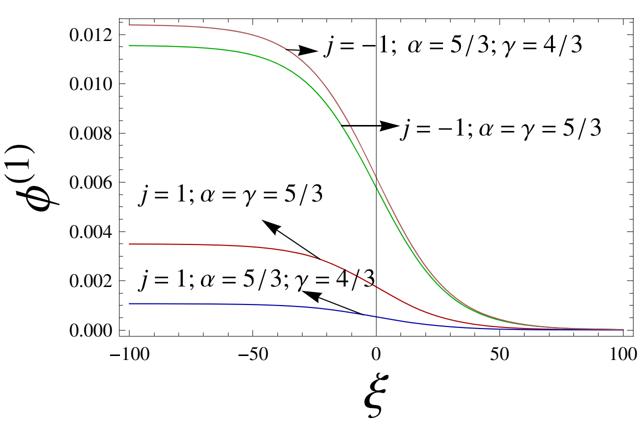

Fig. 1 Showing the variation of amplitude of IA shock waves in the presence of arbitrarily charged static heavy ions for $u_0 = 0.01$, $\alpha_e = 0.4$, $\alpha_n = 0.3$, and $\beta = 0.2$

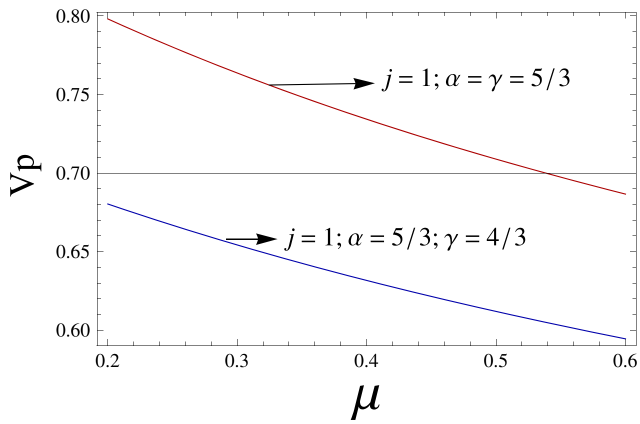

Fig. 2 (*Color online*) Showing the effects of μ on the phase velocity of IA shock waves for $u_0 = 0.01$, $\alpha_e = 0.4$, $\alpha_n = 0.3$, $\beta = 0.2$, and $j = +1$

4 Numerical Analysis and Results

We have numerically solved (24) and studied the effects of cylindrical ($\nu = 1$) and spherical ($\nu = 2$) geometries on time dependent IA shock waves. An exact analytic solution of (24) is not possible. However, for clear understanding, we first briefly discuss about the stationary shock wave solution for (24) with $\nu = 0$. We should note that for large value of τ, the term $\frac{\nu\phi^{(1)}}{2\tau}$ is negligible. So, in our numerical analysis, we start with a large value of τ (viz. $\tau = -14$), and at this large (negative) value of τ, we choose the stationary shock wave solution of (24) [without the term $\frac{\nu\phi^{(1)}}{2\tau}$] as our initial pulse. The stationary shock wave solution of standard Burgers equation is obtained by considering a frame $\xi = \zeta - u_0\tau$ (moving with speed u_0) and the solution is

$$\phi^{(1)}(\nu = 0) = \phi_m^{(1)}\left[1 - tanh\left(\frac{\xi}{\delta}\right)\right], \tag{27}$$

where $\phi_m^{(1)} = u_0/A$ and $\delta = 2B/u_0$.

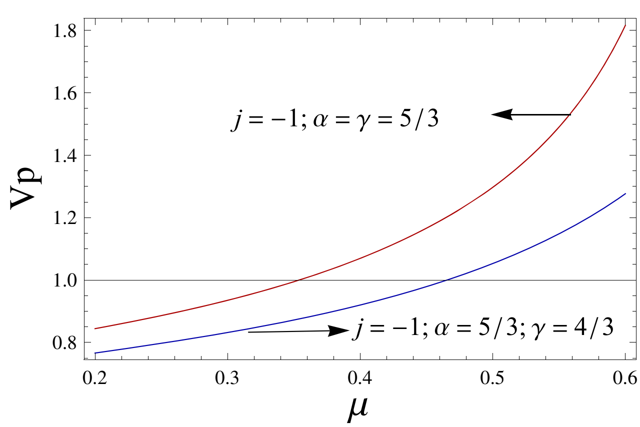

Fig. 3 (*Color online*) Showing the effects of μ on the phase velocity of IA shock waves for $u_0 = 0.01$, $\alpha_e = 0.4$, $\alpha_n = 0.3$, $\beta = 0.2$, and $j = -1$

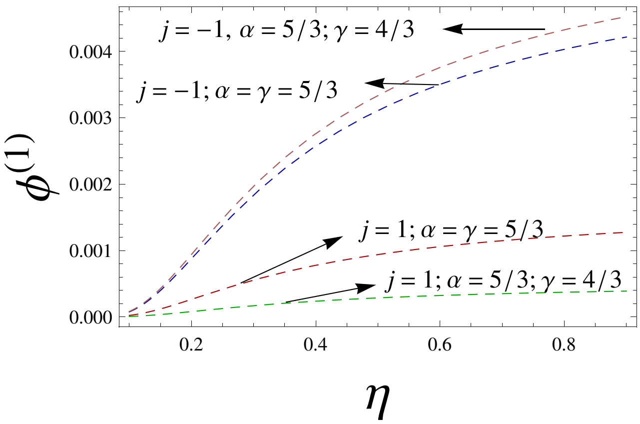

Fig. 4 (*Color online*) Showing the effects of η on IA shock waves for $u_0 = 0.01$, $\alpha_e = 0.4$, $\alpha_n = 0.3$, $\beta = 0.2$, and ξ=+25

Shock waves often arise in nature due to the balance between wave-breaking nonlinearity and wave-damping dissipative forces [34]. The profiles are displayed in Figs. 6–13, which show how the effects of cylindrical (ν=1) and spherical (ν=2) geometries modify the time dependent IA shock structures. We have considered $u_0 = 0.01$ for our numerical analysis of IA waves for the plasma system under investigation here. We have considered that the values of β, α_e, α_n are 0.2, 0.4, 0.3, respectively [35, 36]. We first graphically represented the effects of arbitrarily charged static heavy ions and the degenerate plasma pressure on the amplitude of IA shock waves (see Fig. 1). Then, we graphically represented the effects of μ on the phase speed of IA waves (shown in Figs. 2 and 3). The variation of the amplitude with the dissipative parameter, i.e., the coefficient of viscosity (η) is shown in Figs. 4 and 5. It is notable that, if we compare the non-relativistic (both electrons and ions are non-relativistic degenerate) and ultra-relativistic case (electrons are ultra-relativistic degenerate and ions are non-relativistic degenerate) for IA waves, one cylindrical and one spherical

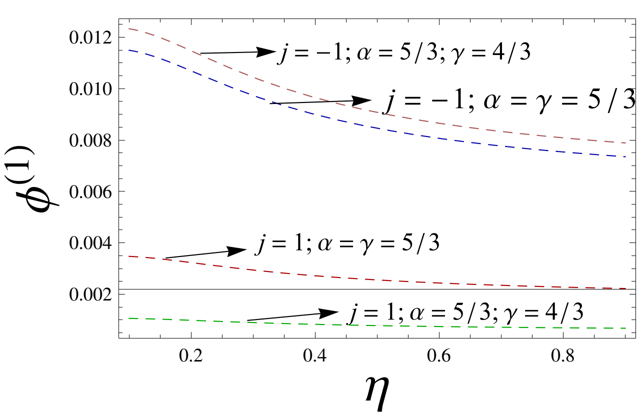

Fig. 5 (*Color online*) Showing the effects of η on IA shock waves for $u_0 = 0.01$, $\alpha_e = 0.4$, $\alpha_n = 0.3$, $\beta = 0.2$, and ξ=-25

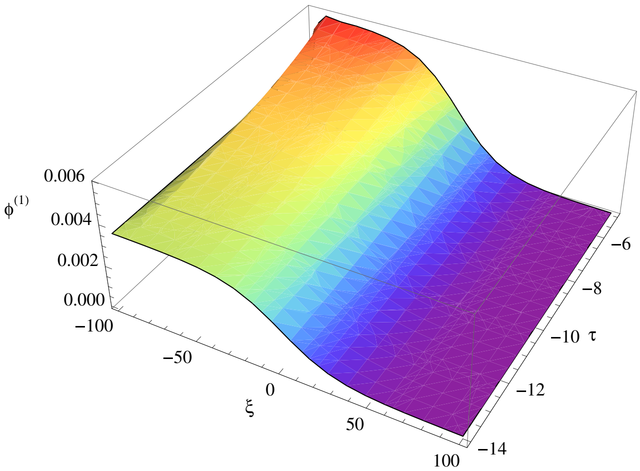

Fig. 6 (*Color online*) Effects of cylindrical geometry on IA shock waves in the presence of static heavy ions when both electrons and ions are non-relativistic degenerate for $u_0 = 0.01$, $\mu = 0.45$, $\eta = 0.3$, $j = +1$, $\alpha = \gamma = \frac{5}{3}$, $\alpha_e = 0.4$, $\alpha_n = 0.3$, $\beta = 0.2$

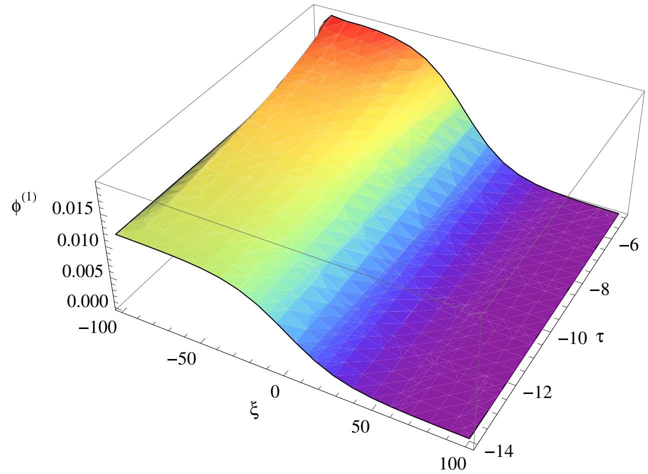

Fig. 8 (*Color online*) Effects of cylindrical geometry on IA shock waves in the presence of static heavy ions when both electrons and ions are non-relativistic degenerate for $u_0 = 0.01$, $\mu = 0.45$, $\eta = 0.3$, $j = -1$, $\alpha = \gamma = \frac{5}{3}$, $\alpha_e = 0.4$, $\alpha_n = 0.3$, and $\beta = 0.2$

shock structure will be found for every case (see Figs. 6, 7, 8 9, 10, 11, 12 and 13). Finally, the results that we have found in this investigation can be summarized as follows:

1. The basic properties (speed, amplitude, and width) of the IA shock waves are found to be significantly modified due to the presence of arbitrarily charged static heavy ions.
2. The amplitude of the IA shock waves significantly differ with the polarity of heavy ions and relativistic parameters. It is found from Fig. 1 that the amplitude of IA wave is always higher (lower) for negatively (positively) charged static heavy ions, for both non-relativistic and ultra-relativistic limits. For negatively (positively) charged heavy ions, we can say that the amplitude is higher (lower) for ultra-relativistic case than for non-relativistic case. It is also valid for all other cylindrical and spherical graphs of IA shock waves (see Figs. 6–13).
3. We can compare the effects of μ on the phase speed of IA waves (see Figs. 2–3) for arbitrarily charged static heavy ions. For negatively (positively) charged heavy ions, the phase speed increases (decreases) with the increase of the value of μ.

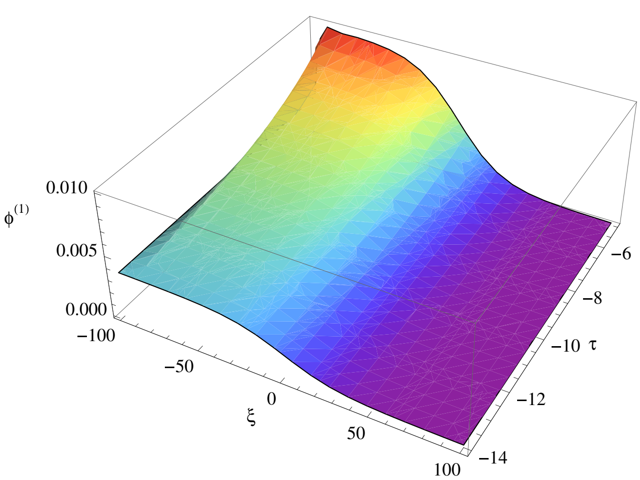

Fig. 7 (*Color online*) Effects of spherical geometry on IA shock waves in the presence of static heavy ions when both electrons and ions are non-relativistic degenerate for $u_0 = 0.01$, $\mu = 0.45$, $\eta = 0.3$, $j = +1$, $\alpha = \gamma = \frac{5}{3}$, $\alpha_e = 0.4$, $\alpha_n = 0.3$, $\beta = 0.2$

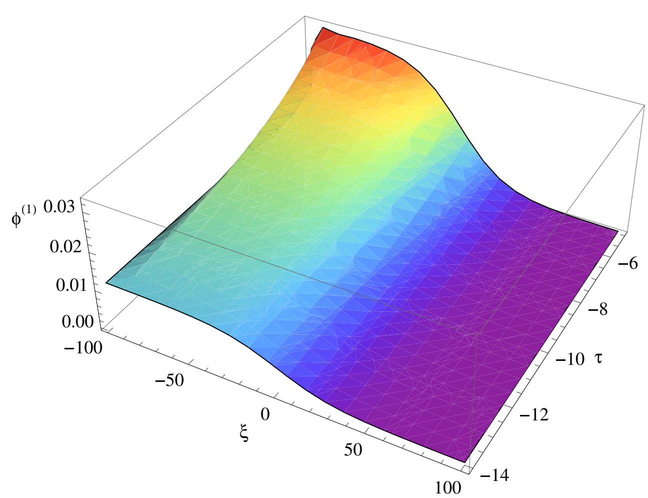

Fig. 9 (*Color online*) Effects of spherical geometry on IA shock waves in the presence of static heavy ions when both electrons and ions are non-relativistic degenerate for $u_0 = 0.01$, $\mu = 0.45$, $\eta = 0.3$, $j = -1$, $\alpha = \gamma = \frac{5}{3}$, $\alpha_e = 0.4$, $\alpha_n = 0.3$, and $\beta = 0.2$

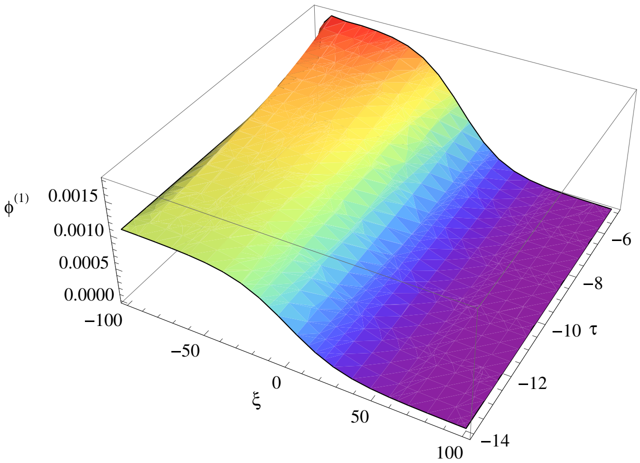

Fig. 10 (*Color online*) Effects of cylindrical geometry on IA shock waves in the presence of static heavy ions when ions are non-relativistic degenerate and electrons are ultra relativistic degenerate for $u_0 = 0.01$, $\mu = 0.45$, $\eta = 0.3$, $j = +1$, $\alpha = \frac{5}{3}$, $\gamma = \frac{4}{3}$, $\alpha_e = 0.4$, $\alpha_n = 0.3$, and $\beta = 0.2$

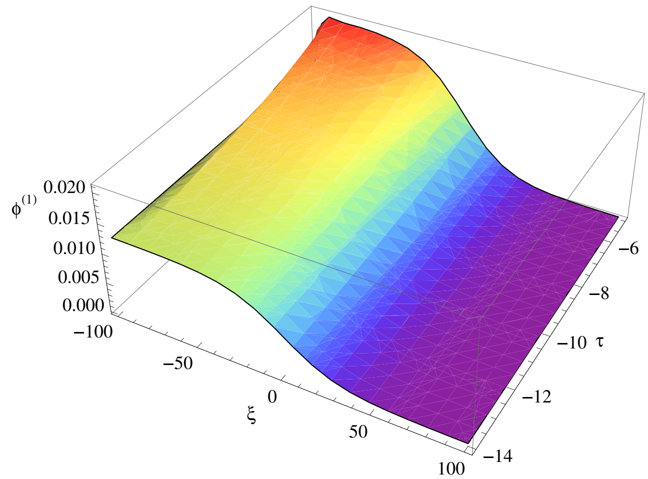

Fig. 12 (Color online) Effects of cylindrical geometry on IA shock waves in the presence of static heavy ions when ions are non-relativistic degenerate and electrons are ultra relativistic degenerate for $u_0 = 0.01$, $\mu = 0.45$, $\eta = 0.3$, $j = -1$, $\alpha = \frac{5}{3}$, $\gamma = \frac{4}{3}$, $\alpha_e = 0.4$, $\alpha_n = 0.3$, and $\beta = 0.2$

4. The amplitude also significantly varies with the value of the coefficient of viscosity (η). It is observed that for positive (negative) value of ξ ($\xi = +25, -25$), the wave potential increases (decreases) with the dissipative parameter η (see Figs. 4–5). From the observation of Figs. 4–5, we found that the amplitude is higher (lower) for negatively (positively) charged ultra-relativistic case.
5. The large value of τ kills the possibility of formation of nonplanar shock waves. It is found that as the value of τ decreases the amplitude of these localized pulses increases for IA waves (see Figs. 6–13).
6. The comparison between cylindrical ($\nu = 1$) and spherical ($\nu = 2$) geometries is also important. If we compare the $\nu = 1$ graphs with the $\nu = 2$ ones, we observe that the amplitude of the potential is always distinctly higher for $\nu = 2$ case than for $\nu = 1$ case, for IA waves (see Figs. 6–13).

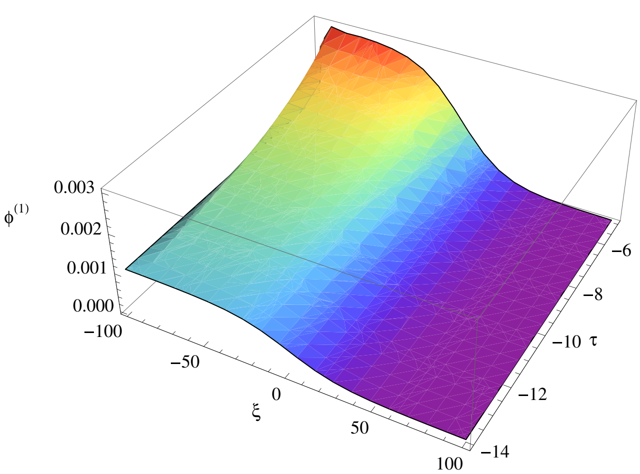

Fig. 11 (*Color online*) Effects of spherical geometry on IA shock waves in the presence of static heavy ions when ions are non-relativistic degenerate and electrons are ultra relativistic degenerate for $u_0 = 0.01$, $\mu = 0.45$, $\eta = 0.3$, $j = +1$, $\alpha = \frac{5}{3}$, $\gamma = \frac{4}{3}$, $\alpha_e = 0.4$, $\alpha_n = 0.3$, and $\beta = 0.2$

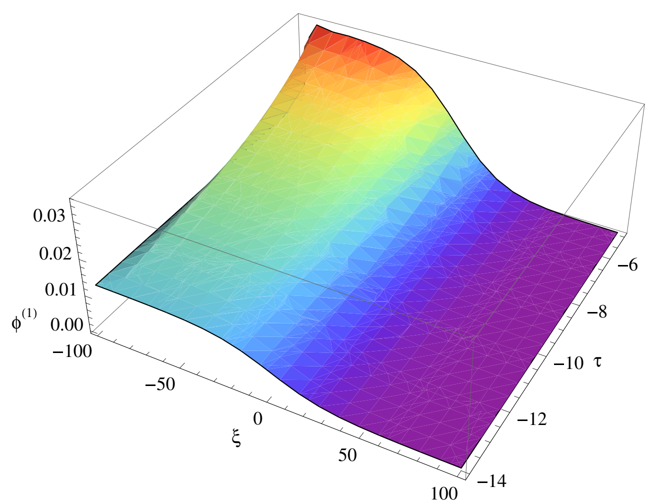

Fig. 13 (*Color online*) Effects of spherical geometry on IA shock waves in the presence of static heavy ions when ions are non-relativistic degenerate and electrons are ultra relativistic degenerate for $u_0 = 0.01$, $\mu = 0.45$, $\eta = 0.3$, $j = -1$, $\alpha = \frac{5}{3}$, $\gamma = \frac{4}{3}$, $\alpha_e = 0.4$, $\alpha_n = 0.3$, and $\beta = 0.2$

5 Discussion

We have presented a rigorous theoretical investigation of the nonlinear propagation of IA shock waves in an unmagnetized, collisionless multi-ion plasma (containing degenerate electron fluids, inertial positively as well as negatively charged ions, and arbitrarily charged static heavy ions). It is found that as time increases the amplitude of the cylindrical and spherical IA shock waves increases. In our numerical analysis, we have tried to give the idea of the non-relativistic and ultra-relativistic degenerate plasma pressure, effects of nonplanar geometry, effects of μ on the phase velocity, and the effects of dissipative parameter η on the wave potential, which make our present work significant to understand the localized electrostatic shock waves, which are formed due to the balance between nonlinearity and dissipation, in many space and astrophysical plasma environments [37–41] (viz. white dwarfs, neutron stars, compact planets like massive Jupiter, other exotic dense stars, and black holes). It may be stressed here that the results of this investigation should be useful for understanding the nonlinear features of localized electrostatic shock waves in space plasmas, in which arbitrarily charged heavy ions, nonplanar geometry, and degenerate plasma pressure play a vital role.

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