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CONDENSED MATTER

Modified Ion-Acoustic Shock Waves and Double Layers in a Degenerate Electron-Positron-Ion Plasma in Presence of Heavy Negative Ions

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Abstract A general theory for nonlinear propagation of one dimensional modified ion-acoustic waves in an unmagnetized electron-positron-ion (e-p-i) degenerate plasma is investigated. This plasma system is assumed to contain relativistic electron and positron fluids, non-degenerate viscous positive ions, and negatively charged static heavy ions. The modified Burgers and Gardner equations have been derived by employing the reductive perturbation method and analyzed in order to identify the basic features (polarity, width, speed, etc.) of shock and double layer (DL) structures. It is observed that the basic features of these shock and DL structures obtained from this analysis are significantly different from those obtained from the analysis of standard Gardner or Burgers equations. The implications of these results in space and interstellar compact objects (viz. non-rotating white dwarfs, neutron stars, etc.) are also briefly mentioned.

Keywords Shock waves · Double layers · Degenerate pressure · Relativistic effect · Compact objects

1 Introduction

There has been an enormous interest in understanding the physics of plasmas having electrons and positrons (e-p) as their constituents for the last few years because of its presence in earth's atmosphere [1], in Van Allen radiation belts [1, 2], in neutron star magnetosphere [3, 4], in active galactic nuclei [5], at the center of Milky Way galaxy [6], and in the early universe [7]. Mostly, the electrons and positrons present in space and astrophysical plasma environments are highly energetic [8, 9]. The e-p plasmas are thought to be generated by pair production in a high-energy process [10, 11]. Surko et al. [12, 13] have reported that due to sufficient lifetime of positrons, the two-component electron-ion (e-i) plasma behaves as three-component electron-positron-ion (e-p-i) plasma. Comparative studies of the properties of the wave motion in e-p-i plasmas have major differences from the two-component e-i and e-p plasmas.

For describing and analyzing the different astrophysical environments [14–16] (where particle velocities are close to the speed of light), relativistic degeneracy of plasmas has received a great attention and relativistic effects [17–21] play an important role in understanding the different electrostatic nonlinear phenomena. Plasmas in different astrophysical compact objects such as white dwarfs, neutron stars, and so on are examples where relativistic degeneracy is a dominant phenomenon. Astrophysical compact objects have degenerate electron number density so high (in white dwarfs it can be of the order of 10^{30} cm^{-3}, even more [22, 23]), that their cores consist of strongly coupled non-degenerate ion lattices immersed in degenerate electron fluids that follow the Fermi-Dirac distribution function [24]. Chandrasekhar mathematically derived the equation of state for degenerate electrons in stellar compact objects [25]. Chandrasekhar [25, 26] developed a general expression for the relativistic electron pressures in his classical papers. The electron fluid pressure can be given by the following equation

$$P_e = K_e n_e^{\gamma}, \tag{1}$$

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and while for the positron fluid

$$P_p = K_p n_p^{\gamma}, \tag{2}$$

for the non-relativistic limit [25–29]

$$\gamma = \frac{5}{3};\ \ K_e = K_p = K = \frac{3}{5}\left(\frac{\pi}{3}\right)^{\frac{1}{3}}\frac{\pi\hbar^2}{m} \simeq \frac{3}{5}\Lambda_c\hbar c, \tag{3}$$

(where $\Lambda_c = \pi\hbar/mc = 1.2\times10^{-10}\ cm$, and $\hbar$ is the Planck constant divided by 2π) and for the ultra-relativistic limit [25–29]

$$\gamma = \frac{4}{3};\ \ K_e = K_p = K = \frac{3}{4}\left(\frac{\pi^2}{9}\right)^{\frac{1}{3}}\hbar c \simeq \frac{3}{4}\hbar c. \tag{4}$$

Asif and Saeed [30] considered an unmagnetized e-p-i plasma and obtained that the amplitude and steepness of the shock wave decrease with the increase of the relativistic streaming factor and the positron concentration. By investigating quantum e-p-i plasma, Masud et al. [31] observed that the strength and the steepness of the quantum ion acoustic shock wave increase with decreasing stretched time coordinate. Pakzad [32] investigated the effects of relativistic ions, ion temperature, and the quantum Bohm potential on the shock waves in a dense quantum plasma, whose constituents are electrons, positrons, and positive ions. Eliasson and Shukla [33] examined a super-dense quantum plasma consisting of relativistically degenerate electrons and fully ionized ions, and figured out the formation of electrostatic shock structures. Rahman et al. [34] also investigated an e-p-i plasma and studied the salient features of compressive shock waves. Mamun and Shukla [28] analyzed a quantum plasma with non-relativistic and ultra-relativistic degenerate electron fluids and strongly coupled degenerate ion fluids and rigorously investigated the salient features of shock waves. The dense astrophysical quantum plasmas can be confined by negatively charged stationary heavy ions. Therefore, the effect of the negatively charged static heavy ions has to be taken into account, especially for astrophysical observations (such as white dwarfs, neutron stars, black holes etc) where the degenerate plasma pressure and negatively charged static heavy ions [35] play an important role in the formation and stability of the existing waves. Zobaer et al. [36, 37] studied an unmagnetized electron-positron-ion (e-p-i) plasma and analyzed the properties of the propagating electrostatic shock waves.

More recently, a large number of authors [27, 38–43] studied the basic properties of solitary, and shock waves by deriving the Korteweg-de Vries (K-dV), modified K-dV (comes from higher order of K-dV equation), Gardner (comes from higher term of modified K-dV equation), and Burgers equation for planar and nonplanar cases in considering different types of effects [34, 35, 44, 45]. But the interesting point is to note that the Burgers equation for nonplanar case is called modified Burgers equation [46–48]. The modified Burgers and Gardner (with additional dissipative term) equations were not derived as mK-dV and Gardner equations considering higher order nonlinearity earlier. Therefore, in our present work, we take the higher order nonlinearity and derive the Gardner equation to study the basic properties of shock and double layer structures. Finally, we hope that this investigation will be useful to study the effects of degenerate pressure in interstellar and space plasmas where the effects of dissipation, higher order nonlinearity, and negatively charged heavy ions play a vital role. The present theory may have application to explain the basic features of nonlinear localized electrostatic disturbances for non-rotating white dwarfs [4], neutron stars [5], acceleration charged particles in aurora [49], neutron magnetosphere [50], and laboratory plasmas [12, 51, 52].

This manuscript is organized as follows. In Section 2, we present the basic sets of equations for the system we investigated. In Section 3, the modified Burgers equation is derived. In Section 4, the Gardner equation is obtained. In Section 5, the small but finite amplitude shock and double layer are numerically analyzed. In Section 6, final discussion and results of the investigation are given.

2 Governing Equations

We consider the propagation of electrostatic perturbation mode in an unmagnetized collisionless degenerate dense plasma containing degenerate electron and positron fluids (in both non-relativistic and ultra-relativistic limits), inertial viscous ion fluids and negatively charged static heavy ions. Thus, at equilibrium, we have $n_{p0} + n_{i0} = n_{e0} + Z_h n_{h0}$. We have also considered the number density of electron and positron is equal at equilibrium, i.e., $n_{e0} = n_{p0}$. The dynamics of the one dimensional electrostatic wave propagating in such a plasma system are governed by the following normalized equations:

$$\frac{\partial n_s}{\partial t} + \frac{\partial}{\partial x}(n_s u_s) = 0, \tag{5}$$

$$\frac{\partial u_i}{\partial t} + u_i\frac{\partial u_i}{\partial x} + \frac{\partial \phi}{\partial x} - \eta\frac{\partial^2 u_i}{\partial x^2} = 0, \tag{6}$$

$$n_e\frac{\partial \phi}{\partial x} - K\frac{\partial n_e^{\gamma}}{\partial x} = 0, \tag{7}$$

$$n_p\frac{\partial \phi}{\partial x} - K\frac{\partial n_p^{\gamma}}{\partial x} = 0, \tag{8}$$

$$\frac{\partial^2 \phi}{\partial x^2} = -\rho, \tag{9}$$

$$\rho = \alpha_e n_e - n_i - \alpha_p n_p + \mu, \tag{10}$$

where n_s (s = i,e,p; i for ion, e for electron, and p for positron) is the plasma species number density normalized by their equilibrium value n_{s0}, u_s is the plasma ion fluid speed normalized by $C_{im} = (m_e c^2/m_i)^{1/2}$ with m_e (m_i) being the electron (plasma ion species) rest mass and c being the speed of light in vacuum, ϕ is the electrostatic wave potential normalized by $m_e c^2/e$ with e being the magnitude of the charge of an electron. The time variable (t) is normalized by $\omega_{pi} = (4\pi e^2 n_0/m_i)^{1/2}$, and the space variable (x) is normalized by $\lambda_m = (m_e c^2/4\pi e^2 n_{e0})^{1/2}$, $\alpha_e (= n_{e0}/n_{i0})$ is the ratio of electron to ion number density, $\alpha_p (= n_{p0}/n_{i0})$ is the ratio of positron to ion number density, and $\mu (= Z_h n_{h0}/n_{i0})$ is the ratio of the number density of negatively charged heavy ions to positive ion multiplied by Z_h. The coefficient of viscosity η is a normalized quantity given by $\omega_i \lambda_m^2 m_i n_{i0}$. We have defined as $K = n_{e0}^{\gamma-1} K_e/m_e c^2 = n_{p0}^{\gamma-1} K_p/m_e c^2$.

3 Derivation of Modified Burgers Equation

Now, we will derive a dynamical modified Burgers (mB) equation for the nonlinear propagation of the modified ion-acoustic (mIA) waves by using (5)–(10). To do so, we employ a reductive perturbation technique to examine the electrostatic perturbation propagating in a dense electron-positron degenerate plasma system due to the effect of dissipation. Now, we first introduce the stretched coordinates [15]

$$\zeta = \epsilon^2 (x - V_p t), \tag{11}$$
$$\tau = \epsilon^4 t, \tag{12}$$

where V_p is the wave phase speed (ω/k with ω being angular frequency and k being the wave number of the perturbation mode), and ϵ is a smallness parameter measuring the weakness of the dissipation ($0 < \epsilon < 1$). We then expand n_s, u_s, and ϕ, in power series of ϵ:

$$n_s = 1 + \epsilon n_s^{(1)} + \epsilon^2 n_s^{(2)} + \epsilon^3 n_s^{(3)} + \cdots, \tag{13}$$
$$u_s = \epsilon u_s^{(1)} + \epsilon^2 u_s^{(2)} + \epsilon^3 u_s^{(3)} + \cdots, \tag{14}$$
$$\phi = \epsilon \phi^{(1)} + \epsilon^2 \phi^{(2)} + \epsilon^3 \phi^{(3)} + \cdots, \tag{15}$$
$$\rho = \epsilon \rho^{(1)} + \epsilon^2 \rho^{(2)} + \epsilon^3 \rho^{(3)} + \cdots, \tag{16}$$

and develop equations in various powers of ϵ. To the lowest order in ϵ, (5)–(10) give $u_s^{(1)} = V_p n_s^{(1)}$, $n_i^{(1)} = \phi^{(1)}/V_p^2$, $n_e^{(1)} = \phi^{(1)}/K'$, $n_p^{(1)} = \phi^{(1)}/K'$, and $V_p = \sqrt{[\frac{K'}{(\alpha_e - \alpha_p)}]}$ where $K' = \gamma K$. The relation $V_p = \sqrt{[\frac{K'}{(\alpha_e - \alpha_p)}]}$ represents the dispersion relation for the mIA type electrostatic waves in a degenerate e-p-i plasma under consideration.

Substituting (11)–(12) and (13)–(16) in (5)–(8), and equating the coefficient of ϵ^4 and also taking the coefficient of ϵ^2, we obtain a set of equations that can be simplified as

$$u_i^{(2)} = \frac{\phi^{(2)}}{V_p} + \frac{(\phi^{(1)})^2}{2V_p^3}, \tag{17}$$
$$n_i^{(2)} = \frac{\phi^{(2)}}{V_p^2} + \frac{3(\phi^{(1)})^2}{2V_p^4}, \tag{18}$$
$$n_e^{(2)} = \frac{\phi^{(2)}}{K'} - \frac{(\gamma-2)(\phi^{(1)})^2}{2K'^2}, \tag{19}$$
$$n_p^{(2)} = \frac{\phi^{(2)}}{K'} - \frac{(\gamma-2)(\phi^{(1)})^2}{2K'^2}, \tag{20}$$
$$\rho^{(2)} = \frac{1}{2} A (\phi^{(1)})^2 = 0, \tag{21}$$

where

$$A = \left[\frac{\alpha_e(\gamma-2)}{K'^2} - \frac{\alpha_p(\gamma-2)}{K'^2} + \frac{3}{V_p^4} \right], \tag{22}$$

To the next higher order of ϵ, i.e., equating the coefficients of ϵ^5 from (5)–(8) and ϵ^3 from (9) by using (11)–(16), one can derive the following sets of equations

$$\frac{\partial n_s^{(1)}}{\partial \tau} - V_p \frac{\partial n_s^{(3)}}{\partial \zeta} + \frac{\partial u_s^{(3)}}{\partial \zeta} + \frac{\partial}{\partial \zeta}\left[u_s^{(2)} n_s^{(1)} + n_s^{(2)} u_s^{(1)} \right] = 0, \tag{23}$$

$$\frac{\partial u_i^{(1)}}{\partial \tau} - V_p \frac{\partial u_i^{(3)}}{\partial \zeta} + \frac{\partial}{\partial \zeta}\left[u_i^{(1)} u_i^{(2)} \right] + \frac{\partial \phi^{(3)}}{\partial \zeta} - \eta \frac{\partial^2 u_i}{\partial \zeta^2} = 0, \tag{24}$$

$$\frac{\partial}{\partial \zeta}\left[\phi^{(3)} - K' \left\{ n_e^{(3)} + \frac{(\gamma-2)(\phi^{(1)}\phi^{(2)})}{K'^2} - \frac{(2\gamma^2 - 5\gamma + 3)}{6K'^3}(\phi^{(1)})^3 \right\} \right] = 0, \tag{25}$$

$$\frac{\partial}{\partial \zeta}\left[\phi^{(3)} - K' \left\{ n_p^{(3)} + \frac{(\gamma-2)(\phi^{(1)}\phi^{(2)})}{K'^2} - \frac{(2\gamma^2 - 5\gamma + 3)}{6K'^3}(\phi^{(1)})^3 \right\} \right] = 0, \tag{26}$$

$$\frac{\partial \phi^{(1)}}{\partial \zeta} = -\rho^{(3)}, \tag{27}$$

$$\rho = n_i + \alpha_p n_p - \alpha_e n_e - \mu. \tag{28}$$

Now combining (23)–(28), we obtain an equation of the form

$$\frac{\partial \phi^{(1)}}{\partial \tau} + \beta \{\phi^{(1)}\}^2 \frac{\partial \phi^{(1)}}{\partial \zeta} = C \frac{\partial^2 \phi^{(1)}}{\partial \zeta^2}, \tag{29}$$

where

$$\beta = ab, \tag{30}$$

$$a = \frac{15}{2V_p^6} + \frac{(\alpha_p - \alpha_e)(\gamma - 2)(2\gamma - 1)}{2K'^3} \tag{31}$$

$$b = \frac{V_p^3}{2} \tag{32}$$

$$C = \frac{\eta}{2}. \tag{33}$$

Equation (29) is called the modified Burgers equation for planar geometry. The stationary shock solution of (29) is given by

$$\phi^{(1)} = \left[\phi_m \left\{1 - tanh\left(\frac{\xi}{\Delta}\right)\right\}\right]^{\frac{1}{2}}, \tag{34}$$

where the special coordinate, $\xi = \zeta - u_0\tau$, the amplitude is $\phi_m = \frac{3u_0}{2\beta}$, the width is $\Delta = \sqrt{\frac{C}{u_0}}$, and u_0 is the plasma species speed at equilibrium.

4 Derivation of Gardner Equation

It is obvious from (21) that $A = 0$ since $\phi^{(1)} \neq 0$. One can find that $A = 0$ at its critical value $\mu = (\mu)_c = 0.67$ (which is a solution of $A = 0$). So, for μ around its critical value μ_c, $A = A_0$ can be expressed as

$$A_0 = s\left(\frac{\partial A}{\partial \mu}\right)_{\mu=\mu_c} = s\epsilon, \tag{35}$$

where $|\mu - \mu_c|[= (\mu = \mu_c)]$ is a small and dimensionless parameter and can be taken as the expansion parameter ϵ, i.e., $|\mu - \mu_c| \simeq \epsilon$, and $s = 1$ for $\mu < \mu_c$ and $s = -1$ for $\mu > \mu_c$. So, $\rho^{(2)}$ can be expressed as

$$\epsilon^2\rho^{(2)} \simeq \epsilon^3\frac{1}{2}s(\phi^{(1)})^2. \tag{36}$$

Now taking the coefficient of ϵ^3 from Poisson's equation, we get

$$\frac{\partial\rho^{(3)}}{\partial\zeta} = \frac{\partial {n_i}^{(3)}}{\partial\zeta} + \alpha_p\frac{\partial {n_p}^{(3)}}{\partial\zeta} - \alpha_e\frac{\partial {n_e}^{(3)}}{\partial\zeta}. \tag{37}$$

Now we can find the value of $\rho^{(3)}$ from (37), where the values of $n_p^{(3)}$, $n_n^{(3)}$ and $n_e^{(3)}$ can be found from using (23)–(26). Therefore, combining these equations into (37), we can finally write the following equation

$$\frac{\partial\phi^{(1)}}{\partial\tau} + sB\phi^{(1)}\frac{\partial\phi^{(1)}}{\partial\zeta} + \beta(\phi^{(1)})^2\frac{\partial\phi^{(1)}}{\partial\zeta} = C\frac{\partial^2\phi^{(1)}}{\partial\zeta^2}, \tag{38}$$

where $B = \frac{V_p^3}{2}$.

Equation (38) is like Gardner equation with additional dissipative term. There are two solutions of this (38); one is for shock wave and another is for double layer from where we can easily make difference and have a clear view between the shock wave and double layer.

We note that the differential equation (38) has different kinds of solution depending on different initial boundary conditions used for the system under consideration. Since we are interested here in looking for shock and double layer solutions, we use appropriate boundary conditions for obtaining such types of solution.

The solution of (38) for shock waves is

$$\phi^{(1)} = \left[\phi_m^2 \left\{1 - tanh\left(\frac{\xi}{\Delta}\right)\right\}\right]^{\frac{1}{4}}, \tag{39}$$

where the values of β, ξ, and Δ are the same as for mB equation. On the other hand, the solution of (44) for double layers is

$$\phi^{(1)} = \left[\phi_m^2 \left\{1 + tanh\left(\frac{\xi}{\Delta}\right)\right\}\right]^{\frac{1}{4}}, \tag{40}$$

where the special coordinate, $\xi = \zeta - u_0\tau$, the amplitude is $\phi_m = \frac{3u_0}{4\beta}$, the width is $\Delta = \sqrt{\frac{4C}{u_0}}$, and u_0 is the plasma species speed at equilibrium.

5 Numerical Analysis

The electrostatic shock profiles, caused by the balance between wave-breaking nonlinearity and wave-damping dissipation, are shown in the Figs. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, and 13. The conditions for the existence of mIA shock structures and their basic features are found to be significantly modified in presence of non-degenerate ions, both non-relativistic and ultra-relativistic electron and positron fluids, and negatively charged static heavy ions. The mIA waves are seen to be modified when e-p being considered non-relativistic degenerate ($\alpha = \gamma = 5/3$) than e-p being considered ultra-relativistic degenerate ($\alpha = 5/3$; $\gamma = 4/3$).

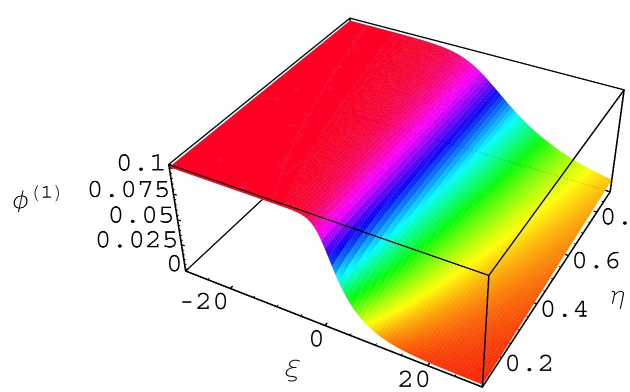

Fig. 1 Variation of η and ξ on $\phi^{(1)}$ of mIA shock waves obtained from (39) for $\mu < \mu_c$ and e-p being non-relativistic

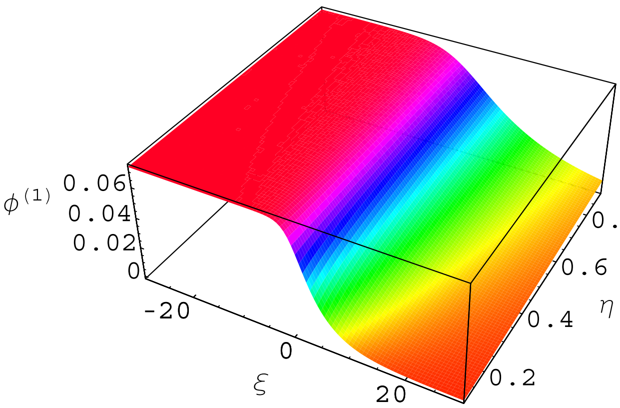

Fig. 2 Variation of η and ξ on $\phi^{(1)}$ of mIA shock waves obtained from (39) for $\mu < \mu_c$ and e-p being ultra-relativistic

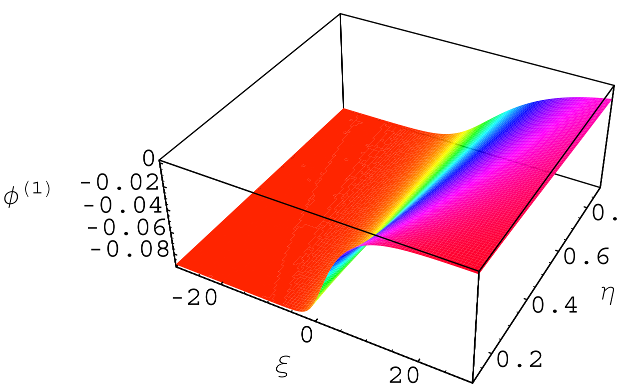

Fig. 3 Variation of η and ξ on $\phi^{(1)}$ of mIA shock waves obtained from (39) for $\mu > \mu_c$ and e-p being non-relativistic

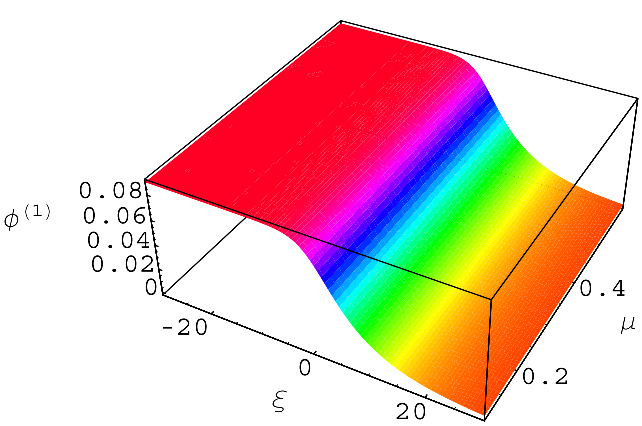

Fig. 4 Variation of μ and ξ on $\phi^{(1)}$ of mIA shock waves obtained from (39) for $\mu < \mu_c$ and e-p being non-relativistic

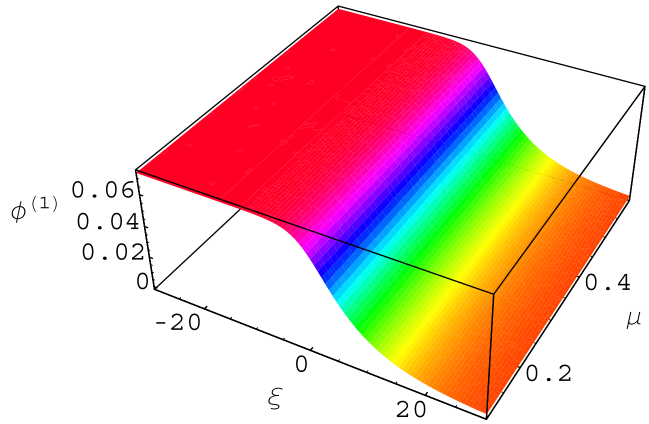

Fig. 5 Variation of μ and ξ on $\phi^{(1)}$ of mIA shock waves obtained from (39) for $\mu < \mu_c$ and e-p being ultra-relativistic

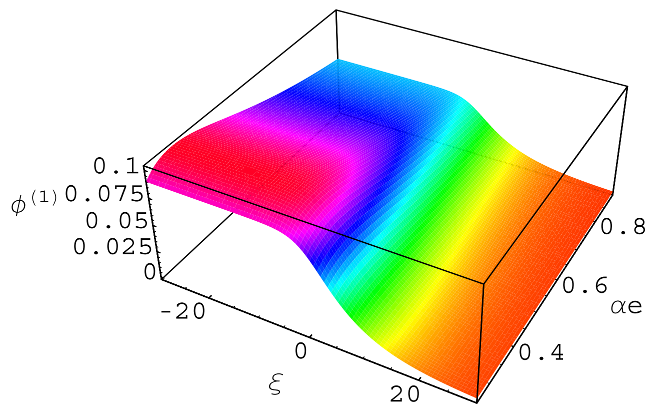

Fig. 6 Variation of α_e and ξ on $\phi^{(1)}$ of mIA shock waves obtained from (39) for $\mu < \mu_c$ and e-p being non-relativistic

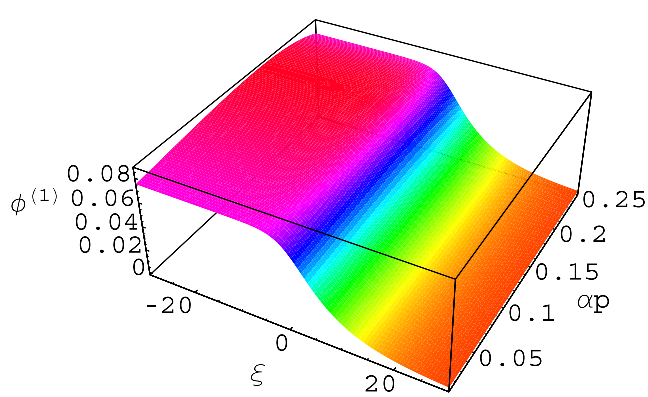

Fig. 7 Variation of α_p and ξ on $\phi^{(1)}$ of mIA shock waves obtained from (39) for $\mu < \mu_c$ and e-p being non-relativistic

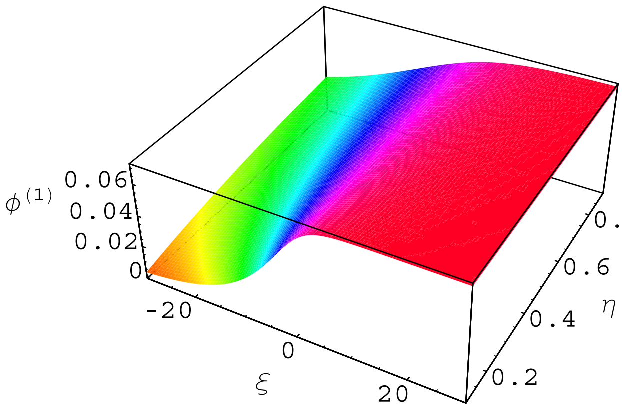

Fig. 8 Variation of η and ξ on $\phi^{(1)}$ of mIA DLs obtained from (40) for $\mu < \mu_c$ and e-p being non-relativistic

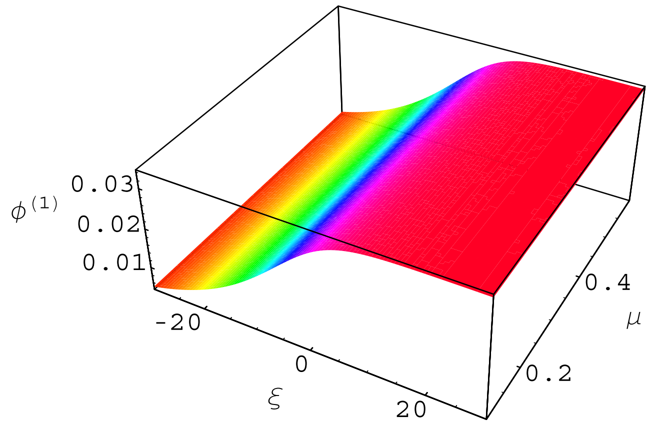

Fig. 11 Variation of μ and ξ on $\phi^{(1)}$ of mIA DLs obtained from (40) for $\mu < \mu_c$ and e-p being ultra-relativistic

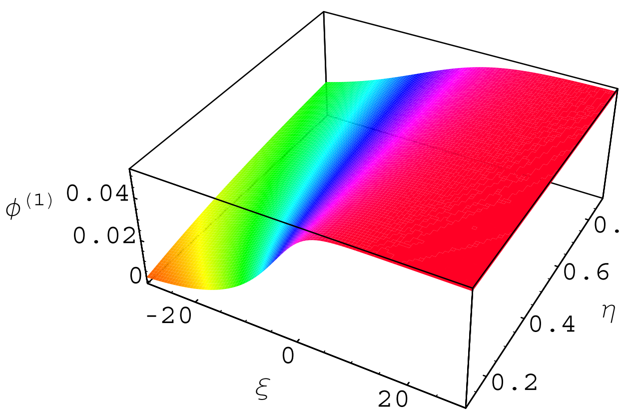

Fig. 9 Variation of η and ξ on $\phi^{(1)}$ of mIA DLs obtained from (40) for $\mu < \mu_c$ and e-p being ultra-relativistic

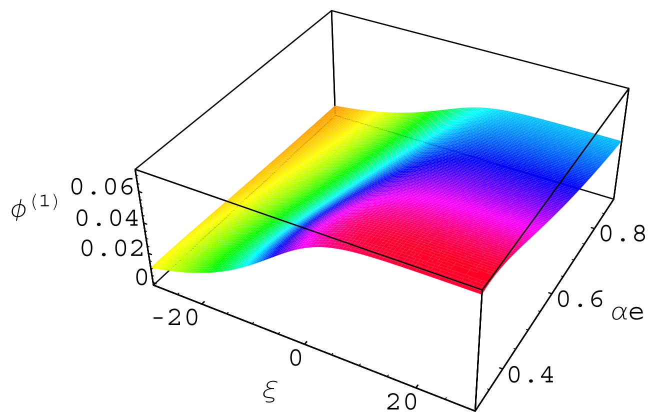

Fig. 12 Variation of α_e and ξ on $\phi^{(1)}$ of mIA DLs obtained from (40) for $\mu < \mu_c$ and e-p being non-relativistic

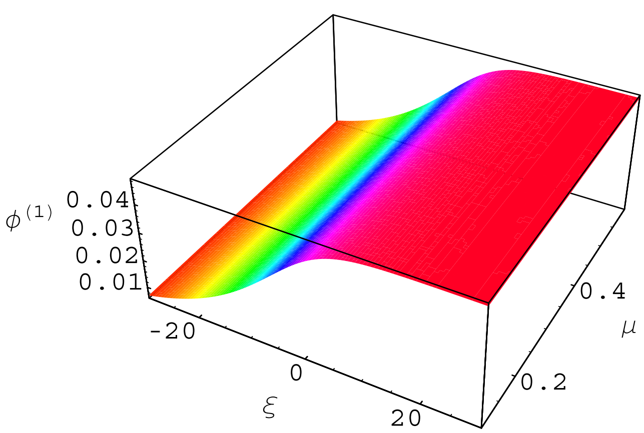

Fig. 10 Variation of μ and ξ on $\phi^{(1)}$ of mIA DLs obtained from (40) for $\mu < \mu_c$ and e-p being non-relativistic

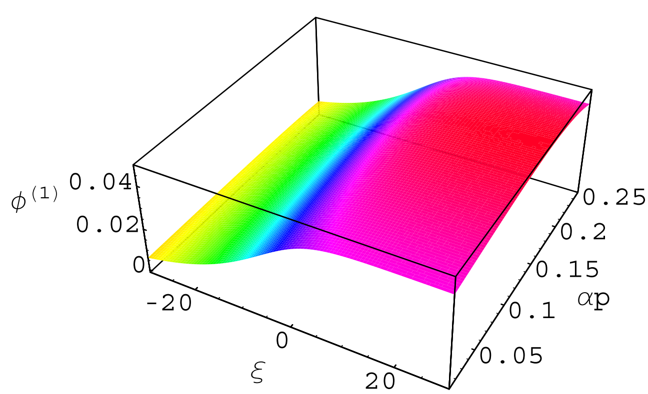

Fig. 13 Variation of α_p and ξ on $\phi^{(1)}$ of mIA DLs obtained from (40) for $\mu < \mu_c$ and e-p being non-relativistic

We considered $u_0 = 0.01$ for our numerical analysis of mIA waves for the plasma system investigated here. We also considered the values of η, α_e, and α_p are 0.3, 0.3, and 0.2, respectively [53–55]. We first graphically represented the effect of η on the amplitude $\phi^{(1)}$ of shock structures (see Figs. 1–3). Figures 4–9 show the variation of μ, α_e, and α_p on the amplitude $\phi^{(1)}$ of shock structures. Then, we graphically represented the effects of η, μ, α_e, and α_p on the amplitude $\phi^{(1)}$ of DLs (see Figs. 10–13). The variation of non-relativistic and ultra-relativistic limits was also identified and shown in case of both shock and DL structures. It is observed that the plasma system under consideration supports Gardner shocks with either negative ($\mu > \mu_c$) or positive potential ($\mu < \mu_c$) but double layers (DLs) only for positive potential ($\mu < \mu_c$).

6 Discussion and Results

In this paper, we investigated the electrostatic perturbation mode for relativistic degenerate e-p-i plasma in the presence of negatively charged heavy ions. The formation of nonlinear electrostatic propagation modes, particularly shock profiles, was theoretically investigated. We found that the non-degenerate ions, degenerate electron and positron fluids, and negatively charged static heavy ions as well as relativistic effects have significant effects on the propagation of shock and DL structures. The results obtained from this investigation can be summarized as follows:

1. The plasma system under consideration supports small but finite amplitude Gardner shocks and DLs, whose basic properties (polarity, width, speed etc.) are found to be significantly modified due to the effects of relativity, degeneracy, and plasma particle number densities.
2. It is observed that the plasma system under consideration supports Gardner shocks with either positive or negative potential (depending on the plasma parameters), but DLs with only positive potential.
3. It is found that the phase speed (V_p) of these electrostatic shocks and DLs is inversely proportional to the square root of heavy ion to light ion number densities ratio (μ).
4. The magnitude of the amplitude of shocks and DLs increases with the increasing values of μ and η but decreases with α_e (see Figs. 1–6 and 8–10).
5. It is also observed that the amplitude of the mIA shocks and DLs is always significantly higher for e-p being non-relativistic degenerate than e-p being ultra-relativistic degenerate (see Figs. 1–13).
6. The comparison between shock and DL structures is also important. It is found that the amplitude of shocks is distinctly higher than DLs for both non-relativistic and ultra-relativistic limits (see Figs. 1–7 and 8–11).

We represented a theory for mIA shock and DL structures in a degenerate e-p-i plasma medium and investigated their basic features (polarity, width, speed, etc). It may also be added here that our investigation is valid for small amplitude mIA shock waves in unmagnetized and uniform dense plasma system. However, arbitrary relativistic case, transition between the two regimes, and external magnetic field are also problems of recent interest for many space and laboratory plasma situations, but beyond the scope of our present investigation. We finally hope that our present investigation will be very helpful for understanding the basic features of the localized electrostatic disturbances in a relativistic degenerate e-p-i plasma which occurs in some astrophysical compact objects, e.g., non-rotating white dwarf stars, neutron stars, and so on [3–7].

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