Braz J Phys (2015) 45:54–58
DOI 10.1007/s13538-014-0289-4

CONDENSED MATTER

Anisotropic Magnus Force in Type-II Superconductors with Planar Defects

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Received: 16 November 2011 / Published online: 19 December 2014

Abstract The effect of planar defects on the Magnus force in type-II superconductors is studied. It is shown that the deformation of the vortex due to the presence of a planar defect leads to a local decrease in the mean free path of electrons in the vortex. This effect reduces the effective Magnus coefficient in normal direction to the planar defect, leading to an anisotropic regime of the Hall effect. The presented developments here can qualitatively explain experimental observations of the anisotropic Hall effect in high-T_c superconductors in the mixed state.

Keywords Type-II superconductors · Planar pinning centers · Magnus force · Hall anisotropy

1 Introduction

In the last years, the interest in the vortex motion in high-T_c superconductors has been emphasized by the contribution of this dynamics in a great variety of transport phenomena, which are associated to non-peculiar properties of such materials. Some new features, not found in conventional type-II superconductors as the Hall anomaly in high-temperature superconductor (HTSC) compounds [1, 2], become a motor for the development of this area of the solid-state physics. The Hall anisotropy in high-T_c superconductors is another aspect observed in several works [3, 4], which have reported a violation of the Onsager principle.

The equation of motion, which governs the vortex movement in type-II superconductors, has been subject of several researches, helping us to understand many characteristics of these systems. Generally, the vortex dynamics has been considered in the hydrodynamical two-fluid model [5, 6], where the relative displacement between the superfluid and the vortex generates the Magnus force, which arises when a vortex with flow circulation around it moves through the medium, affecting the vortex trajectory. In this connection, the normal component reacts to this motion, producing both the longitudinal viscous drag and the transversal Iordanski forces, which are the two components of the medium's force.

Today, there are several attempts to construct a unified theory about the vortex motion. Some approximations use the sophisticated many-body formalism [7–9]. Other authors apply a simpler theory based on the kinetic Boltzmann equation to study the dynamic behavior of the vortex structures [10, 11]. However, so far there are many open questions about the balance equation in type-II superconductors and therefore the vortex dynamics still is an unsolved problem for the solid-state physics community. An example of the foregoing is the problem of the Magnus force acting on a moving vortex. This aspect is a discussion source between some points of view in several researches [7–11]. In particular, the impact of pinning centers on the vortex dynamics in high-T_c superconductors has been object of fundamental interest [12–14] because defects, like twin boundaries, can be created in HTSC materials, in a non-intensional way during their growing. Equally, planar defects occur naturally in layered superconducting structures, so that vortices are intrinsically pinned in an easy form. In this sense, the importance of planar defects is based on their ability to create localized states which reduce the order parameter when the vortices approach them. The clout of twin boundaries on the vortex dynamics has been analyzed in many works [15, 16], but so far it is normally

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believed that planar defects can affect the vortex through the pinning force and the renormalized viscous drag force. In this connection, the goal of this paper is to study the Magnus force acting on vortices in the nearness of planar defects such as twin boundaries and somehow to contribute to a better understanding of the fascinating phenomenon of the vortex motion. Likewise, the purpose of this paper is to show in a qualitative manner the origin of the reported, in high-T_c superconductors [3, 4], Hall anisotropy.

The paper is organized as follows: In Section 2, we obtain a general expression for the Magnus force in the nearness of a planar defect. In Section 3, we study the local coherence length in the vortex, and in Section 4, we analyze and compare, with experimental observations, the main conclusions of the paper.

2 Magnus Force in the Nearness of a Planar Defect

The studied system is a type-II superconductor under the action of a magnetic field H, which is oriented along the z-axis. We limit ourselves to the case where $H_{c1} < H \ll H_{c2}$, which allows analyzing the individual behavior of the vortex. In this model, the many-body state function, for a superconductor in presence of one vortex, has the form

$$\Psi\left(\mathbf{r};\mathbf{r}_0\right)=\exp\left[\frac{i}{2}\theta\left(\mathbf{r}-\mathbf{r}_0\right)\right]\Psi_0\left(\mathbf{r};\mathbf{r}_0\right), \quad (1)$$

where the parameters $\theta\left(\mathbf{r}-\mathbf{r}_0\right)=\sum_{j=1}^{N}\theta\left(\mathbf{r}_j-\mathbf{r}_0\right)$, and $\Psi_0\left(\mathbf{r};\mathbf{r}_0\right)=\Psi_0\left(\mathbf{r}_1,\ldots,\mathbf{r}_N;\mathbf{r}_0\right)$ are the phase and amplitude of the state function; and $\mathbf{r}$ and $\mathbf{r}_0$ are the electronic and vortex (in the laboratory frame) position vectors, respectively. In order to study the evolution of the system, we take the time derivative to the expression (1) as follows:

$$\left\langle\Psi\left|\frac{d}{dt}\right|\Psi\right\rangle=\left\langle\Psi|\nabla_{\mathbf{r}}\Psi\right\rangle\cdot\mathbf{v}_s+\left\langle\Psi|\,\nabla_{\mathbf{r}_0}\Psi\right\rangle\cdot\mathbf{v}_L. \quad (2)$$

Here, $\mathbf{v}_s=d\mathbf{r}/dt$ and $\mathbf{v}_L=d\mathbf{r}_o/dt$ are the electron and vortex velocity, respectively, and $\langle\Psi|\,\nabla_{\mathbf{r}(\mathbf{r}_o)}\Psi\rangle$ is defined as follows:

$$\langle\Psi|\,\nabla_{\mathbf{r}(\mathbf{r}_o)}\Psi\rangle=\frac{i}{2}\int d^2\mathbf{r}\rho\left(\mathbf{r},\mathbf{r}_0\right)\nabla_{\mathbf{r}(\mathbf{r}_o)}\theta\left(\mathbf{r}-\mathbf{r}_0\right)+\int d^2\mathbf{r}\Psi_0^*\nabla_{\mathbf{r}(\mathbf{r}_0)}\Psi_0, \quad (3)$$

where $\rho\left(\mathbf{r},\mathbf{r}_0\right)$ is the superfluid electron number density. On the other hand, we can assume $\nabla_{\mathbf{r}_0}=-\nabla_{\mathbf{r}}$, because it operates on a function of argument $\left(\mathbf{r}-\mathbf{r}_0\right)$. In this sense, deriving the (2) with respect to the vortex line position, $\mathbf{r}_0$, we obtain the Magnus force:

$$\mathbf{F}_{\mathrm{M}}=\frac{\hbar}{2}\nabla_{\mathbf{r}_0}\left\{\left[\int d^2\mathbf{r}\rho\left(\mathbf{r},\mathbf{r}_0\right)\nabla_{\mathbf{r}}\theta\right]\cdot\left(\mathbf{v}_s-\mathbf{v}_L\right)\right\}. \quad (4)$$

For a more compact expression, we can use the vector identities $\nabla\left(\mathbf{c}\cdot\mathbf{d}\right)=\mathbf{c}\times\left[\nabla\times\mathbf{d}\right]+\left(\mathbf{c}\cdot\nabla\right)\mathbf{d}$ and $\nabla\times\left(\varphi\mathbf{a}\right)=\left(\nabla\varphi\right)\times\mathbf{a}+\varphi\nabla\times\mathbf{a}$. Since the Magnus force is perpendicular to the relative velocity $\left(\mathbf{v}_s-\mathbf{v}_L\right)$, then the term $\left(\mathbf{v}_s-\mathbf{v}_L\right)\cdot\nabla_{\mathbf{r}_0}$ is zero. In the same mode, taking into account that $\nabla_{\mathbf{r}}\rho\left(\mathbf{r},\mathbf{r}_0\right)\times\nabla_{\mathbf{r}}\theta\left(\mathbf{r}-\mathbf{r}_0\right)$ is also zero for every vector $\mathbf{r}$, we rewrite the expression (4) as follows:

$$\mathbf{F}_{\mathrm{M}}=-\frac{\hbar}{2}\left(\mathbf{v}_s-\mathbf{v}_L\right)\times\int d^2\mathbf{r}\rho\left(\mathbf{r},\mathbf{r}_0\right)\nabla_{\mathbf{r}}\times\nabla_{\mathbf{r}}\theta. \quad (5)$$

The function $\nabla_{\mathbf{r}}\times\nabla_{\mathbf{r}}\theta\left(\mathbf{r}-\mathbf{r}_0\right)$ is zero everywhere except for the center of the vortex, where the integral of this function is 2π. This corresponds to the behavior of the δ function, so it can be replaced by $2\pi\delta(\mathbf{r}-\mathbf{r_0})\hat{z}$, where $\hat{z}$ is the unit vector along the vortex. On the other hand, the functional form of $\rho\left(\mathbf{r},\mathbf{r}_0\right)$ depends on the profile of the order parameter in the vortex. In a general form, $\rho\left(\mathbf{r},\mathbf{r}_0\right)$ can be written as $\rho\left(\mathbf{r},\mathbf{r}_0\right)=\rho_s\left(\mathbf{r}\right)+\delta\rho\left(\mathbf{r},\mathbf{r}_0\right)$, where $\rho_s(\mathbf{r})$ is the electron number density without vortex, while $\delta\rho$ is its variation due to the presence of the vortex. In particular, in the Ginzburg-Landau limit, one takes some conjectures about the profile of the density inside the vortex. An approach of the deviation $\delta\rho$, which we will use in the present analysis, was suggested by Welch [17]:

$$\delta\rho\left(\mathbf{r},\mathbf{r}_0\right)=\rho_s\left(1-\exp\left(-\frac{\mathbf{r}'^2}{2\xi_{\mathrm{b}}^2}\right)\right). \quad (6)$$

In the above expression, ξ_{b} is the bulk Ginzburg-Landau coherence length and $\mathbf{r}'$ is the electron position vector in the vortex frame. Moreover, this paper examines the effect of a planar defect onto the Magnus force. The role of planar defects in type-II superconductors can be played by twin boundaries as well as grain boundaries. In this context, to define the variation $\delta\rho$, it is necessary to take into account the contribution from the planar defect, which deform the vortex in direction normal to them, while in parallel direction to the defect, the vortex does not suffer substantial deformation. In Fig. 1, we have pictured the analyzed situation in a very demonstrative form. The contribution $\rho_s(\mathbf{r})$ is constant and equals to the background superfluid density ρ_s for closed loops larger than the size of the vortex core. Therefore, according to the above considerations, we have that inside the vortex, moving electrons parallel to the planar defect do not contribute to the Magnus coefficient because, in this case, simply $\mathbf{r}'=\mathbf{r}-\mathbf{r}_o$, and therefore, replacing the expression (6) in (5), the integral in this equation is zero. Thus, in this orientation, the Magnus force is due to contribution of superfluid flow outside the vortex $\sim\rho_s$. Moreover, moving electrons in the direction to the defect contribute to the definition of a local Magnus coefficient. The foregoing can be showed as follows: The electron position in the vortex frame must be represented through the vectors involved in the problem, i.e., $\mathbf{r}_o$ and $\mathbf{r}_d$ where $\mathbf{r}_d$ is the position

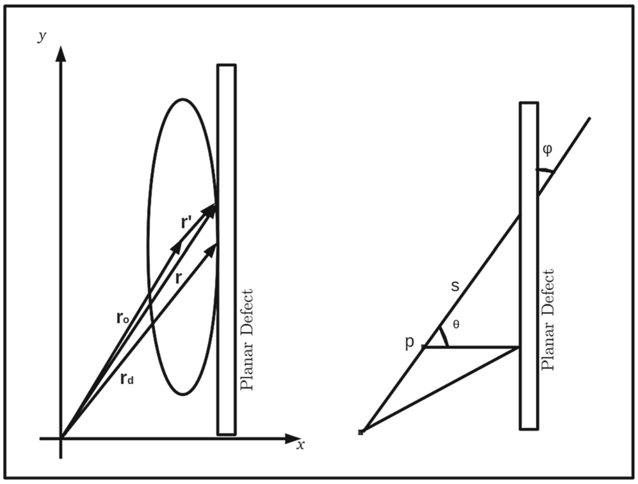

Fig. 1 Geometry of the system

vector of the planar defect (in the laboratory frame) in a normal point to the vortex center. In this sense, the electron position vector normal to the defect becomes

$$\mathbf{r}'_{\perp} \approx (\mathbf{r}_o - \mathbf{r}) + (\mathbf{r} - \mathbf{r}_d) \, . \tag{7}$$

Taking into account the expressions (7) and (6) in the integral (5), finally the Magnus force takes the following functional form:

$$\mathbf{F}_{\mathrm{M}} = q_v \eta'(\xi) \, (\mathbf{v}_s - \mathbf{v}_L) \times \hat{z}. \tag{8}$$

$q_v = 1(-1)$ for a vortex parallel (antiparallel) to the z-direction and $\eta'(\xi)$ is the Magnus force coefficient, which in parallel direction to the defect takes its bulk value $\eta'_{\parallel} = \eta'$, while in normal direction to the defect takes the local value

$$\eta'_{\perp} = \eta' \left[1 - \exp\left(-\frac{\xi^2}{2\xi_b^2} \right) \right], \tag{9}$$

where $\eta' = h\rho_s/2$. Here, $\xi = \mathbf{r}_0 - \mathbf{r}_d$ is the local "radio" of the vortex in direction of deformation due to the planar defect. Such kind of considerations demonstrate the anisotropic nature of the Magnus coefficient in the presence of a planar defect. This fact is evident in the direction of vortex deformation, where the coefficient is a function of the relative variation of the coherence length due to the defect. These arguments show that planar pinning centers dramatically affect the amplitude of the Magnus force transforming it in anisotropic.

3 Local Coherence Length

In order to obtain an explicit formulation for the Magnus coefficient, it is necessary to determine the analytic form of the local coherence length in the deformed vortex. Such kind of expression can be derived from electron scattering mechanism near a planar defect, i.e., the presence of the planar pinning center leads to electron scattering at the boundary, and therefore, the features of the system can be changed both in the normal and superconducting states. In this sense, for this purpose, we recall the known local relation between the coherence length ξ and the mean free path ℓ as follows [18]:

$$\frac{1}{\xi} = \frac{1}{0.74\xi_0} + \frac{1}{\ell}, \tag{10}$$

where ξ_0 is the BCS coherence length. In the above expression, we shall confine ourselves to the $\xi \ll \xi_o$ limit, which leads to $\xi \approx \ell$. In this regard, in this section, we will use the developments put forward by Zerweck [19], considering the fact that the vortex is pinned by a planar defect located in the y-z-plane. For a flux line parallel to the z-direction, it can be estimated that the vortex "radius" in the y-direction almost does not vary and it takes the Ginsburg-Landau bulk value, while in the x-direction, perpendicular to the planar defect, the length ξ is a function of the distance x to the defect. Figure 1 shows the proposed scheme. In this connection, we will calculate the mean free path ℓ_θ along an arbitrary straight line which passes through a point p and crosses the defect forming an angle $\varphi = \pi/2 - \theta$ with its plane. This model supposes, in the simplest case of diffuse isotropic dispersion, that electrons cannot pass the defect without being scattered. Defining the probability for the electron to travel a distance r' without collisions by $\exp\left(-r'/\ell_0\right)$, where ℓ_0 is the mean free path without defect, then with the help of Fig. 1, we obtain the following relation:

$$s = \frac{x}{\cos\theta}, \tag{11}$$

where s is the distance between the point p and the defect along the considered line. Now we take an average over the solid angle, where it should be noted that the relation (11) is only valid for $\theta < \frac{\pi}{2}$ and that the mean free path does not change for $\theta > \frac{\pi}{2}$:

$$\ell = \frac{1}{2}\int_0^{\pi/2} \ell_\theta \sin\theta d\theta + \frac{\ell_0}{2}\int_{\pi/2}^{\pi} \sin\theta d\theta, \tag{12}$$

where $\ell_\theta = \int_0^\infty r' P(r') dr'$ and the distribution function $P(r')$ has the form

$$P(r') = (1/\ell_0) \int_0^{r'} \exp\left(-r''/\ell_0\right) dr''. \tag{13}$$

Taking into account the above relations and the form of $P(r')$, evaluated at s, the mean free path ℓ takes the following form:

$$\ell = \ell_0 - \frac{\ell_0}{2}\int_0^{\pi/2} \exp\left(-\frac{x}{\ell_0\cos\theta}\right) \sin\theta d\theta. \tag{14}$$

If we consider the limit when the distance between the point p and the defect is small compared to the mean free path without defect, i.e., $x \ll \ell_0$, then from (10) and (14), the local coherence length takes the following form:

$$\xi \approx \frac{\ell_0}{2}\left[1 - \frac{x}{\ell_0}\ln(x/\ell_0)\right]. \qquad (15)$$

According to the above expression and considerations, it is important to note the following: in the y-direction, the Magnus coefficient takes its bulk value in the absence of disorder, whereas in the x-direction, the Magnus coefficient $\eta'(x)$ is a function of distance from the vortex center to the planar defect (see Fig. 1), propitiated by a local value of the coherence length in the vortex, via reduction of the electron's free mean path due to electron scattering mechanism. Figure 2 plots the Magnus coefficient as a function of distance from the vortex center to the defect, for some values of the impurity parameter $\alpha = 2\sqrt{2}\xi_b/\ell_0$.

4 Discussion and Conclusions

In this paper, we determined the effect of planar defects on the Magnus force in type-II superconductors. In a local model, we obtained an explicit expression for the Magnus force and showed that planar defects reduce the mean free path of electrons and therefore the coherence length in the vortex, so leading to an anisotropic regime, where the Magnus coefficient depends on the direction to the defect. From Fig. 2, we see that in samples with higher impurity parameter, the effect of the Magnus coefficient reduction is greater. The foregoing can be better understood from (9), in the $\xi \ll \xi_b$ limit, where $\eta'_\perp$ acquires the following compact form:

$$\eta'_\perp = \frac{\eta'\xi^2}{2\xi_b^2}. \qquad (16)$$

The expression (16) and Fig. 2 clearly show the Magnus coefficient dependence from the relative reduction of the coherence length due to the defect. These results can be important in the understanding of some features associated

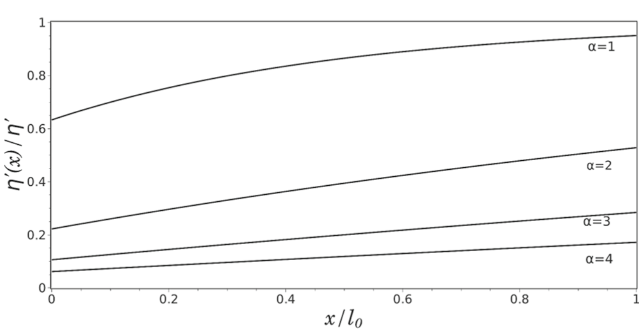

Fig. 2 Magnus coefficient as a function of distance from the vortex center to the defect

to vortex dynamics like the Hall anomaly. The analysis presented here shows that planar defects can affect not only the effective drag force as was pointed out by Schklovski [20], but can also affect the effective Magnus force in type-II superconductors. In this regard, in the Magnus force coefficient, we can identify two contributions: one from the superfluid far from the vortex core and the second one which is local and comes from electron dispersion in direction to the vortex deformation. These results can qualitatively explain experimental observations of the anisotropic Hall effect in high-T_c superconductors in the mixed state [3, 4], where the transversal components of the resistivity tensor do not satisfy the Onsager principle $\varrho_{xy} \neq -\varrho_{yx}$. To demonstrate the above affirmation, we start from the equation, which governs the local vortex dynamics in the presence of a pinning center [21]

$$\eta'(\xi)(\hat{z} \times \mathbf{v}_L) - \tilde{\eta}\mathbf{v}_L = \rho_s(\hat{\kappa} \times \mathbf{v}_s). \qquad (17)$$

Here, κ and $\eta'(\xi)$ are the quantum of circulation and the Magnus coefficient respectively and $\tilde{\eta}$ is the renormalized coefficient of the viscous drag force

$$\tilde{\eta} = \eta + \gamma, \qquad (18)$$

where η is the viscous drag coefficient, which arises from normal dissipation in the vortex core, and γ is the nonlinear phenomenological vortex pinning viscosity, which depends on the magnitude of $\mathbf{v}_L$. In the formulation of (17), we took into account from (8) that the Lorentz force per unit length acts upon an individual vortex far from the vortex core, where the superfluid velocity $\mathbf{v}_s$ is uniform. Expressing the components of the vortex velocity through the components of the electron velocity and using the Josephson relation $\mathbf{E} = \mathbf{v}_L \times \mathbf{B}$, we can obtain the local Ohm law in the matrix form $E_\alpha = \sum_\beta \rho_{\alpha\beta} J_\beta$, where the components of the resistivity tensor, in the proposed geometry of previous section, have the following form:

$$\rho_{xx} = \rho_{yy} = \frac{\tilde{\eta}\kappa B}{\eta'^2 + \tilde{\eta}^2}, \qquad (19)$$

$$\rho_{xy} = \frac{\eta'_\perp \kappa B}{\eta'^2 + \tilde{\eta}^2}, \quad \rho_{yx} = -\frac{\eta'_\parallel \kappa B}{\eta'^2 + \tilde{\eta}^2}. \qquad (20)$$

We note in previous relationships that the key factor which determines the shape of the transverse components of the resistivity is the Magnus coefficient. As is known for a system, where a magnetic field is applied, the property of time-reversal symmetry implies that transport coefficients as the Hall components of the electrical resistivity require a configuration $\varrho_{xy}(B) = \varrho_{yx}(-B)$, which favors the fulfillment of the reciprocity of the Onsager principle, which is closely related to the detailed balance near the equilibrium. Inhomogeneities, as shown by relations in (16) and (20), break down the symmetry properties and consequently a local violation of the Onsager relation can be observed

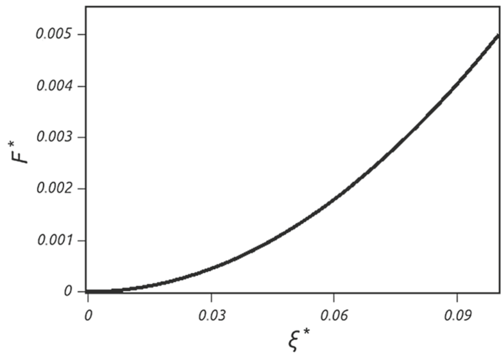

Fig. 3 Ratio of the Magnus force components vs. coherence length. $F^* = F_\perp/F_\parallel$, $\xi^* = \xi/\xi_b$

$\varrho_{xy} \neq -\varrho_{yx}$. Even the field reversal in (20) does not lead to satisfying the equality due the anisotropy of Magnus coefficient. This fact can be explained qualitatively within a theory of transition of the Magnus force to an anisotropic regime due to the presence of defects in the sample as is the case of planar defects, which, in type-II superconductors, can be twin boundaries as well as grain boundaries, etc. From the point of view of vortex dynamics, this effect is equivalent to the existence of the anisotropic effective Magnus force, which takes different values for the Hall components of the resistivity tensor due to the reduction of the electron coherence length normal to the planar defect. In Fig. 3, we show the ratio $F_\perp/F_\parallel$ of the Magnus force components as a function of the coherence length.

Summarizing the above developments, we have shown that the deformation of the vortex due to the presence of a planar defect leads to a local decrease in the mean free path of electrons in the vortex, leading to the anisotropy of the Magnus force and favoring a transition to the anisotropic regime of the Hall effect.

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