Critical Behavior of the Fully Frustrated Two Dimensional XY model

A.B. Lima and B.V. Costa

Departamento de Física ICEX, Universidade Federal de Minas Gerais

Caixa Postal 702, 30123-970 Belo Horizonte, MG, Brazil

Received on 8 August, 2004

Using Monte Carlo simulations we have investigated the critical behavior of the classical fully frustrated XY model in two dimensions in a square lattice. There are two phase transitions in the model, one of the Berezinskii-Kosterlitz-Thouless type and a Z_2 transition at higher temperature. We show that the vortex anti-vortex density has a clear signature of the Z_2 phase transition at exactly the percolation threshold.

1 Introduction

Classical continuous spin models with short range interactions in two dimensions are a prototype for systems which exhibit topological excitations [1]. It is well known that this kind of model undergoes a phase transition at a finite temperature T_{BKT}, from a high-temperature phase where the spin-spin correlations decay exponentially to a low temperature phase where they have a power-law decay. This phase transition is believed to be driven by a vortex-antivortex unbinding mechanism [2]. A vortex (antivortex) is a topological excitation in which spins on a closed path around the excitation have a positive (negative) chirality, f

$$f = \sum_{plaquete} (\theta_i - \theta_j), \quad (1)$$

where θ is the angle that the XY spin vector component makes with some fixed direction in the plane. The models we are interested in can be described by the following Hamiltonian

$$H = \sum_{<i,j>} J_{i,j}(S_i^x S_j^x + S_i^y S_j^y), \quad (2)$$

where $S_i = \{S_i^x, S_i^y, S_i^x\}$ is a spin vector at site i, $J_{i,j}$ is an exchange coupling which is ferromagnetic in all lines in the x direction and is alternately ferromagnetic and anti-ferromagnetic in the y direction. The coupling distribution leads the ground state of the model to have a checkerboard pattern of plaquetes with positive (vortex) or negative (anti-vortex) chirality $f/\pi = \pm 1$. Due to this symmetry the model has a Z_2 transition at T_{Z_2} [3]. In a recent work we have shown that $T_{BKT} = 0.3655(5)J$ and $T_{Z_2} = 0.3690(3)J$ for the XY model [4]. The vortex density at $T = 0$ is $\rho = 1$. Once the temperature grows, pairs vortex anti-vortex begin to annihilate so that we have a diluted Ising model to deal with. Using Monte Carlo simulation we have calculated the vortex anti-vortex density and the percolation probability for the model. The vortex anti-vortex density ρ is just the number of vortices divided by the lattice volume L^2 [1]. The percolation probability P, must be a step function: for $\rho < \rho_c$, $P = 0$ and $P = 1$ for $\rho > \rho_c$, where ρ_c is the critical concentration.[5] We carried out simulations in square lattices of sizes $L \times L$ with $L = 20, 40, 80$ and 100. Each point in our simulation is the result of the average over 5×10^4 independent configurations. Fig. 1 shows the vortex anti-vortex density as a function of temperature.

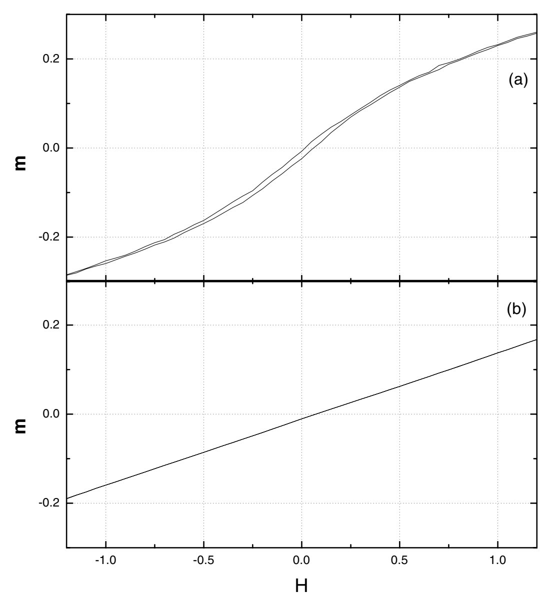

Figure 1. The vortex density as a function of temperature. The insert shows the derivative of the vortex density. The position of the maxima coincides with the percolation temperature.

We observe that close to the transitions there is a steep drop on the vortex density. The insert shows the derivative, $d\rho/dT$, for several lattice sizes. At some value T_L each curve presents a maximum. An extrapolation for $L \to \infty$ gives $T_L = 0.368(3)J$,which matches T_{Z_2} inside the error bars. At the Ising transition we expect $\rho(T_{Z_2}) = \rho_c$, where ρ_c is to be identified with the percolation threshold. Using the percolation probability P we obtain an estimate for the critical point as the intercept of the curves for different lattice sizes.

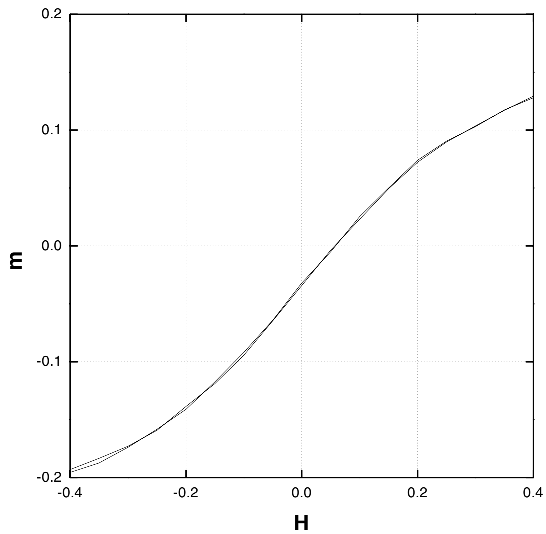

Figure 2. Percolation probability as a function of temperature. The percolation temperature is obtained as the intersection of the curves for several lattice sizes.

From Fig. 2 we get $T_c = 0.365(5)$, in excellent agreement with the results above. In short, we have performed Monte Carlo simulation in the fully frustrated XY model defined by equation 2. The model has two phase transitions, one BKT and other of the Ising type. We have shown that the Ising transition can be obtained from the percolation probability which coincides with the inflection point of the vortex anti-vortex density as a function of temperature. One should notice that the temperatures $T_{BKT} = 0.3655(5)J$ and $T_c = 0.365(5)$ seems to coincide and we are compelled to say that the BKT and the Ising transition occur at the same temperature as suggested by earlier works.[6] However, we can not conclude this from our data, since due to the error bars T_c matches both T_{Z_2} and T_{BKT}. A more intensive simulation have to be done to decide about that.

Financial support from the Brazilian agencies CNPq, CAPES, FAPEMIG and CIAM-02 49.0101/03-8 (CNPq) are gratefully acknowledged.

References

[1] J.E.R. Costa, D.P. Landau and B.V. Costa, Phys. Rev.B **57**, (1998)11510.

[2] J. Villan, J. Phys. C**10**, (1977)4793.

[3] S. Teitel and C. Jayaprakash, Phys. Rev. Lett. **51** (1983)1999.

[4] A.B. Lima and B.V. Costa, J. Magn. Magn. Mater. **263**, (2003)324-331.

[5] A. Coniglio, C.R. Nappi, F. Peruggi and L. Russo, Journal of Physics A:Mathematical and General **10**, (1977)205 and references therein.

[6] C. Ebner and D. Stroud, Phys. Rev. B **28** (1983)5053; M.Y. Choi and S. Doniach, Phys. Rev. B **31** (1985)4516.