^{3}S$_1$ and ^{1}S$_0$ Meson Spectra in a Renormalized QCD-Inspired Model

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Received on 17 May, 2005

In this work we study an extension of a light-cone QCD-inspired model, where the interacting part of the mass squared operator consists of a harmonic oscillator potential as confinement, a Coulomb-like interaction and a zero-range spin interaction acting on the $^3S_0$ states. The renormalization of the model is performed by using as a input the pion mass to fix the strength of the spin interaction. We apply the extended model to study the splitting of the ^{3}S$_1$ and ^{3}S$_0$ ground state mesons and the spectrum in the light meson sector. We show that the experimental values of the splitting between the masses of ^{3}S$_1$ and ^{3}S$_0$ ground state mesons as a function of the ground state pseudoscalar mass is reproduced by the model. In the botonium case our result is consistent with other theoretical estimates.

I. INTRODUCTION

The lowest Fock component of the light-front hadron wave function is an eigenfunction of an effective mass squared operator, which is phenomenologically described with constituent quark degrees of freedom [1–5]. The interaction in the mass squared operator comes from an effective one-gluon-exchange where the Dirac-delta term corresponds to the hyperfine interaction which is projected only on the singlet state and a harmonic confinement [5]. The harmonic confining interaction is included to describe the observation of the quasi-linear dependence of the squared meson masses with the radial quantum number [6, 7]. The hyperfine interaction has the role to split the pion and rho meson masses, however as the pion has a Goldstone boson nature the spin interaction should arise from collective effects much beyond the simple one-gluon exchange, in view of that we parameterize the spin-spin interaction as acting only on the singlet state. We have also used a hyperfine spin interaction and it provides essentially the same results, as we verified.

In a simpler version of the model where confinement was not included the masses of the ground state of the pseudoscalar mesons and in particular the pion structure [4] were described with a small number of free parameters, which is only the canonical number plus one the renormalized strength of the Dirac-delta interaction.

The mass squared operator model with harmonic confinement and without Coulomb-like interaction was used to study the S-wave meson spectra from π-ρ to η_b-Υ addressing the splitting of the excited $^1S_0$ and $^3S_1$ meson states and as a function of the ground state pseudoscalar mass [4, 5]. The universal flavor-independent parameters of the confining interaction in the mass squared operator, were fitted to the $^3S_1$-meson ground state mass and to the slope of the trajectory of excited states with the radial quantum number [6, 7]. The linear relationship between the mass squared of excited states with radial quantum number was found to be qualitatively valid even for heavy mesons like Υ, and moreover the model is in reasonable agreement with available data [8] and/or with the meson mass spectra given by Godfrey and Isgur [9].

In the model the $^3S_1 -^1 S_0$ mass splitting is due to an attractive Dirac-delta interaction acting on the $^1S_0$ spin state with strengths parameterized by the masses of the pseudoscalar ground state in each meson family. The pseudoscalar ground state mass defines the renormalization condition of the model for each meson quark content. However, the adopted renormalization scheme for the confining model is not flavor independent, and in practice we had one strength parameter for each meson sector. In the case of nonconfining calculation within the model [4] such goal was achieved and the ground state mass splitting originated by the singular spin-spin interaction was obtained only from the pion mass. Later on, a consistent treatment of the hyperfine interaction within the confining model was discussed in Ref. [10], although the calculations were simplified without addressing the singularity of the spin-spin operator itself in the square mass operator equation [4].

In this work, we treat numerically renormalization of the confining mass squared operator model with the singular spin-spin interaction acting in the singlet channel. At small relative distances between the quark and aniquark the harmonic potential is just flat and due to that the ultraviolet divergence of the confining model is exactly the same as found in the non-confining version, which was shown to be renormalizable [4]. Then, we proceed as in Ref. [4] and use the pion mass value to supply the necessary ultra-violet information to calculate the spectra and the universal correlation between the $^3S_1 -^1 S_0$ ground state mass splitting with the pseudoscalar mass [4].

This paper is organized as follows. In section II, we describe the extended Light-Cone model. In section III, we present the results for the mass splitting of the ground state of the pseudoscalar and vector mesons together with results for the meson spectra. In section IV, we conclude.

II. EXTENDED LIGHT-CONE QCD-INSPIRED THEORY

In this section we review a previous work [4], which has extended the renormalized effective QCD-theory of Ref. [4] to include confinement. In the effective theory the bare squared mass operator equation for the lowest Light-Front Fock-state component of a bound system of a constituent quark and antiquark of masses m_1 and m_2, is written as:

$$M^2\psi(x,\vec{k}_\perp) = \left[\frac{\vec{k}_\perp^2+m_1^2}{x}+\frac{\vec{k}_\perp^2+m_2^2}{1-x}\right]\psi(x,\vec{k}_\perp)$$
$$-\int\frac{dx'd\vec{k}'_\perp\theta(x')\theta(1-x')}{\sqrt{x(1-x)x'(1-x')}}\times$$
$$\left(\frac{4m_1m_2}{3\pi^2}\frac{\alpha(Q^2)}{Q^2}-\lambda\widehat{P}_0-W_{conf}(Q^2)\right)\psi(x',\vec{k}'_\perp)\ , \quad (1)$$

where M is the mass of the bound-state, $\widehat{P}_0$ is the projection onto the singlet spin state and ψ is the valence component of the light-front wave-function, i.e., the quark-antiquark Fock-state component. The confining interaction is included in the model by $W_{conf}(Q^2)$. The momentum transfer Q is the space-part of the four momentum transfer, the strength of the Coulomb-like potential is α and λ is the bare coupling constant of the singular spin interaction.

For convenience, the mass operator equation is written in the instant form representation using the transformation [3]:

$$x(k_z) = \frac{(E_1+k_z)}{E_1+E_2}\ , \quad (2)$$

and the Jacobian of $(x,\vec{k}_\perp)$ to $\vec{k}$ is:

$$dxd\vec{k}_\perp = \frac{x(1-x)}{m_rA(k)}d\vec{k}\ , \quad (3)$$

with the phase-space factor:

$$A(k) = \frac{1}{m_r}\frac{E_1E_2}{E_1+E_2}\ , \quad (4)$$

where the reduced mass is $m_r = m_1m_2/(m_1+m_2)$. The individual energies are $E_i = \sqrt{m_i^2+k^2}$ (i=1,2) and $k \equiv |\vec{k}|$.

The instant form representation of the squared mass operator eigenvalue equation is:

$$\left(M_0^2+V_{\rm conf}+V+V^\delta\widehat{P}_0\right)|\varphi\rangle = M^2|\varphi\rangle\ , \quad (5)$$

where the free mass operator, M_0 $(=E_1+E_2)$, is the sum of the free energies of quark 1 and 2, V is the Coulomb-like potential, V^δ is the short-range singular interaction and $V_{\rm conf}$ gives the quark confinement.

The matrix elements of the interaction operators V and V^δ appearing in Eq. (5) are given by [4]

$$\langle\vec{k}|V|\vec{k}'\rangle = -\frac{4m_s}{3\pi^2}\frac{\alpha}{\sqrt{A(k)}Q^2\sqrt{A(k')}}\ , \quad (6)$$

with $m_s = m_1+m_2$ and

$$\langle\vec{k}|V^\delta|\vec{k}'\rangle = \langle\vec{k}|\chi\rangle\frac{\lambda}{m_r}\langle\chi|\vec{k}'\rangle = \frac{\lambda}{m_r}\frac{1}{\sqrt{A(k)}}\frac{1}{\sqrt{A(k')}}; \quad (7)$$

where the phase-space factor $A(k)$ is defined by Eq.(4) and $Q^2 = |\vec{k}'-\vec{k}|$. We have introduced a form-factor of the separable singular interaction defined by $\langle\vec{k}|\chi\rangle = 1/\sqrt{A(k)}$.

We rewrite Eq. (5) in the form

$$\left(M_{\rm ho}^2+V+V^\delta\widehat{P}_0\right)|\varphi\rangle = M^2|\varphi\rangle\ , \quad (8)$$

where the confining harmonic oscillator interaction is included in the operator

$$M_{\rm ho}^2 = \left(\sqrt{\widehat{k}^2+m_1^2}+\sqrt{\widehat{k}^2+m_2^2}\right)^2+2m_sv(\widehat{r})-c_0\ , \quad (9)$$

where the hat denotes the operator character and

$$v(\widehat{r}) = \frac{1}{2}c_2\widehat{r}^2\ , \quad (10)$$

where c_0 and c_2 are two universal parameters valid for all of the mesons.

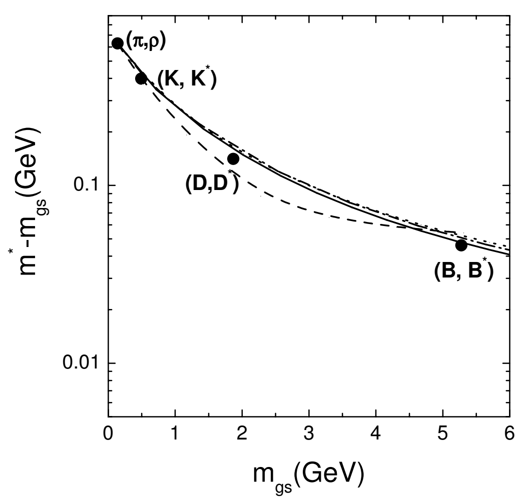

FIG. 1: $^3S_1-^1S_0$ ground state meson mass splitting as a function of the pseudoscalar ground state mass for the light-quark family. Model calculations: Coulomb-like plus singlet spin-spin interaction [4] (dashed line); singlet spin-spin interaction plus confinement (solid line); hyperfine interaction plus confinement (dotted line); singlet spin-spin interaction, Coulomb-like potential and confinement (dot-dashed line). Experimental values [8] (full circle).

The square mass operator given by Eq. (8) is diagonalized in the Hilbert space basis defined by the eigenvalue equation

$$\left(-4\nabla^2-c_0+m_sc_2r^2\right)\Psi_n(\vec{r}) = M_n^2\Psi_n(\vec{r}), \quad (11)$$

which take into account the ultraviolet behavior of the free mass operator and the confining interaction of the squared mass operator of our model. $\Psi_n(\vec{r})$ is the eigenstate of the harmonic oscillator potential with the corresponding eigenvalue

$$M_n^2 = 4\sqrt{m_sc_2}\left(2n+\frac{3}{2}\right)-c_0$$
$$= w(n+\frac{3}{4})-c_0, \quad (12)$$

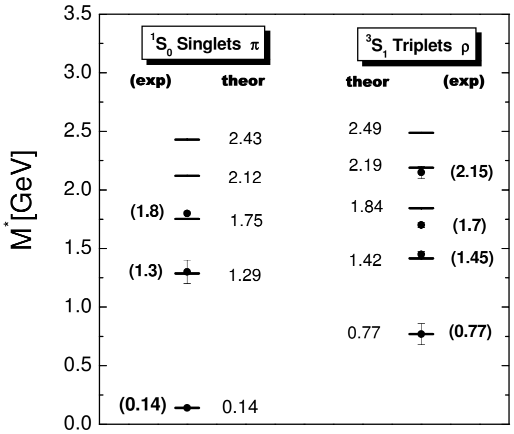

FIG. 2: S-wave light quark meson spectrum. Left: pseudoscalar mesons; right: vector mesons. Results for the model given by Eq. (8). The experimental values [8] are given between parenthesis. ($\rho(1700)$ might be *D*-wave dominant [7]).

where n (0, 1, 2, $\cdots$) is the radial quantum number and $w = 8\sqrt{m_s c_2}$. For $\alpha = 0$, when the Coulomb-like interaction is turned off, Eq. (12) gives the vector meson spectrum with w as the slope of the linear trajectory of excited states with the radial quantum number [6, 7].

III. RESULTS AND DISCUSSION

The parameters of the present model are w, the ground state mass of the $^3S_1$ meson, μ, the renormalized strength of the Dirac-Delta interaction and the running strength of the Coulomb-like interaction, α from Eq.(1), which depends of Q^2, is given by [8]:

$$\alpha(Q^2) = \frac{12\pi}{(33-2n_f)\log\left(\frac{Q^2}{\Lambda^2}\right)}, \tag{13}$$

where $\Lambda = 215$MeV is the scale parameter and $n_f = 5$ is the flavor number. We consider α constant for $Q/\Lambda < 100$ [9] and multiplied α by 0.86 to match the experimental values at high Q^2 [8].

The free parameters in the squared mass operator model of Eq. (8) are c_0, c_2 in the confining potential, the constituent quark masses and the strength of the singular singlet spin interaction. The corresponding values are determined from $w = 1.39$ GeV^2 [7], the masses of π and ρ fixing m_u =265 MeV [5]. (Of course one could change w and m_u.) The model is renormalizable and in our calculation we have adjusted numerically the strength of the singular spin interaction to reproduce the experimental pion mass, the strength is finite because we are using a finite basis ($n = 20$) to diagonalize the mass operator. In that sense, due to the finite truncation of the basis, the model is already regularized. Finally, the constituent quark masses are determined from the masses of the pseudoscalar mesons K, D and B: m_s=0.39 GeV, m_c =1.401 GeV and m_b=4.633 GeV with Coulomb-like interaction and m_s=0.381 GeV, m_c =1.367 GeV and m_b=4.589 GeV without Coulomb-like interaction.

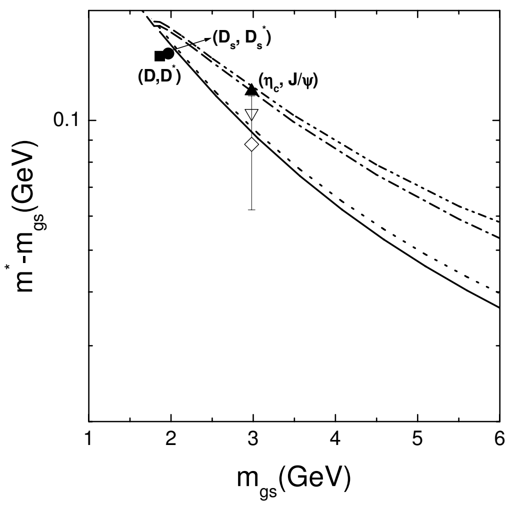

FIG. 3: $^3S_1 -^1 S_0$ ground state meson mass splitting as a function of the pseudoscalar ground state mass for the charm family. Model results are labelled as figure 1 and the dot-dot-dashed line corresponds to the hyperfine interaction, Coulomb-like potential and confinement calculation. Experimental values [8]:$D-D^*$ (full square), $D_s - D_s^*$ (full circle) and $\eta_c - J/\Psi$ (full triangle). Two evaluations of the mass splitting in heavy quark effective theory are given by the inverted triangle [11] and diamond [12].

In Figure 1, we present our results for the correlation between the $^3S_1 -^1 S_0$ ground state meson mass splitting and the pseudoscalar ground state mass for mesons with at least one light-quark. We compare the calculations with the light-front model of the squared mass operator with different interactions and the experimental data [8]. We show the results for the Coulomb-like potential with $\alpha = 0.5$ and spin-spin interaction in the singlet channel [4]. In this case absolute confinement is missing and the correlation curve saturates faster than the experimental data. In the harmonic confining model, we study the effect of the inclusion of the Coulomb-like, singlet-spin and hyperfine interactions (the hyperfine potential in Eq. (8) corresponds to replace $\widehat{P}_0$ by $\vec{S}_1 \cdot \vec{S}_2$). As seen in the figure, the results are not very sensitive to the different confining models with the same w, because all them have in common the pion and rho masses fitted to the experimental values, which brings the physics of the spin-spin interaction to the model, despite the different forms of the spin operator. Also the Coulomb-like interaction has little effect in these mesons once the pion and rho meson masses are fixed.

In figure 2, we present our results with Eq. (8) for the $\pi-\rho$ mass splitting for the first few levels. The large splitting in the ground state is diminished in the excited states, and the model results consistently smaller masses for the $^1S_0$ states compared to the respective $^3S_1$ ones. The $\rho(1700)$ is suggested to be a D-wave meson[7], and it does not fit well in our picture for $^3S_1$ meson resonance.

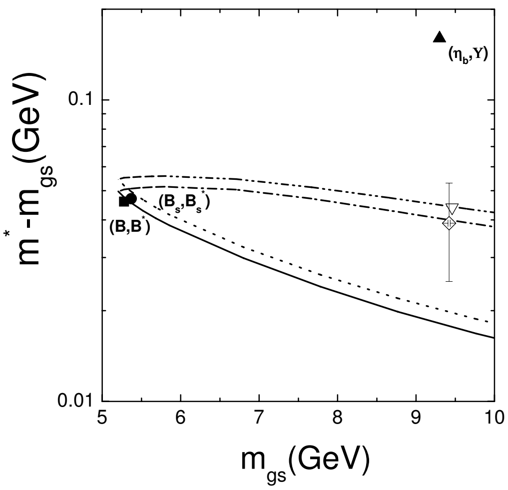

FIG. 4: ${}^3S_1 - {}^1S_0$ ground state meson mass splitting as a function of the pseudoscalar ground state mass for the bottom family. Model results are labelled as figure 3. Experimental values of $B-B^*$ (full square), $B_s - B_s^*$(full circle) and $\eta_b - \Upsilon$(full triangle) [8][a]. Two evaluations of the mass splitting in heavy quark effective theory are given by the inverted triangle [11] and diamond [12].

[a] $\eta_b - \Upsilon$ needs experimental confirmation.

In figures 3 and 4 we show results for the correlation between the ${}^3S_1 - {}^1S_0$ ground state meson mass splitting and the pseudoscalar ground state mass for mesons with at least one heavy quark c or b, respectively. We observe that the Coulomb-like interaction has more important effects when the quark mass increases, which is natural. Our simple model is quite consistent with the recent results from heavy quark effective theory [11, 12].

IV. CONCLUSION

The extended light-cone QCD-inspired effective theory including harmonic confinement, Coulomb-like and a singular spin interaction in the mass squared operator is applied to study the splitting between S-wave pseudoscalar and vector mesons as well the $\pi-\rho$ spectra. The renormalization of the model is possible and has been done numerically by adjusting the strength of the singular spin interaction to reproduce experimental the pion mass. The model reproduces the phenomenological correlation between the ground state ${}^3S_1 - {}^1S_0$ mass splitting as a function pseudoscalar mass [5, 8], for mesons with light and heavy quarks. The mass splitting between the charmonium spin states is quite consistent with the experimental data [8]. For the botonium case our calculation is consistent with a recent estimate within heavy quark effective theory [12]. The model has the large splitting of the ground state of the light ${}^1S_0$ and ${}^3S_1$ meson, its suppression for the excited states and for heavy quark systems, qualitatively consistent with the experimental data. The model provides a phenomenological framework to parameterize the systematics of the $q\bar{q}$-states in the (n, M^2) plane and as well in the (J, M^2) plane and it naturally interpolates meson properties from light to heavy quark systems.

Acknowledgments: We thank FAPESP and CNPq for financial support.

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