The Importance of Strange Mesons in Neutron Star Properties

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In order to obtain the properties of compact stellar objects, appropriate equations of state have to be used. In the literature, strange meson fields, namely the scalar meson field $\sigma^*(975)$ and the vector meson field $\phi(1020)$, had to be considered in order to reproduce the observed strongly attractive $\Lambda\Lambda$ interaction. The introduction of these strange mesons makes the equations of state harder (EOS) due to the repulsive effect of the $\phi(1020)$ meson. In this work the inclusion of these mesons in the equation of state and their influence on the properties of the neutron stars are investigated.

I. INTRODUCTION

In the present work we use the relativistic non-linear Walecka model (NLWM) [1], at zero temperature ($T=0$), with the lowest baryon octet $\{N,\Lambda,\Sigma,\Xi\}$ in β equilibrium with the lightest leptons $\{e^-,\mu^-\}$ and compare the results with the same model plus strange meson fields, $\sigma^*(975)$ and $\phi(1020)$, which introduce strangeness to the interaction according to [2] and [3]. Strange meson fields, namely the scalar meson field $\sigma^*(975)$ and the vector meson field $\phi(1020)$, had to be considered in order to reproduce the observed strongly attractive $\Lambda\Lambda$ interaction. This formalism applied to compact objects like neutron stars, where the energies are such that allow the appearance of the eight lightest baryons. The motivation for this study lies in our interest to describe the interaction between hadrons taking into account a growing number of effects in order to better describe it. Given the difficulty of making comparisons with experimental data we will, for now limit ourselves to verify if the inclusion of these mesons significantly alters some quantities like pressure and energy density in the equation of state and the bulk properties of compact stars.

II. THE FORMALISM

The lagrangian density of the NLWM with the inclusion of the strange meson sector and leptons for β equilibrium is:

$$\mathcal{L}=\sum_{B=1}^{8}\bar{\Psi}_B\left[\gamma_\mu\left(i\partial^\mu-g_{\omega B}V^\mu-g_{\rho B}\vec{\tau}.\vec{b}^\mu\right)-(M_B-g_{\sigma B}\sigma)\right]\Psi_B$$

$$+\frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma-m_\sigma^2\sigma^2\right)-\frac{1}{3!}k\sigma^3-\frac{1}{4!}\lambda\sigma^4-\frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu}+\frac{1}{2}m_\omega^2V_\mu V^\mu$$

$$-\frac{1}{2}\vec{B}_{\mu\nu}\vec{B}^{\mu\nu}+\frac{1}{2}m_\rho^2\vec{b}_\mu\vec{b}^\mu$$

$$+\frac{1}{2}\left(\partial_\mu\sigma^*\partial^\mu\sigma^*-m_{\sigma^*}^2\sigma^{*2}\right)+\frac{1}{2}m_\phi^2\phi_\mu\phi^\mu-\frac{1}{4}S_{\mu\nu}S^{\mu\nu}-\sum_B g_{\sigma^* B}\bar{\Psi}_B\Psi_B\sigma^*-\sum_B g_{\phi B}\bar{\Psi}_B\gamma_\mu\Psi_B\phi^\mu$$

$$+\sum_{l=1}^{2}\bar{\Psi}_l\left(i\gamma_\mu\partial^\mu-M_l\right)\Psi_l, \tag{1}$$

where $\Omega_{\mu\nu}=\partial_\mu V_\nu-\partial_\nu V_\mu$, $\vec{B}_{\mu\nu}=\partial_\mu\vec{b}_\nu-\partial_\nu\vec{b}_\mu-g_\rho\left(\vec{b}_\mu\times\vec{b}_\nu\right)$ and $S_{\mu\nu}=\partial_\mu\phi_\nu-\partial_\nu\phi_\mu$, with B extending over the eight baryons, g_{iB} are the coupling constants of mesons i, $i=\sigma,\omega,\rho$ with baryon B, and m_i is the mass of meson i. λ and k are the weighs of the non-linear scalar terms and $\vec{\tau}$ is the isospin operator. At this point it is worth emphasizing that the strange mesons are not supposed to act at low densities, where the strangeness content is zero. Moreover, the non-linear terms are normally corrections added to the main linear contributions and hence the non-linear terms in the strange sector are disregarded in the present work. The constants

g_{iB} are defined by $g_{iB} = x_B g_i$ where $x_B = \sqrt{2/3}$, for hyperons, [4], $x_B = 1$ for the nucleons, and also $g_\sigma = 8.910$, $g_\omega = 10.626$, $g_\rho = 8.208$, $g_{\sigma^*\Lambda} = g_{\sigma^*\Sigma} = 5.11$, $g_{\sigma^*\Xi} = 9.38$, $g_{\phi\Lambda} = g_{\phi\Sigma} = 4.31, g_{\phi\Xi} = 8.62$, $k = -6.426\ 10^{-4}$, $\lambda = 5.530$ according to [5] and [6]. The strange mesons interact with hyperons only ($g_{\sigma^*\,p} = g_{\sigma^*\,n} = g_{\phi\,p} = g_{\phi\,n} = 0$). The masses of baryons of the octect are: $M_N = 938\,MeV$ (nucleons), $M_\Lambda = 1116\,MeV$, $M_\Sigma = 1193\,MeV$, $M_\Xi = 1318\,MeV$ and the meson masses are: $m_\sigma = 512\,MeV$, $m_\omega = 738\,MeV$, $m_\rho = 770\,MeV$, $m_{\sigma^*} = 975\,MeV$, $m_\phi = 1020\,MeV$. In order to account for the β equilibrium in the star the leptons are also included in the lagrangian density of eq. (1) as a non-interacting Fermi gas. The masses of the leptons are $M_{e^-} = 0.511\,MeV$ and $M_{\mu^-} = 105.66\,MeV$.

Applying the Euler-Lagrange equations to (1) and using the mean-field approximation ($\sigma \to \langle\sigma\rangle = \sigma_0$, $V_\mu \to \langle V_\mu\rangle = \delta_{\mu\,0} V_0$ and $\vec{b}_\mu \to \langle\vec{b}_\mu\rangle = \delta_{\mu 0}\, b^0_\mu \equiv \delta_{\mu 0}\, b_\mu$), we obtain:

$$\sigma_0 = -\frac{k}{2m_\sigma^2}\sigma_0^2 - \frac{\lambda}{6m_\sigma^2}\sigma_0^3 + \sum_B \frac{g_\sigma}{m_\sigma^2} x_B \rho_{\sigma B}\ , \tag{2}$$

$$V_0 = \sum_B \frac{g_\omega}{m_\omega^2} x_B \rho_B\ , \tag{3}$$

$$b_0 = \sum_B \frac{g_\rho}{m_\rho^2} x_B \tau_3 \rho_B\ , \tag{4}$$

$$\sigma_0^* = \sum_B \frac{g_{\sigma^*}}{m_{\sigma^*}^2} x_B \rho_B\ , \tag{5}$$

$$\phi_0 = \sum_B \frac{g_\phi}{m_\phi^2} x_B \rho_B\ , \tag{6}$$

where

$$\rho_{\sigma B} = \frac{M_B^*}{\pi^2}\int_0^{K_{F_B}} \frac{p^2 dp}{\sqrt{p^2 + M_B^*}}\ , \tag{7}$$

$$\rho_B = \frac{1}{3\pi^2} K_{F_B}^3 \quad , \quad M_B^* = M_B - g_{\sigma B}\,\sigma_0 - g_{\sigma^* B}\,\sigma_0^*\ , \tag{8}$$

and the 0 subscripts added to the fields mean that a mean field approximation, where the meson fields were considered as classical fields was performed.

Through the energy-momentum tensor, we obtain:

$$\varepsilon_a = \frac{1}{\pi^2}\left(\sum_{i=B,l}\int_0^{K_{F_i}} p^2 dp \sqrt{p^2 + M_i^{*2}}\right) + \frac{m_\omega^2}{2}V_0^2 + \frac{m_\rho^2}{2}b_0^2 + \frac{m_\sigma^2}{2}\sigma_0^2 + \frac{k}{6}\sigma_0^3 + \frac{\lambda}{24}\sigma_0^4 + \frac{m_{\sigma^*}^2}{2}\sigma_0^{*2} + \frac{m_\phi^2}{2}\phi_0^2, \tag{9}$$

$$P_a = \frac{1}{3\pi^2}\left(\sum_{i=B,l}\int_0^{K_{F_i}} \frac{p^4 dp}{\sqrt{p^2 + M_i^{*2}}}\right) + \frac{m_\omega^2}{2}V_0^2 + \frac{m_\rho^2}{2}b_0^2 - \frac{m_\sigma^2}{2}\sigma_0^2 - \frac{k}{6}\sigma_0^3 - \frac{\lambda}{24}\sigma_0^4 - \frac{m_{\sigma^*}^2}{2}\sigma_0^{*2} + \frac{m_\phi^2}{2}\phi_0^2\ . \tag{10}$$

In a neutron star, charge neutrality and baryon number must be conserved quantities. Moreover, the conditions of chemical equilibrium hold. In terms of the chemical potentials of the constituent particles, these conditions read:

$$\begin{aligned} &\mu_n = \mu_p + \mu_{e^-}\ ,\ \mu_{\mu^-} = \mu_{e^-}\ ,\\ &\mu_{\Sigma^0} = \mu_{\Xi^0} = \mu_\Lambda = \mu_n\ ,\\ &\mu_{\Sigma^-} = \mu_{\Xi^-} = \mu_n + \mu_{e^-}\ ,\\ &\mu_{\Sigma^+} = \mu_p = \mu_n - \mu_{e^-}\ . \end{aligned} \tag{11}$$

III. RESULTS

The inclusion of hyperons and strange mesons alters the equations of state and the particle fractions, as can be seen from Figs 1 and 2.

From Fig. 1 we notice that the inclusion of the hyperons softens the equations of state in comparison with the EOS obtained only with nucleons and leptons. The inclusion of the strange mesons hardens these equations a little at higher energy densities. This indicates that the influence of the strange mesons is significant at higher densities, what can be easily seen in Fig. 2, where we notice a difference in the fractions of heavier hyperons, at densities above $5\rho_0$, where ρ_0 is the saturation density of the nuclear matter.

Neutron star profiles can be obtained by solving the Tolman-Oppenheimer-Volkoff (TOV) equations [7], resulting from the exact solution of Einstein's general relativity equations in the Schwarzschild metric for spherically symmetric, static stars. Applying the equation of states (9) and (10) in TOV equations results in the star properties shown in table I and Fig. 3. In table I the profiles of the stars with the ma-

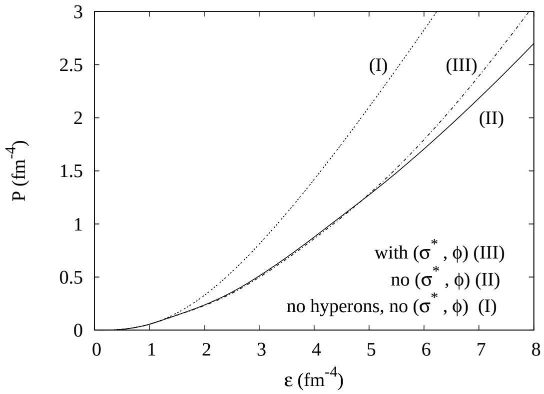

FIG. 1: Pressure versus energy density.

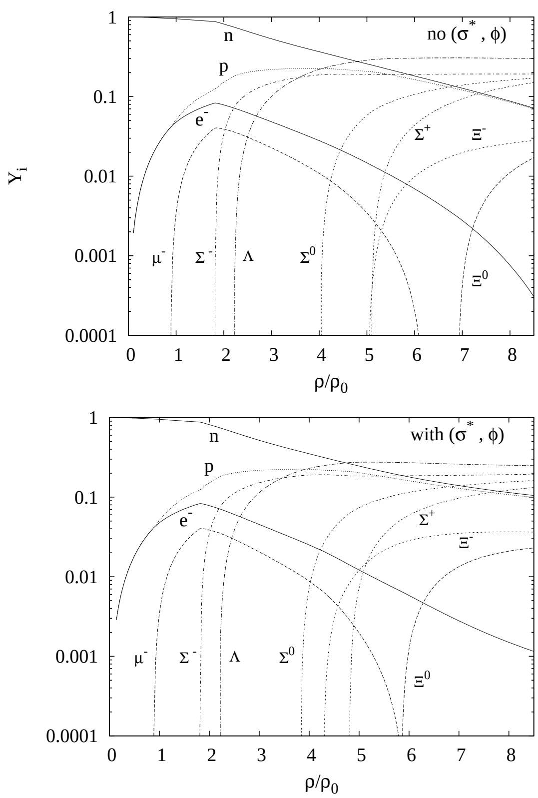

FIG. 2: Particle fractions $Y_i = \rho_i/\rho$, i = B,l, as a function of the total baryon density ρ in units of ρ_0.

ximum gravitational mass and with the maximum radius are shown for two possible EOS: without the strange mesons and with them. In these cases the crust of the stars were not included.

The observed values for the mass of the neutron stars lie between 1.2 to 1.8 M_{sun}. Our results are in the expected range. From table I and Fig. 3, one can see that the differences in the star properties with and without strange meson are not very relevant. Nevertheless, the constitution of the stars at large densities are somewhat different. At about four times the saturation density (see Fig. 2) the inclusion of the strange mesons start playing its role in the constitution of the stars. At this high energy a phase transition to a deconfined phase of quarks or to a system with kaon condensates can already take place. These two possibilities are certainly more important to the properties of neutron stars than a system containing strange mesons. The influence of the inclusion of the strange mesons in protoneutron stars with temperatures around 30 to 40 *MeV* and the their importance when trapped neutrinos are included are under investigation.

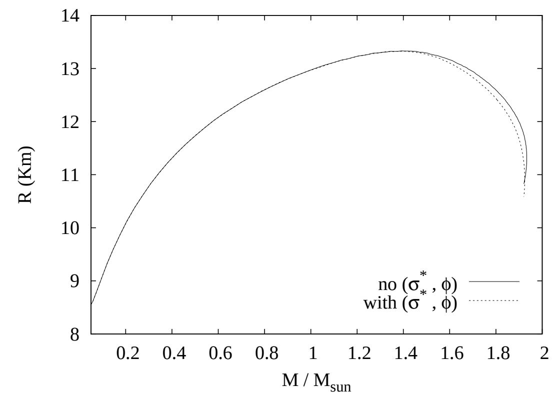

FIG. 3: Radius versus gravitational mass for the families of stars obtained within the NLWM and within the NLWM plus the strange mesons sector after the TOV equations were solved.

	No (σ^*, ϕ)	With (σ^*, ϕ)
M_{max}/M_{sun}	1.93	1.92
R (Km)	11.30	10.91
ε_0 (fm^{-4})	6.39	7.01
R_{max} (Km)	13.33	13.33
M/M_{sun}	1.39	1.39
ε_0 (fm^{-4})	1.75	1.78

TABLE I: Neutron star properties. ε_0 is the central energy density.

ACKNOWLEDGMENTS

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